Theory of Constraints: The Myths and Clarification

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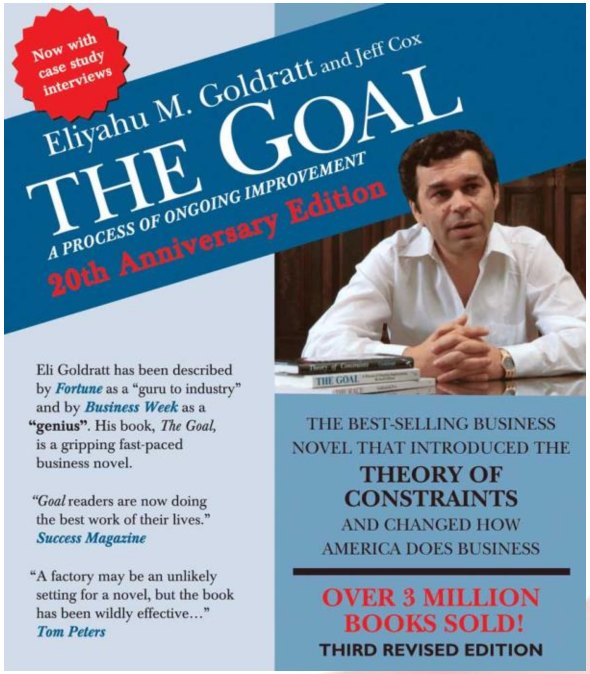
Theory of Constraints

- History
  - An overall management philosophy introduced by Eliyahu M. Goldratt in *The Goal* (1984).

- Key assumption
  - Organizations can be measured by the following three measures:
    - *Throughput*
    - *Operational expense*
    - *Inventory*



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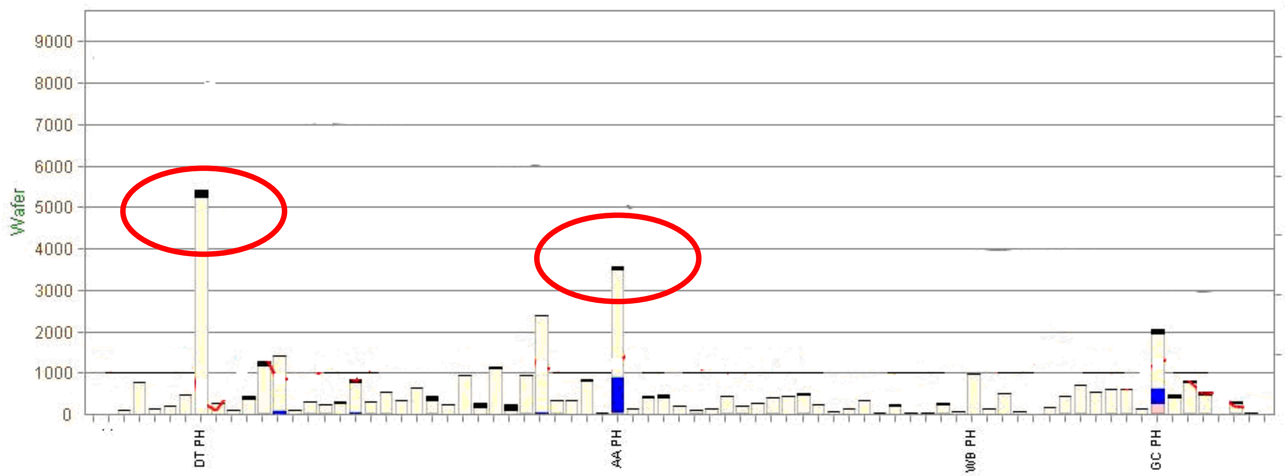
Theory of Constraints (Cont.)

- Constraint
  - Anything that prevents the system from achieving its goal.
- Bottleneck in a production line
  - Any resource whose capacity is equal to or less than the demand placed upon it.



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Process of Ongoing Improvement

- Five focusing steps (POOGI)
  - Identify the system’s constraint(s).
  - Decide how to exploit the system’s constraint(s).
  - Subordinate everything else to the above decision.
  - Elevate the system’s constraint(s).
  - Warning! If in the previous steps a constraint has been broken, go back to step 1, but do not allow inertia to cause a system’s constraint.

The Myth

1. *Should we elevate the system's constraints continuously until there is no more constraint?*
   – Scanner vs. Cleaner

2. *If we want to improve the productivity of a production line, is the bottleneck always the best place to start with?*
   – Scanner vs. Furnace

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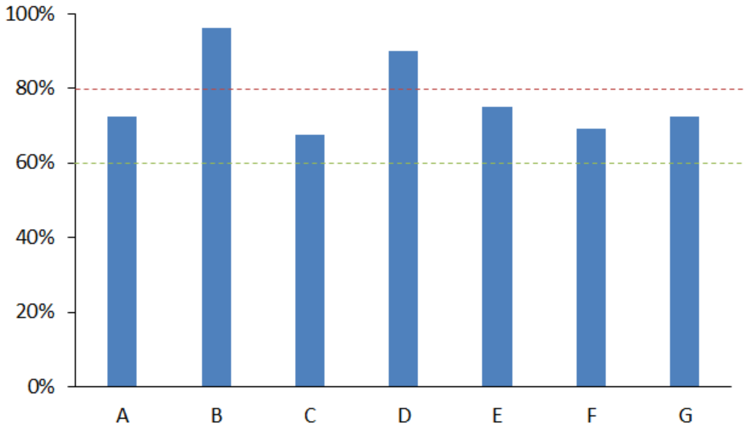
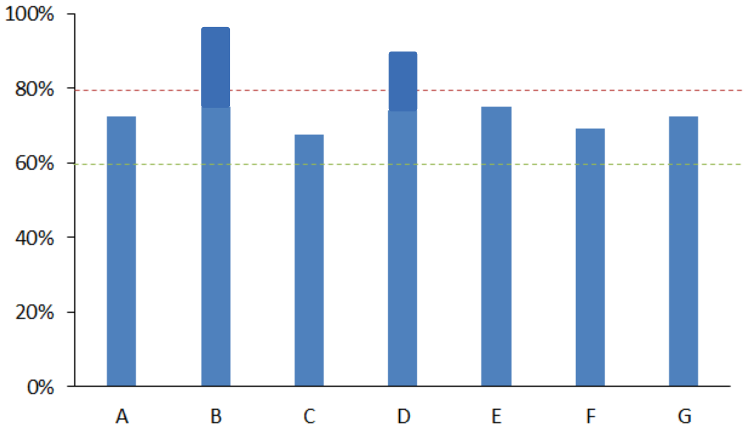
- Question 1:
  - *Should we elevate the system's constraints continuously until there is no more constraint?*





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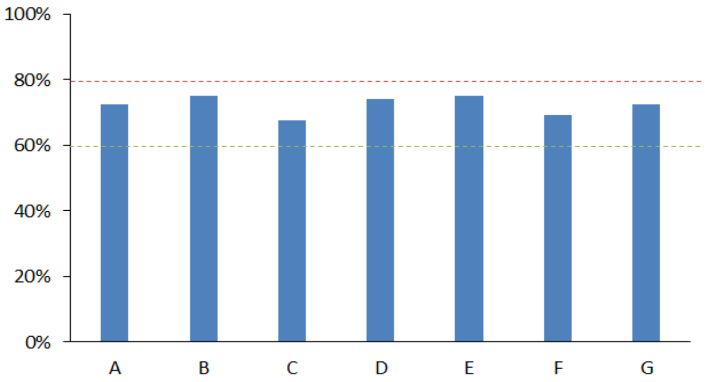
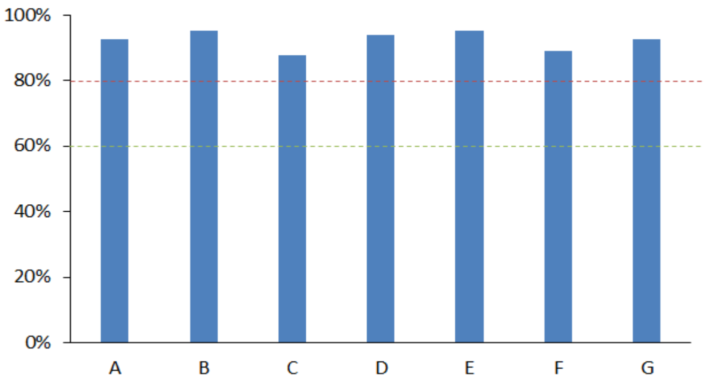
Process of Ongoing Improvement

- If a production line has no specific bottleneck, all stations have similar utilizations.



- During normal situation,

- During hot seasons with increased demand…



All machines become the bottleneck in spite of their costs!

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What is a Bottleneck?

- Definition in the TOC
  - Any resource whose capacity is equal to or less than the demand placed upon it (*The Goal*, 1984).
- However
  - Demand can be greater than capacity in a transient period
    - And induce longer queue time/length
  - Demand must be equal or less than capacity in the long run
    - Otherwise, queue time (or cycle time) diverges to infinite

Hence, *the true constraint of a production line is the total cycle time rather than throughput*.

$$BN = \text{Arg}\max_{i} \left| \frac{\Delta CT_i}{\Delta \mu_i} \right|$$

Performance of a Queueing Network

- Dependence among stations
  - Positive: reduce system queue time
  - Negative: increase system queue time
- Queue time of a production line $\sum_{i=1}^{N} QT_i = \sum_{i=1}^{N} f_{N,i} * QT_i^A$
  - Computation of the Contribution Factor

1. Let $k = N$, $f_{N,i} = 1$ for $i = 1$ to N.
2. If server k is marked as a bottleneck, $f_{N,i} = r_k * f_{N,i}$ for $i = 1$ to $k - 1$.

   Otherwise, $f_{N,k} = r_k * f_{N,k}$. Stop if $k = 2$.
3. Let $k = k - 1$, and go to step 2.

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What is Variability?

- For a random variable *X*,
  - Coefficient of variation (CV) $= c_X = \sigma / E[X]$
  - Variability of *X* is the squared coefficient of variations:

$$c_X^2 = \frac{\sigma_X^2}{E(X)^2}$$

- Variability of a station
  - GI/G/1 queue (Kingman's approximation)

$$QT \le \frac{\sigma_a^2\mu^2 + c_s^2}{2}\frac{\rho}{1-\rho}t \cong \frac{c_a^2 + c_s^2}{2}\frac{\rho}{1-\rho}t = \frac{c_a^2 + c_s^2}{2}QT_{M/M/1},$$

  - VUT in Factory Physics

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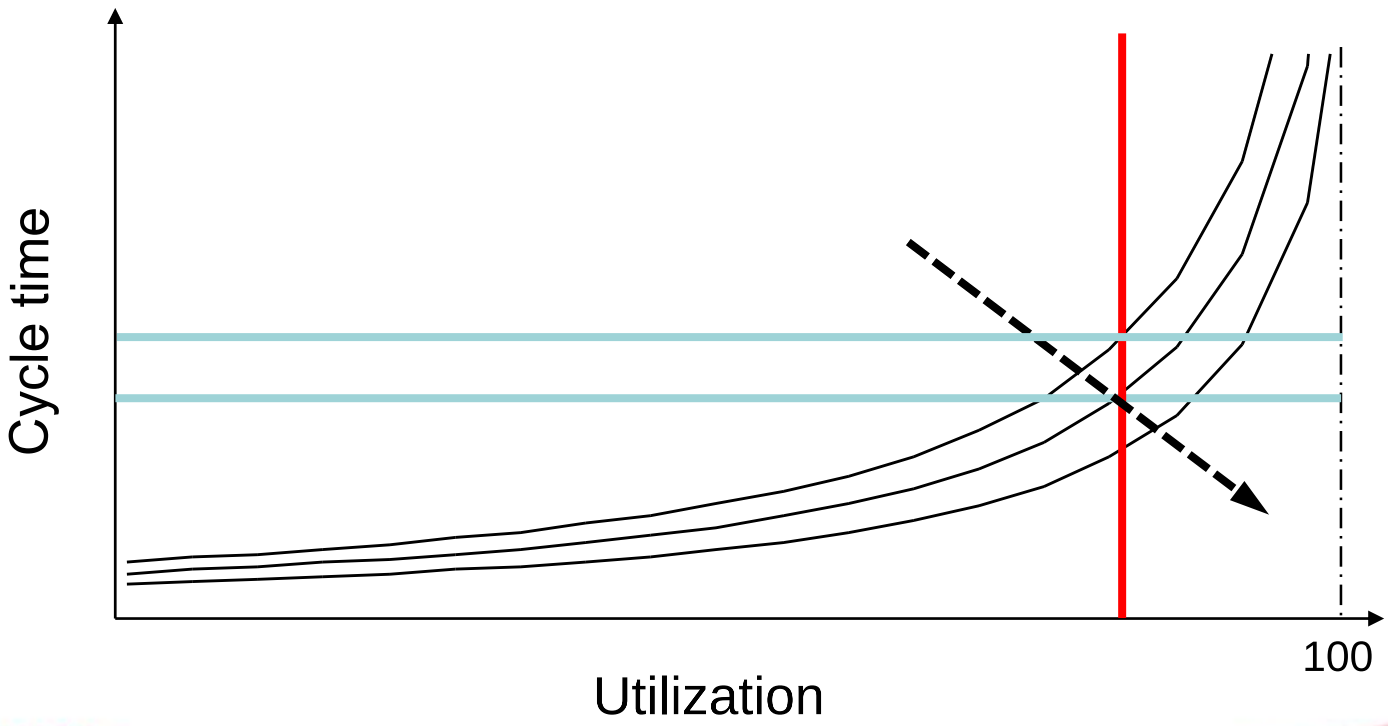
Performance of a Single Station

- GI/G/1 queue: $QT \cong \frac{c_a^2 + c_s^2}{2} QT_{M/M/1}$,
  - Lower variability leads to shorter mean queue/cycle time.



Variability of Production Lines

- GI/G/1 queue: $QT \cong \frac{c_a^2 + c_s^2}{2} QT_{M/M/1}$,

- Let $\alpha_B = (c_a^2 + c_{S_B}^2)/2$, then $\alpha_f = \frac{QT_f}{QT_{M/M_B/1}} = \alpha_B + \frac{1-\rho_B}{\rho_B}\mu_B \sum_{i \neq B} f_i \, \alpha_i \frac{\rho_i}{1-\rho_i}\frac{1}{\mu_i}$

$$= \alpha_B + \left(\frac{\mu_B^2}{\lambda} - \mu_B\right)\sum_{i \neq B} f_i \, \alpha_i \frac{\lambda}{\mu_i - \lambda}\frac{1}{\mu_i},$$

- We use the throughput bottleneck to represent the production line
  - Reduction method (Friedman, 1965)
  - Heavy-traffic bottleneck phenomenon (Iglehart and Whitt, 1970)

Taking derivatives, we can examine its property…

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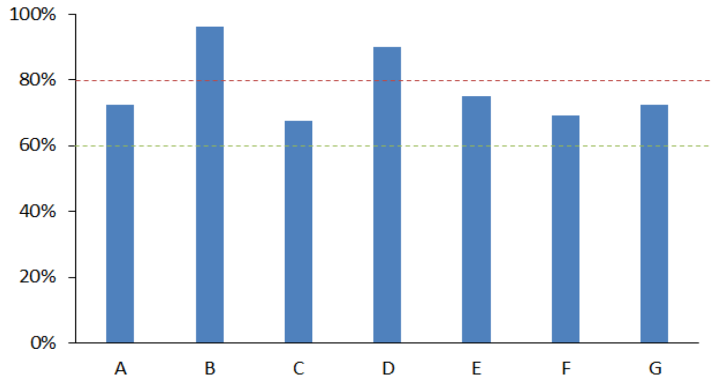
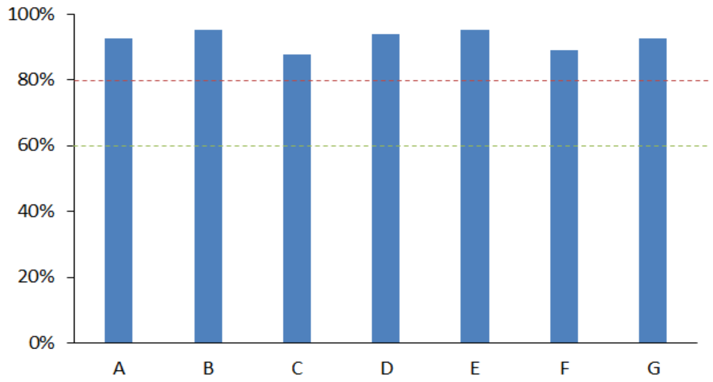
Variability of Production Lines

- Property 1 ~ 3
  - Lower variability for higher arrival rate
  - Lower variability for higher service rates of the non-bottlenecks
  - Higher variability for higher service rate of the bottleneck
- Property 4
  - For two identical production lines…, the one with a larger service rate gap between the bottleneck and non-bottlenecks has the lower variability.





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Observations

- A larger service rate gap has the lower variability
- Lower variability leads to shorter queue/cycle time.
  - The price is to have excess capacity at the non-bottleneck workstations.
    - Buy more scrubbers or cleaners but not scanners…

Hence, *the bottleneck should be planned at the most expansive equipment in terms of capacity*.

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- Question 2:
  - *If we want to improve the productivity of a production line, is the bottleneck always the best place to start with?*

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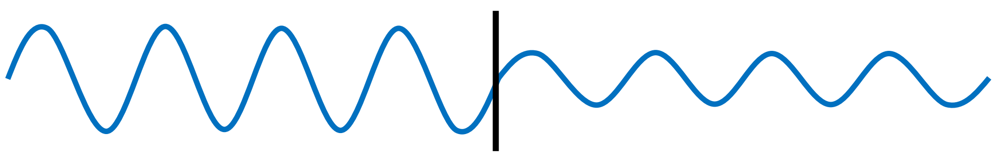
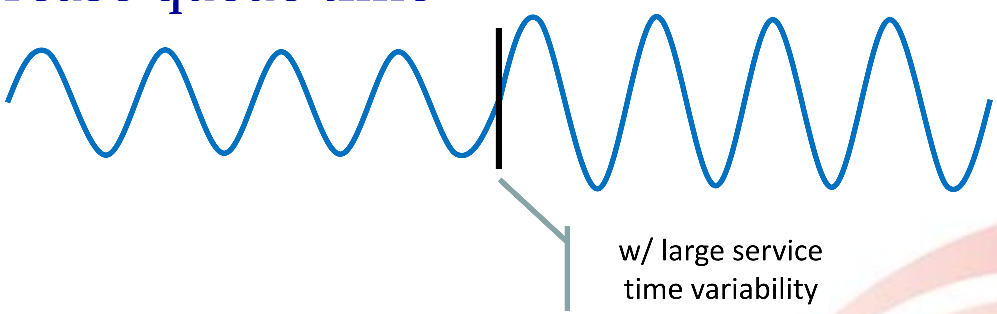
Dependence among Stations

- Positive: reduce queue time

$$c_d^2 \cong \rho^2 c_s^2 + (1-\rho^2) c_a^2$$

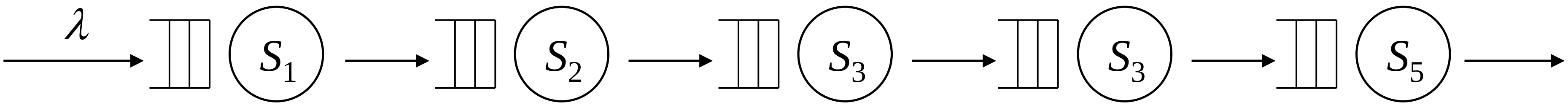


- Negative: increase queue time



Second Moment Result on TOC

- Service times: 28 – 26 – 24 – 22 – 30 with SCVs 0.8
- Poisson arrivals
- If the SCVs can be reduced by half, which one should be the 1st?

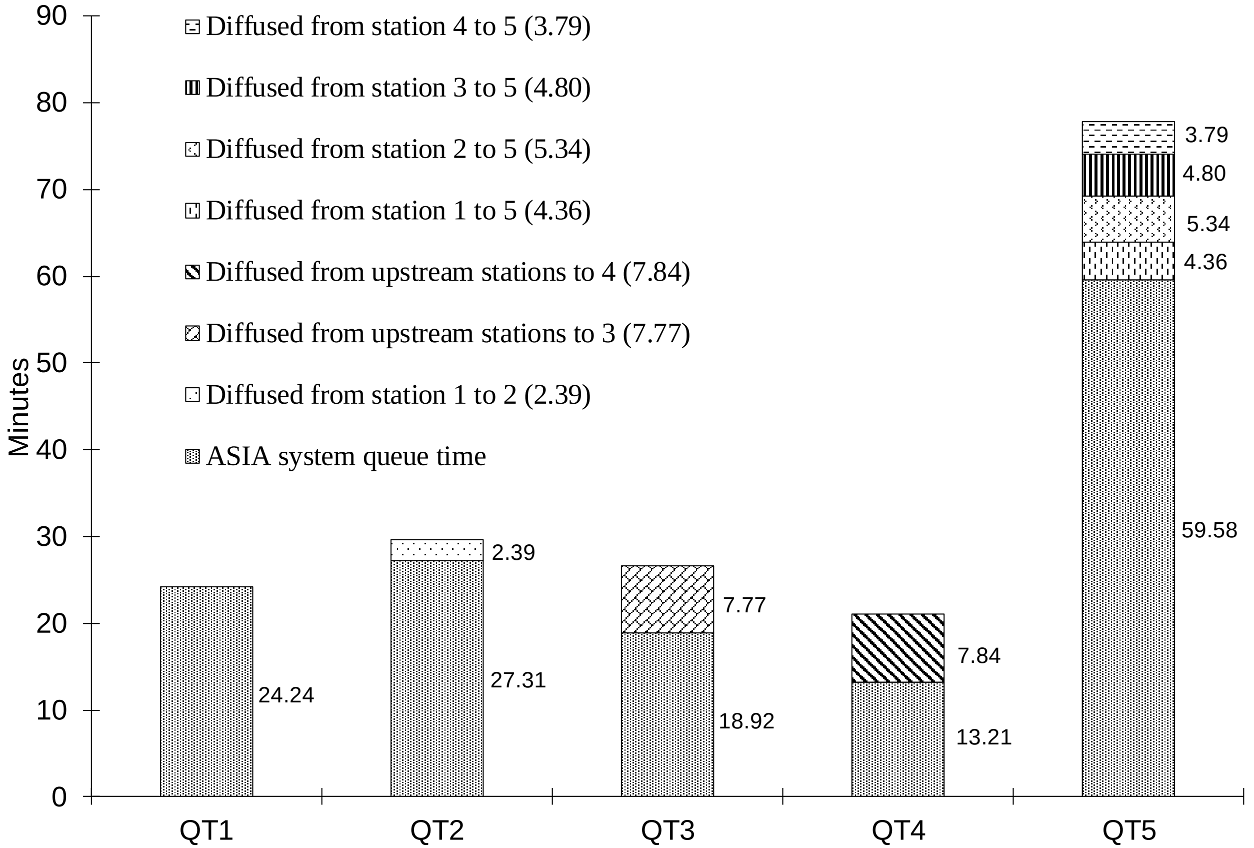


ρ	$QT_f(0,0)$	$QT_f(1,0.5)$	$QT_f(2,0.5)$	$QT_f(3,0.5)$	$QT_f(4,0.5)$	$QT_f(5,0.5)$
0.1	1.63	1.33	1.38	1.43	1.48	1.42
0.2	7.90	6.62	6.88	7.07	7.24	7.05
0.3	18.36	15.57	16.17	16.64	17.04	16.59
0.4	33.23	28.48	29.54	30.35	30.97	30.20
0.5	54.12	46.39	48.27	49.52	50.58	49.15
0.6	84.08	72.13	75.15	77.27	78.79	76.28
0.7	129.37	111.49	116.70	119.78	122.08	116.80
0.8	208.70	179.53	188.95	194.80	197.70	186.96
0.9	388.79	336.96	357.86	368.96	374.53	340.62
0.95	652.69	573.47	610.73	625.14	635.92	548.87

$QT_f(i,p)$: Service time SCV of station *i* is reduced by *p* (in percentage)

Second Moment Result on TOC

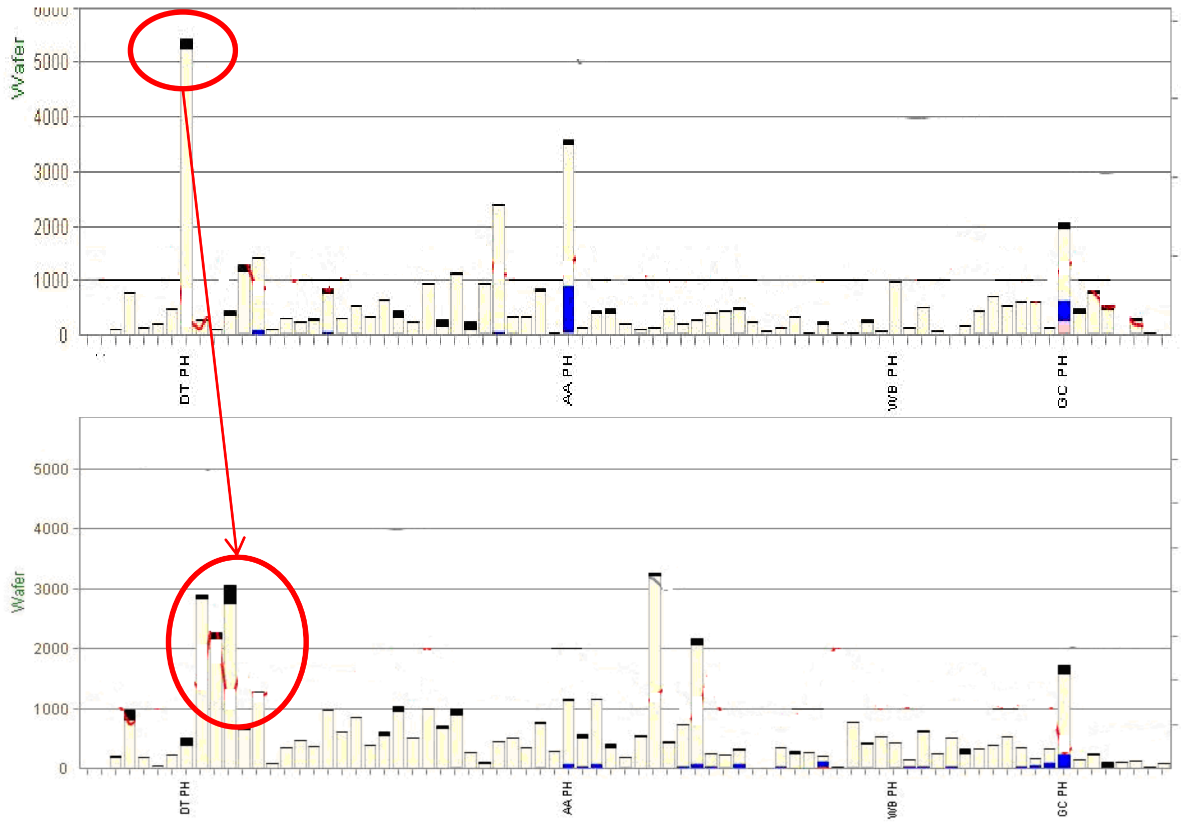
- Service times: 28 – 26 – 24 – 22 – 30 with SCVs 0.8 for all



Queue time analysis for the tandem queue with SCV(1) and $\rho = 0.8$

Observations

- Variability can be transferred to a downstream station.



Cycle time reduction should start from upstream stations or the bottleneck.

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Summary

- Process of ongoing improvement
  - When bottleneck capacity is increased, the non-bottleneck should be increased accordingly.
  - If a system has no bottleneck, every station is a bottleneck.
  - Bottleneck should be designed rather than randomly assigned.
- The 2nd moment result of the Theory of constraints
  - Total cycle time is the true limit.
    - Improving the performance of the system bottleneck may not be the most effective place to reduce system cycle time.
  - Cycle time reduction should start from upstream stations or the throughput bottleneck.

New Process of Ongoing Improvement

- Focusing steps
  1. Determine system's throughput bottleneck based on machine cost and capacity.
  2. Decide the priority of improvement based on the cycle time reduction and cost.

     $$k = Arg_i \min_i \{c_i / \Delta CT_i\}$$

     - The improvement can be reducing the mean or variations of service time.
     - At an upstream station, the throughput bottleneck or a non-bottleneck.
  3. Reduce queue time (or even cycle time) continuously until it becomes zero.

The Toyota production wrings water out of towels that are already dry.
There is nothing more important than planting "trees of will".

– Shingo 1990

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~ If a system has no bottleneck, every station is a bottleneck. ~

Thank you

1. Wu, K., and N. Zhao. (2015). Dependency among single server queues in series and Its applications in productivity improvement. *European Journal of Operational Research*, 247(1), 245-258.
2. Wu, K., Y. Zhou and N. Zhao. (2016). Variability and the Fundamental Properties of a Production Line, *Computer & Industrial Engineering*, 99, 364-371.

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