A Bernoulli model for the single-vendor single-buyer supply chain

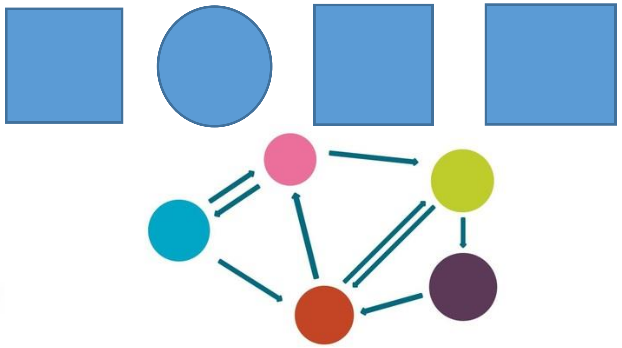
Elisa Gebennini, Andrea Grassi, Bianca Rimini

UNIMORE
UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

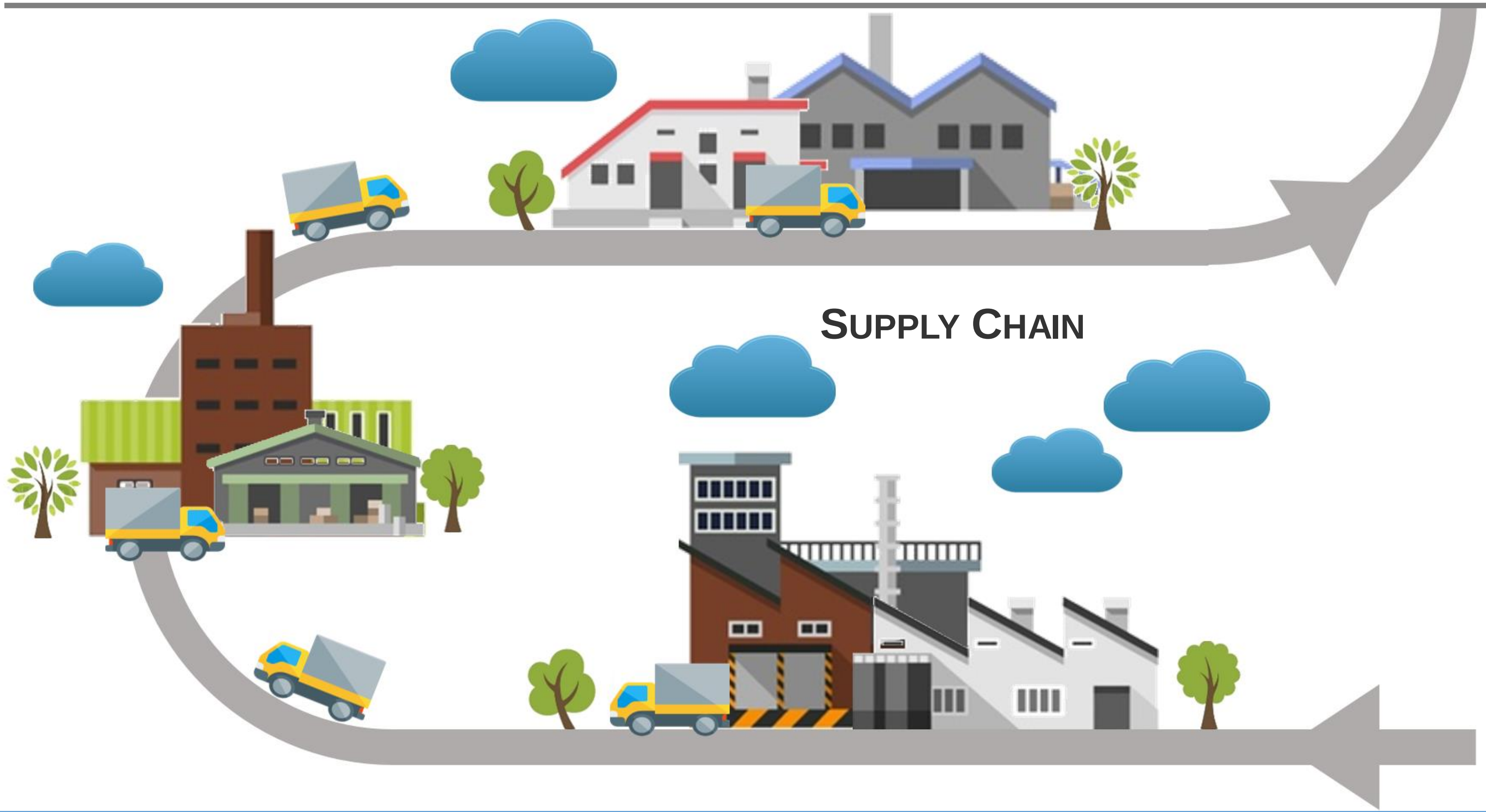
STOCHASTIC MODELS OF MANUFACTURING AND SERVICE OPERATIONS
SMMSO 2017, June 4-9, 2017 - Acaya (Lecce), Italy

Introduction

Stochastic model of a production line

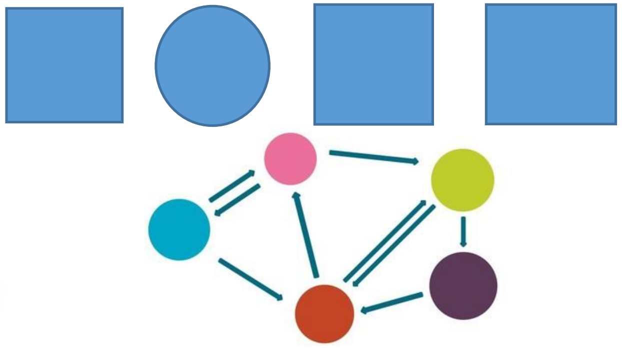


Introduction

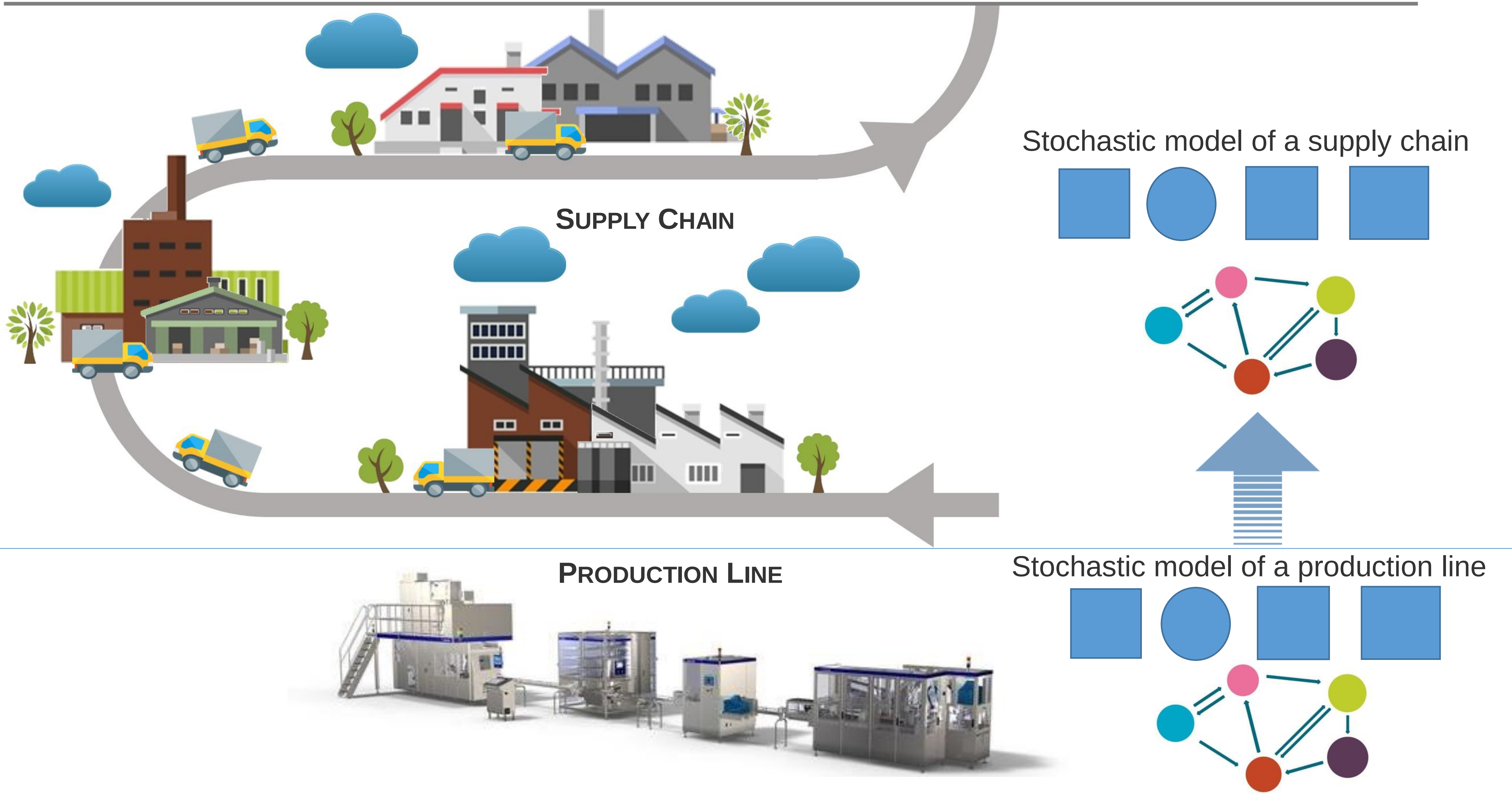


PRODUCTION LINE

Stochastic model of a production line



Introduction

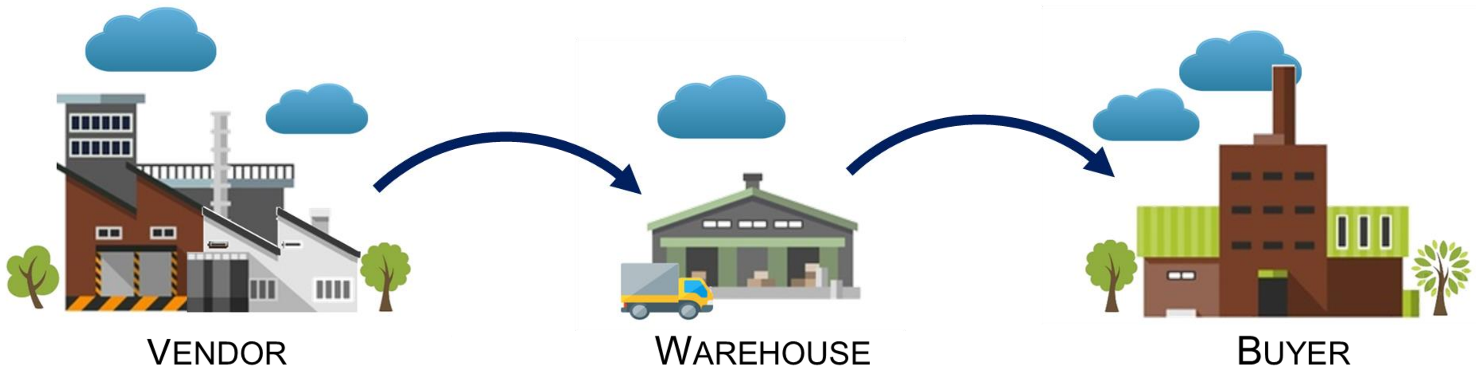


[s, S] Inventory Control Policy

stochastic modeling of a supply chain

- analyse the actual behavior of a certain supply chain (performance measures)
- support strategic decisions for the **whole supply chain**

Single-Vendor Single-Buyer Supply Chain



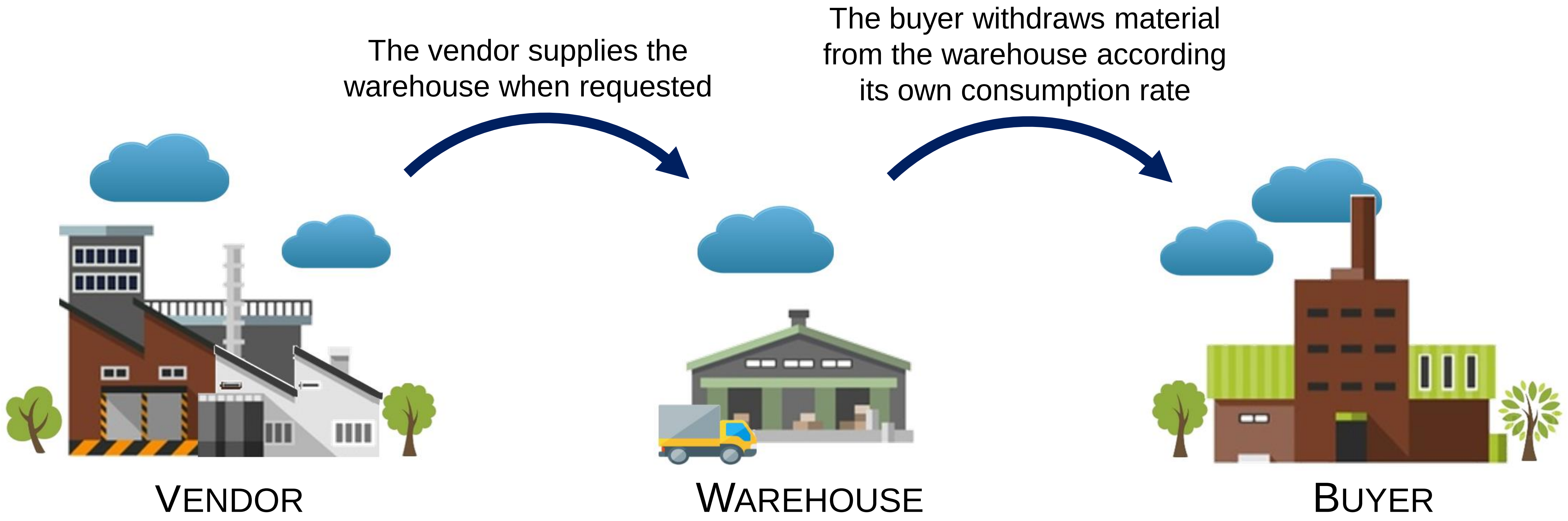
Single-Vendor Single-Buyer System

Vendor

Warehouse

Buyer

Single-Vendor Single-Buyer System



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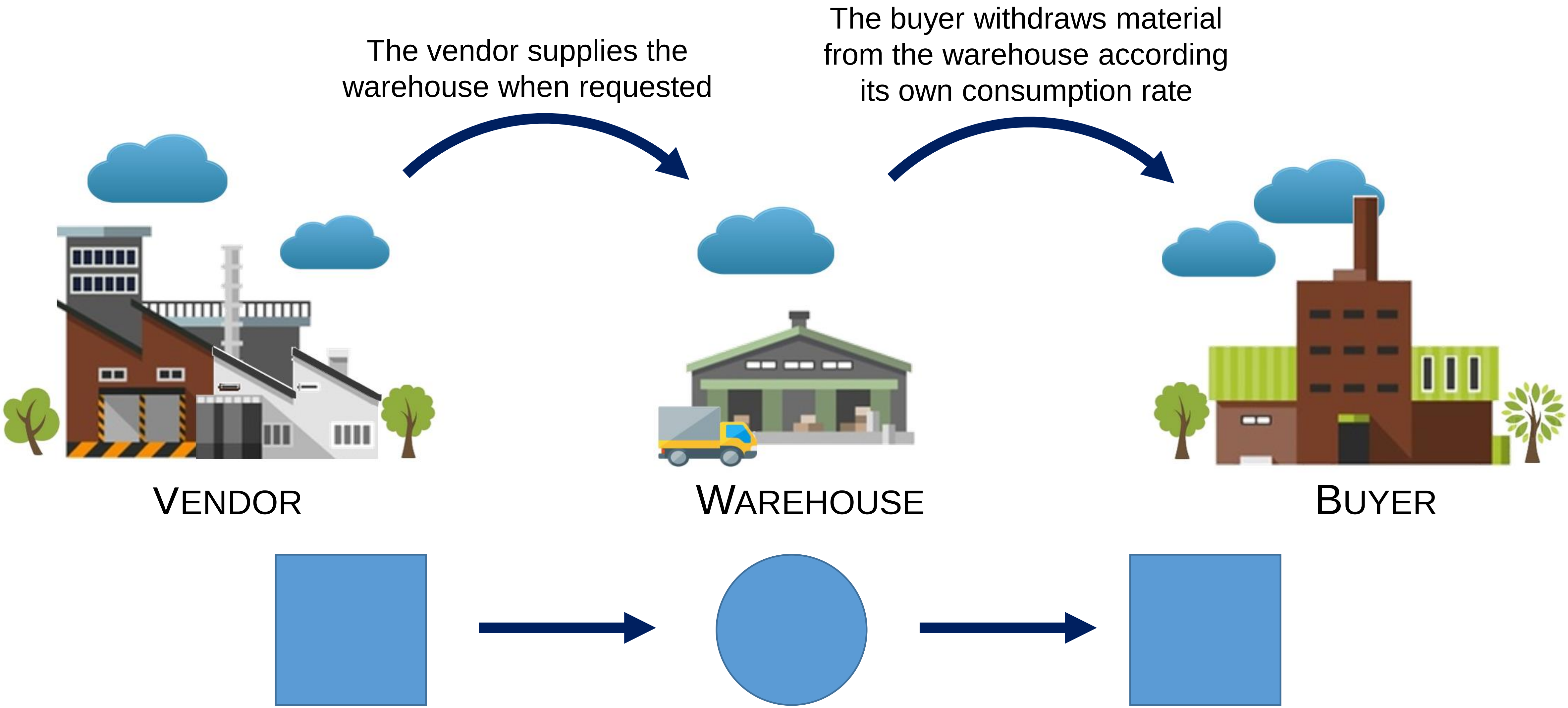

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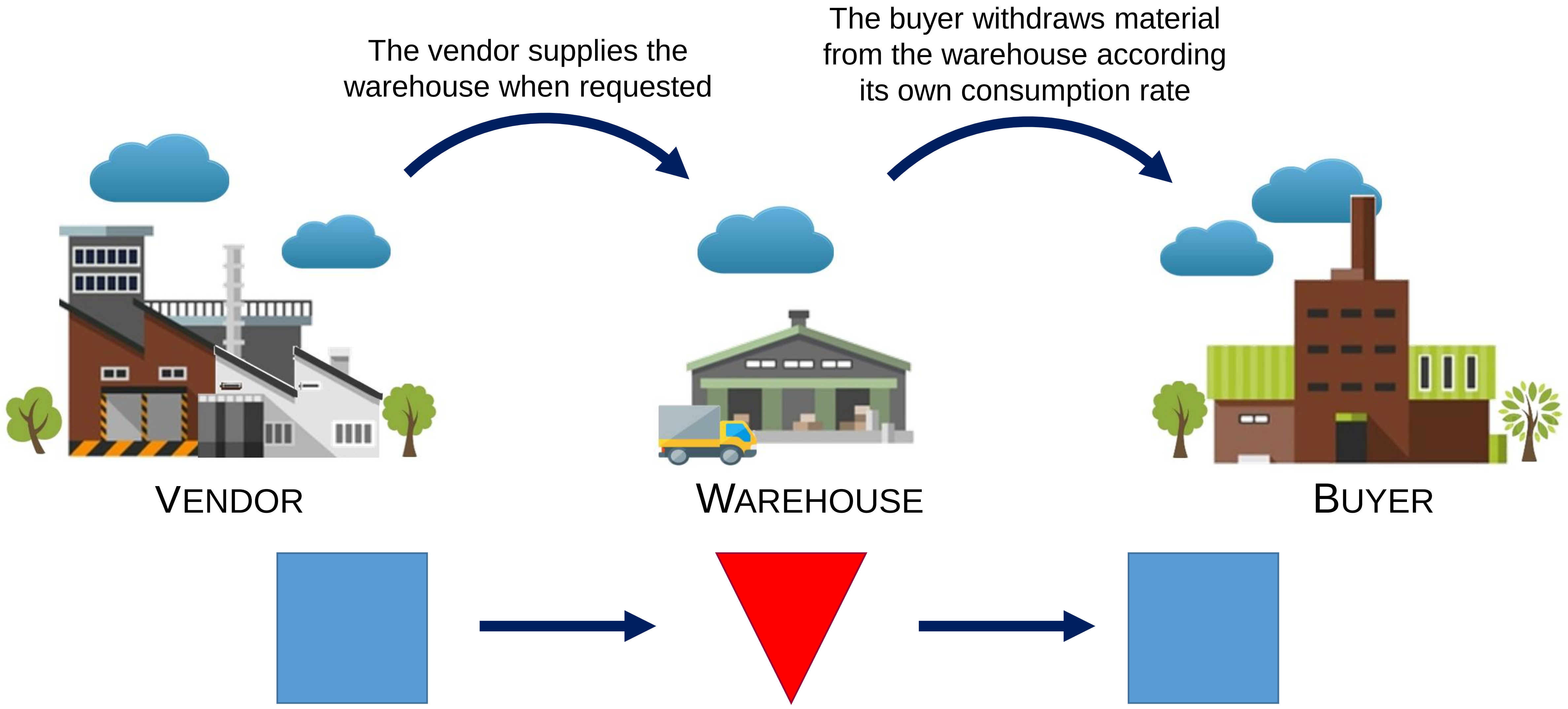
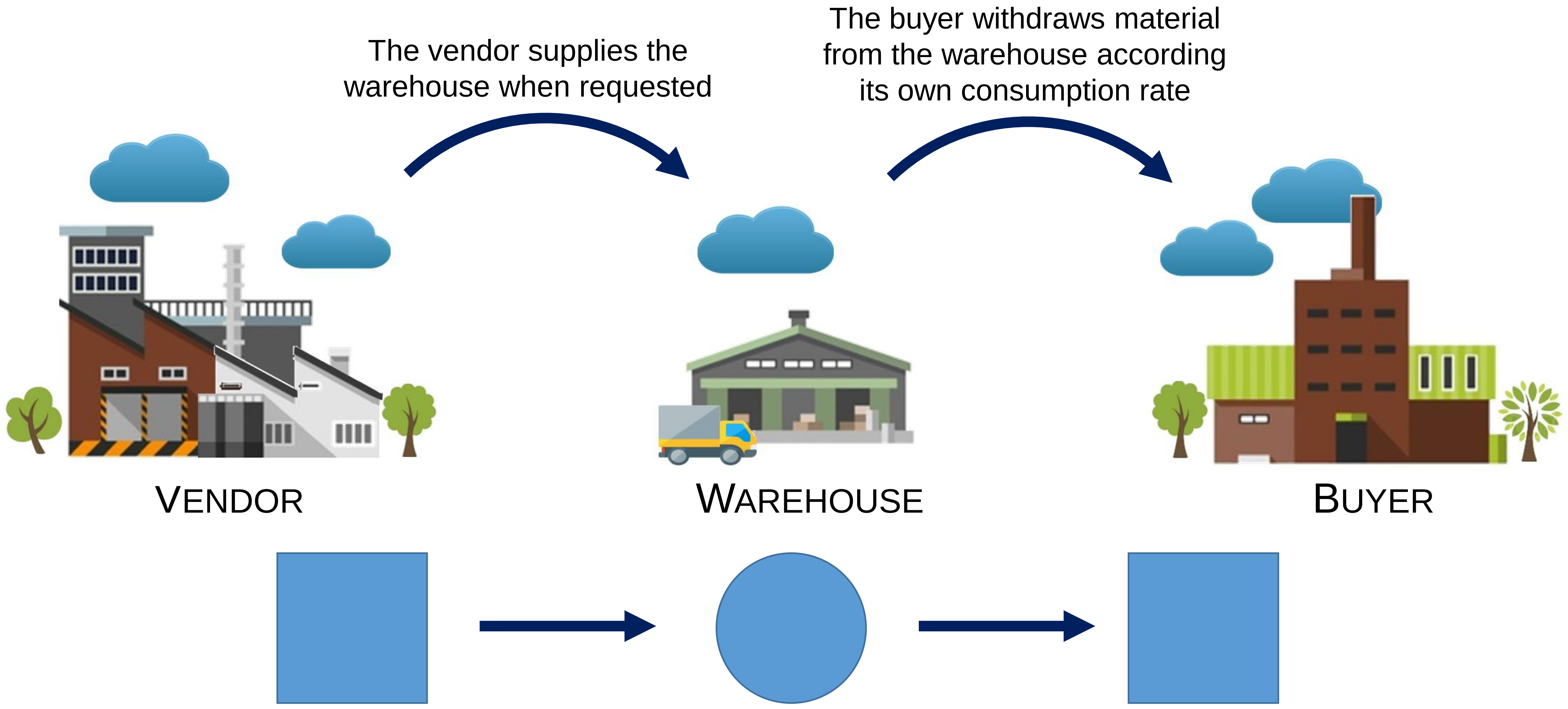


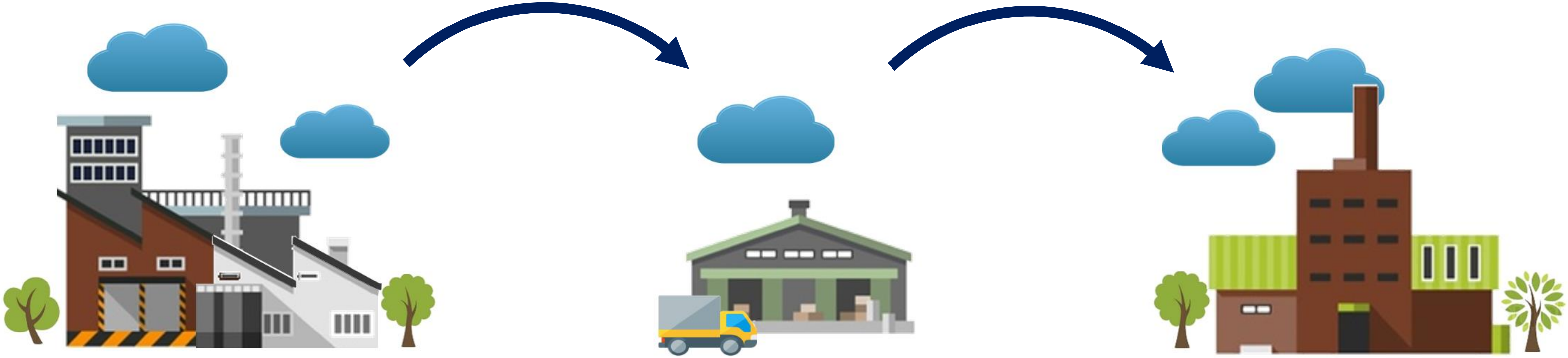
Single-Vendor Single-Buyer System



Single-Vendor Single-Buyer System

The vendor supplies the warehouse when requested

The buyer withdraws material from the warehouse according its own consumption rate

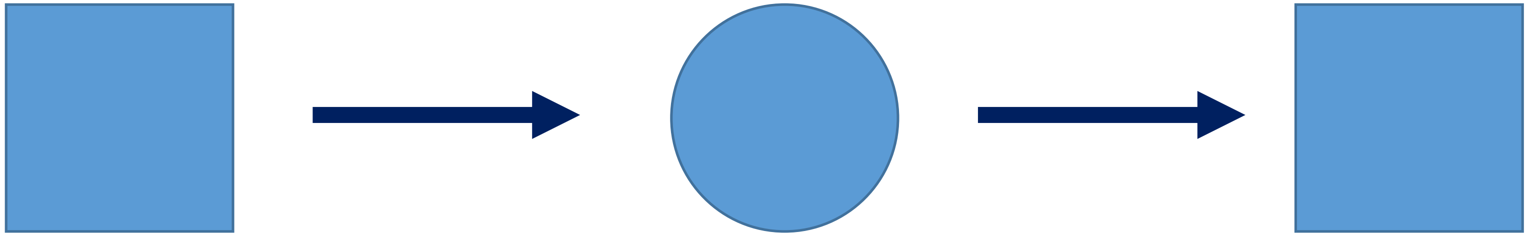


VENDOR

WAREHOUSE

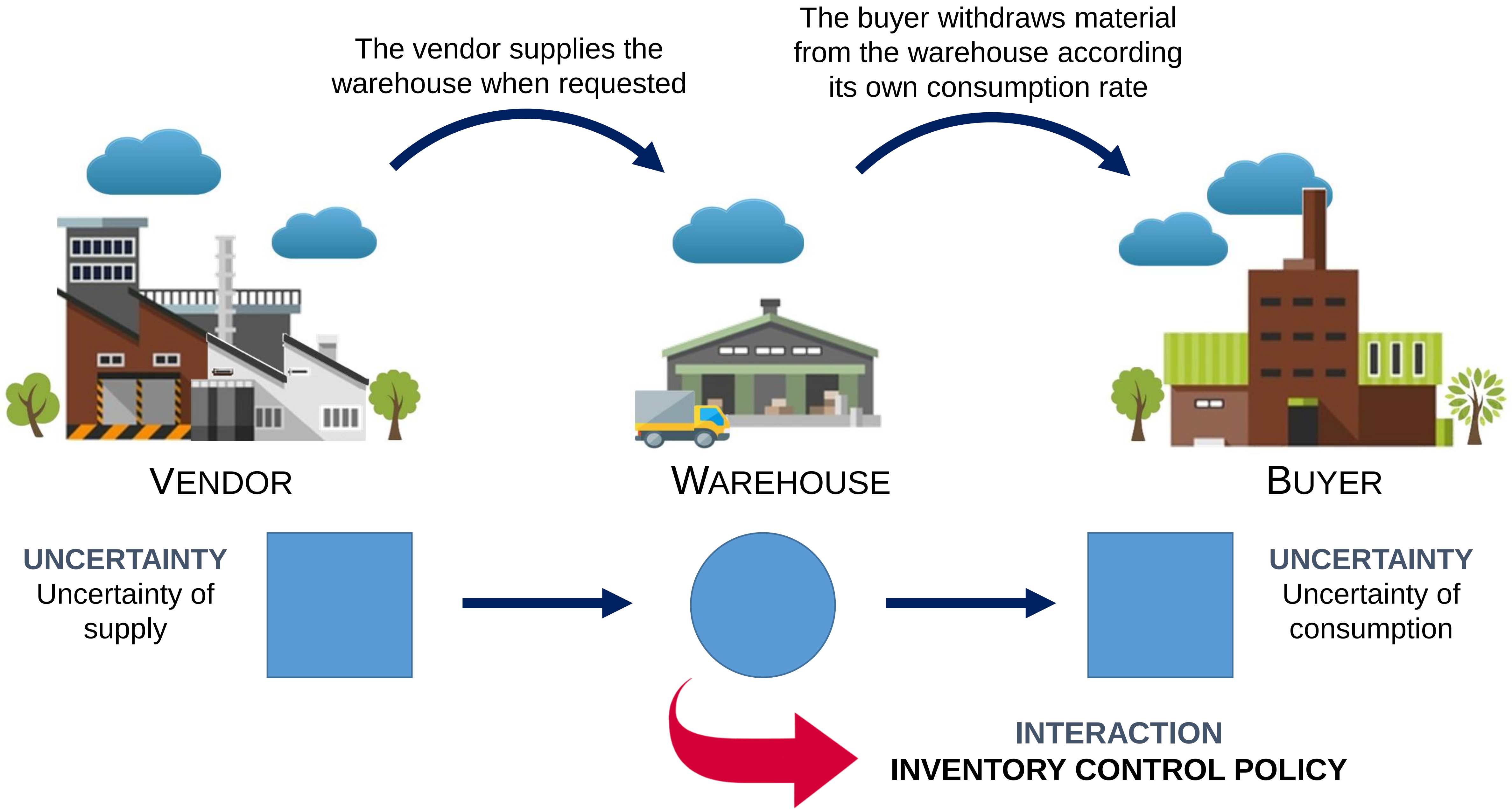
BUYER

UNCERTAINTY
Uncertainty of supply

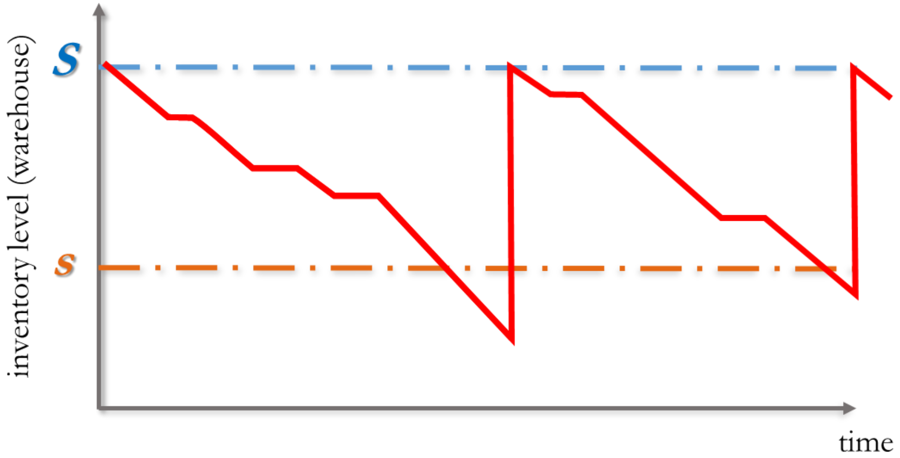


UNCERTAINTY
Uncertainty of consumption

Single-Vendor Single-Buyer System



[s, S] Inventory Control Policy



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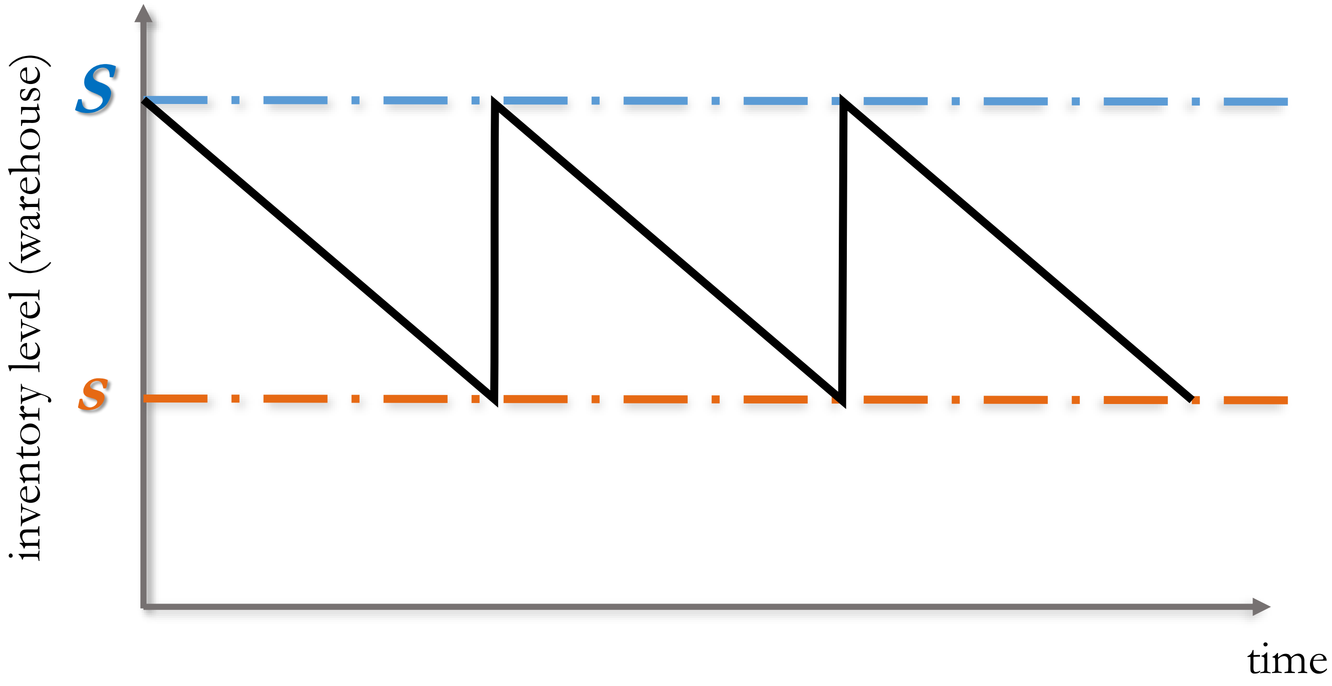


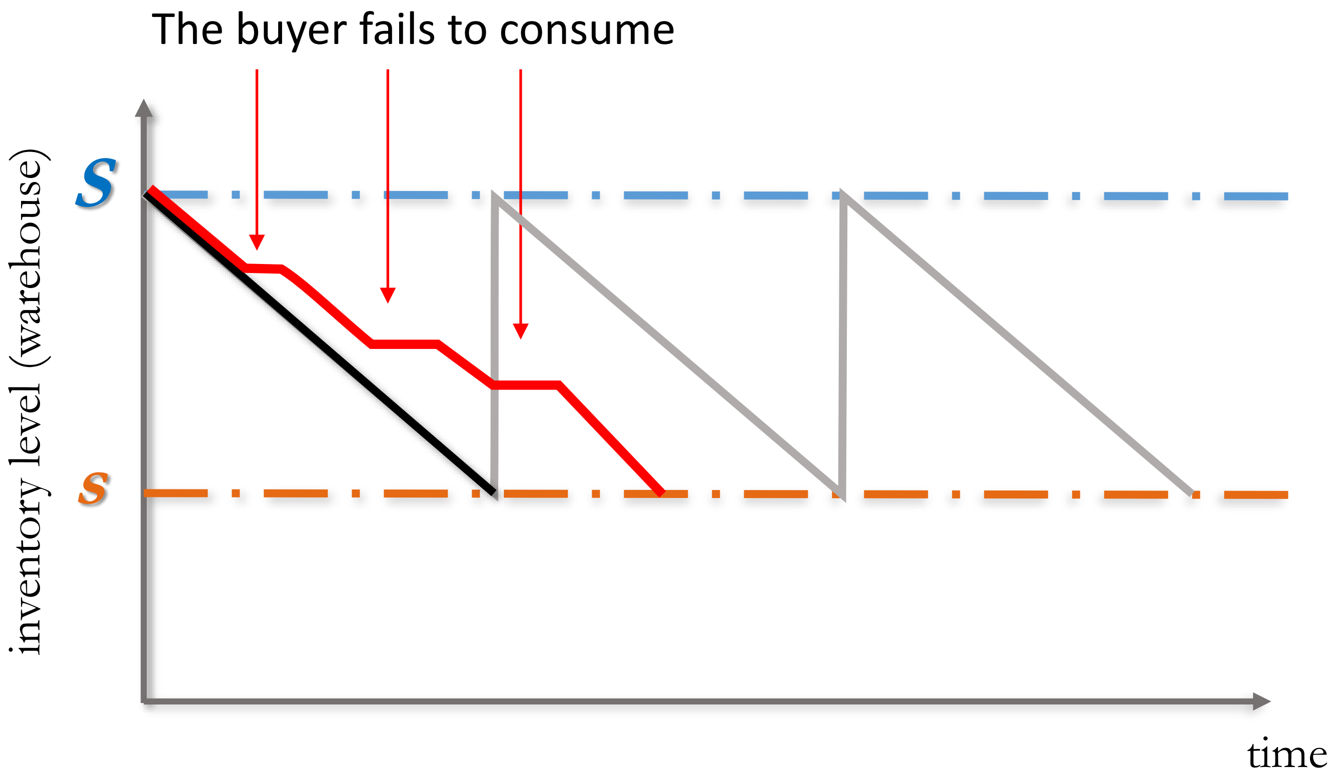
NOTE:

the inventory position is under continuous review by both the vendor and the buyer.

SUPPLY LEAD TIME = 0

at the reorder point s

[s, S] Inventory Control Policy



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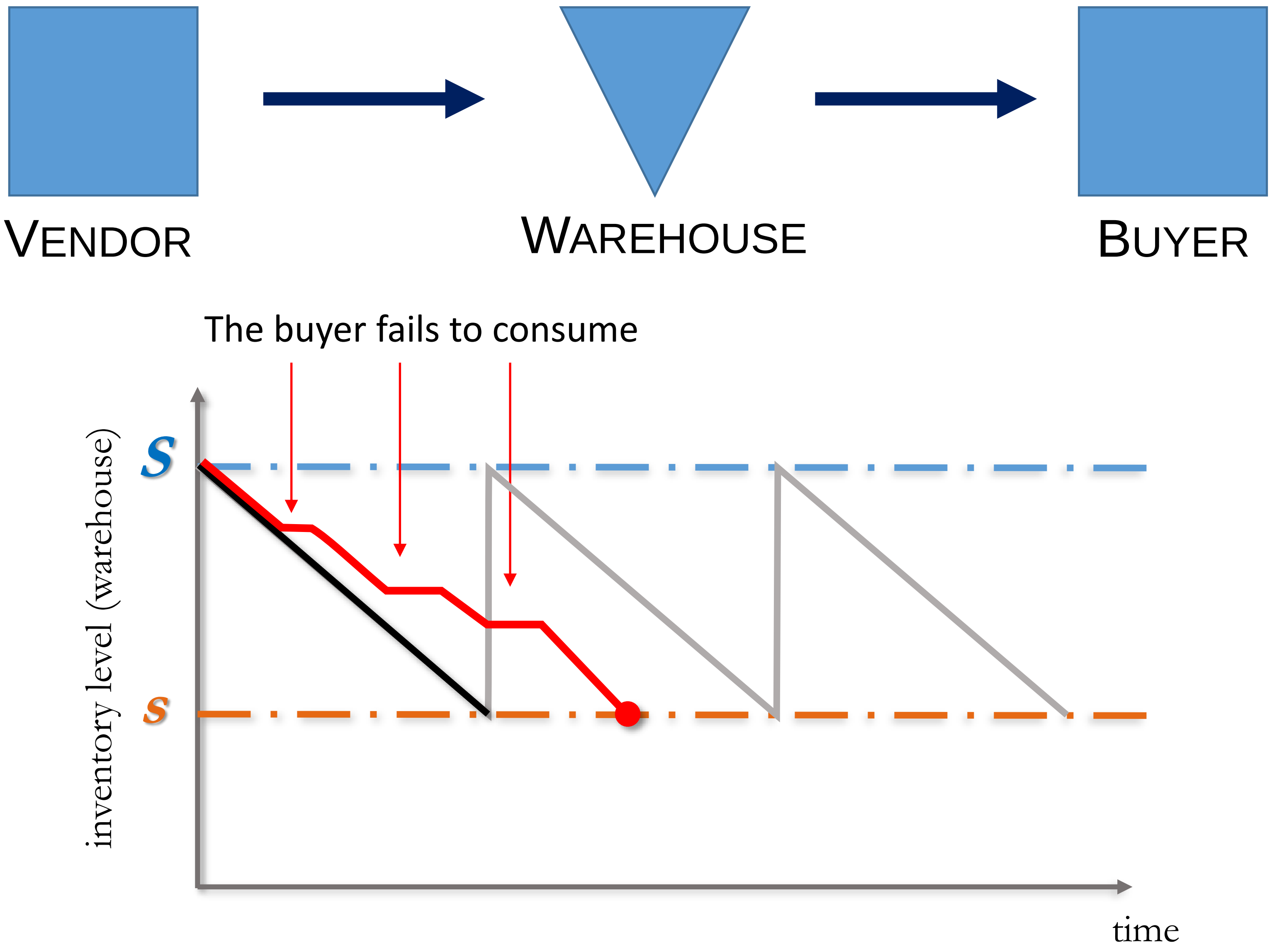

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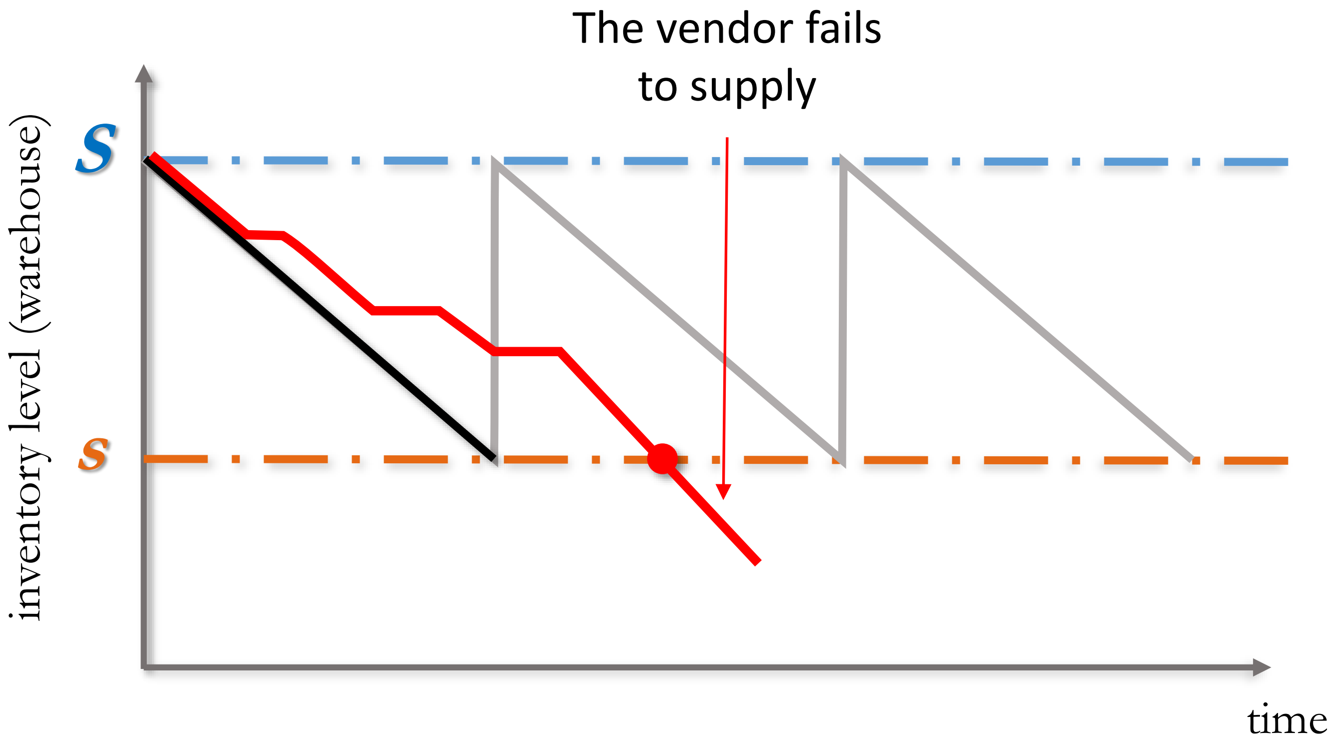

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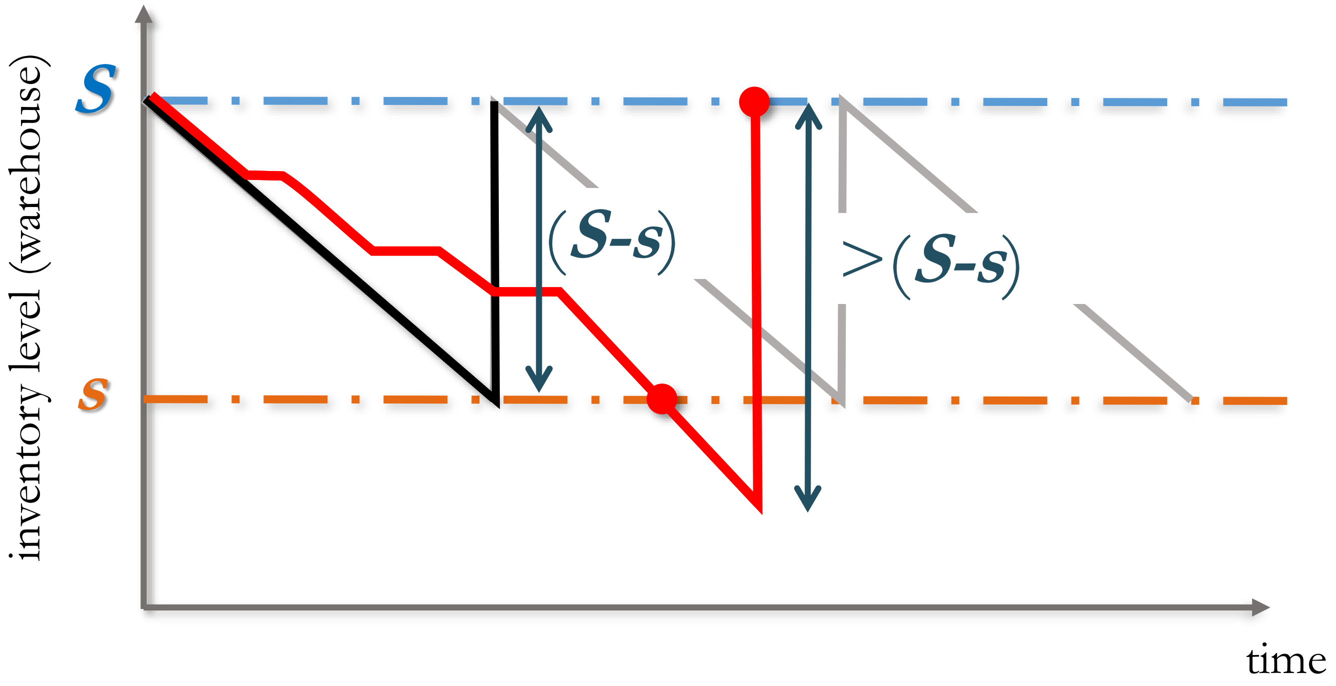

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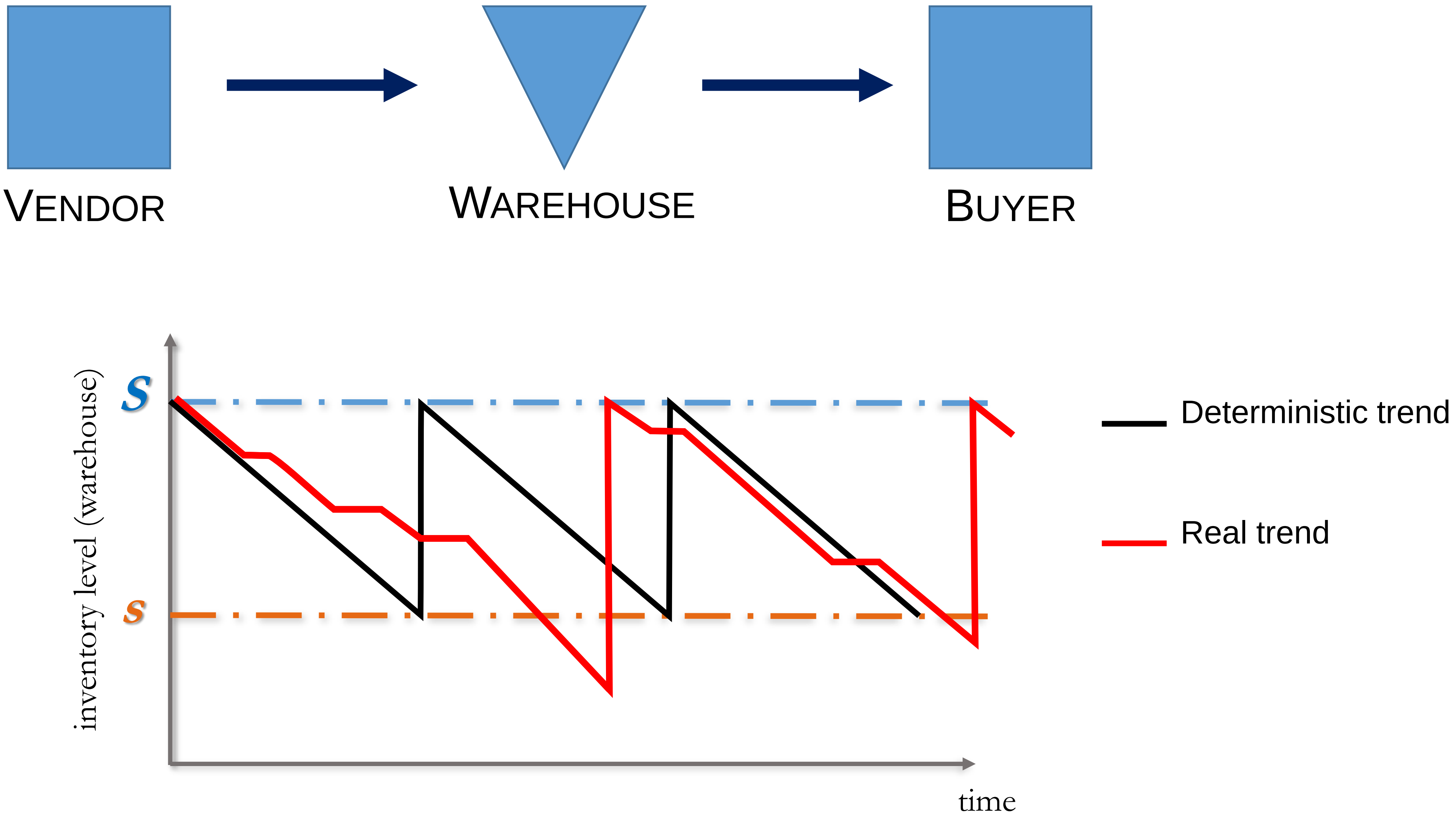

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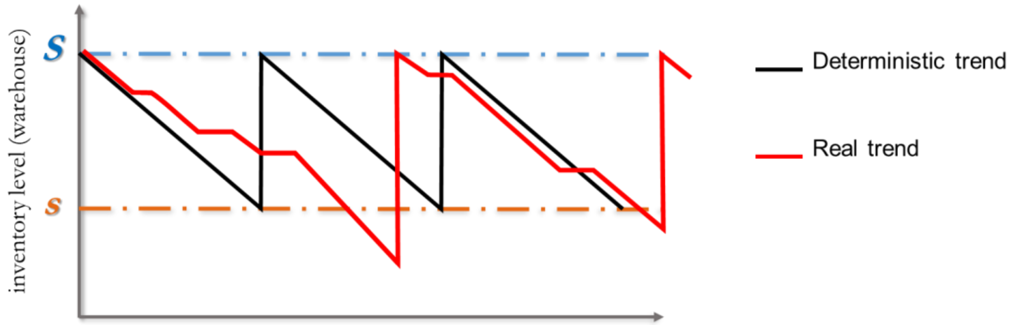
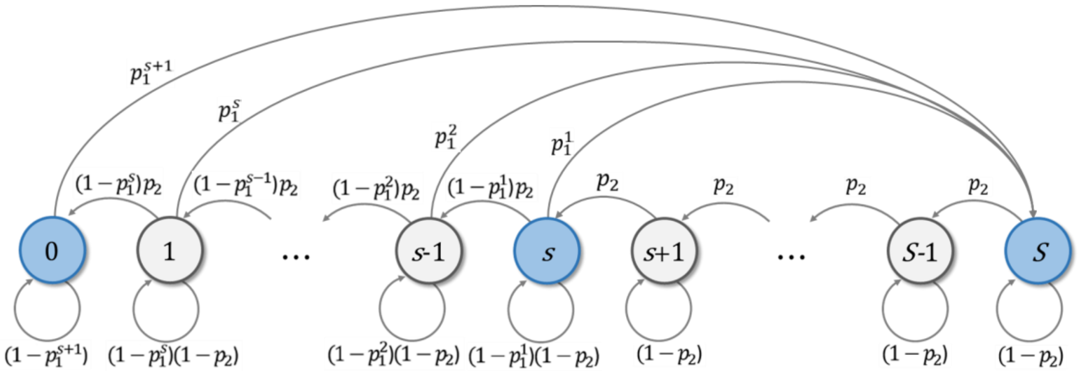


stochastic modeling of a supply chain

- analyse the actual behavior of a certain supply chain (performance measures)
- support strategic decisions for the **whole supply chain** (how to set s and S)

Bernoulli Model of a SV-SB Supply Chain with [s, S] Inventory Control Policy



Bernoulli Model of a SV-SB System with [*s*, *S*] Inventory Control Policy

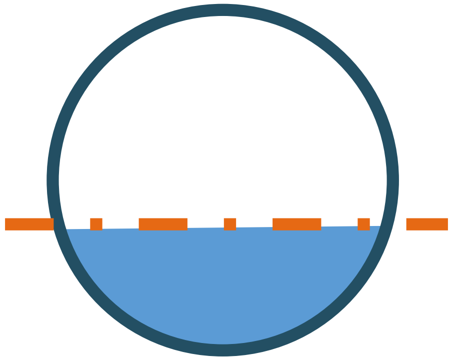
- ✓ Discrete-time discrete-state model Bernoulli model

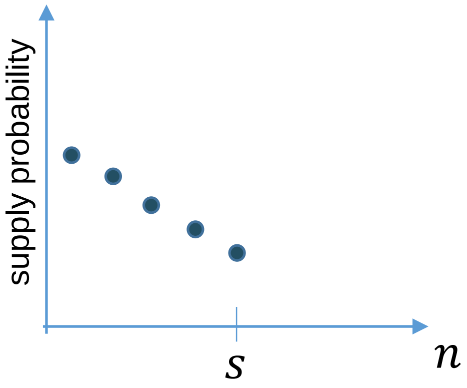
Bernoulli Model of a SV-SB System with [s, S] Inventory Control Policy

✓ Discrete-time discrete-state model Bernoulli model

✓ SUPPLY PROCESS MODELED AS A BERNOULLI PROCESS:

- The vendor can physically access the warehouse only during the replenishment phase ($n \leq s$)
- The vendor supplies a lot of size ($S - n$)
- The supply probability depends on n
  (the supply probability increases as the inventory level falls below the reorder point s)





Bernoulli Model of a SV-SB System with $[s, S]$ Inventory Control Policy

- ✓ Discrete-time discrete-state model Bernoulli model

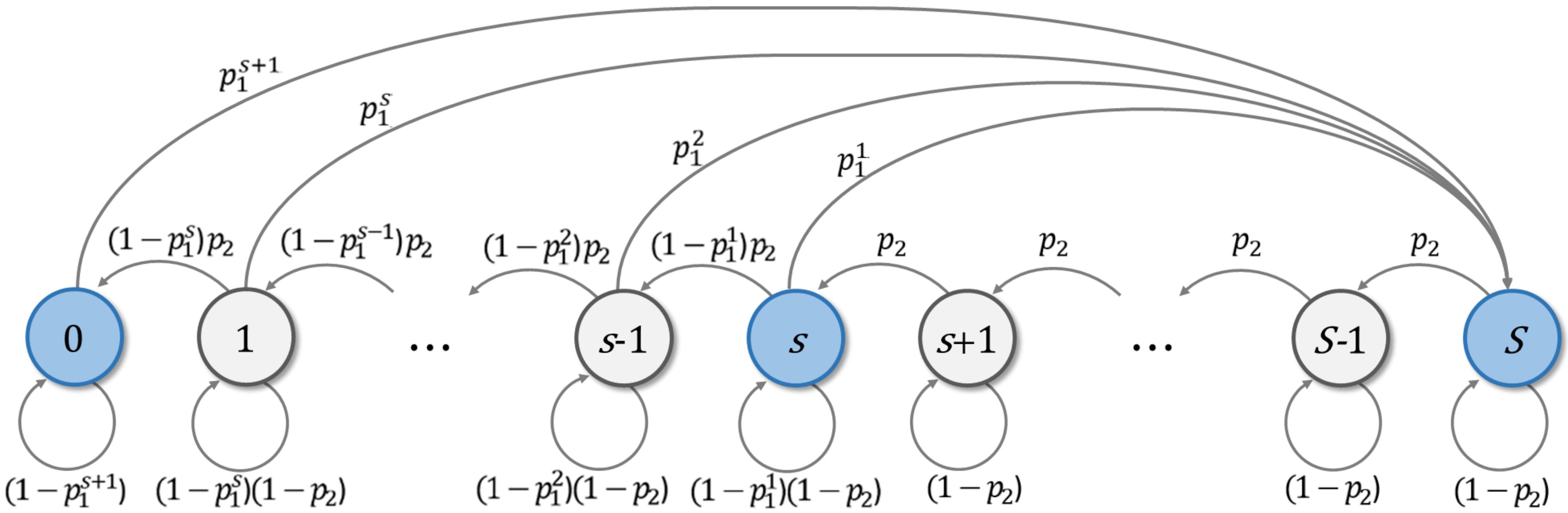
- ✓ SUPPLY PROCESS MODELED AS A BERNOULLI PROCESS:

  - The vendor can physically access the warehouse only during the replenishment phase ($n \leq s$)
  - The vendor supplies a lot of size ($S - n$)
  - The supply probability depends on n
    (the supply probability increases as the inventory level falls below the reorder point s)

- ✓ CONSUMPTION PROCESS MODELED AS A BERNOULLI PROCESS:

  - The buyer can physically access the warehouse in each time unit for consuming a single unit of material.
  - During each cycle time when the inventory position is $n > 0$, the buyer consumes an item with a fixed probability p_2
  - The buyer does not consume in the time unit when the vendor supplies the order

Bernoulli Model of a SV-SB System with [s, S] Inventory Control Policy



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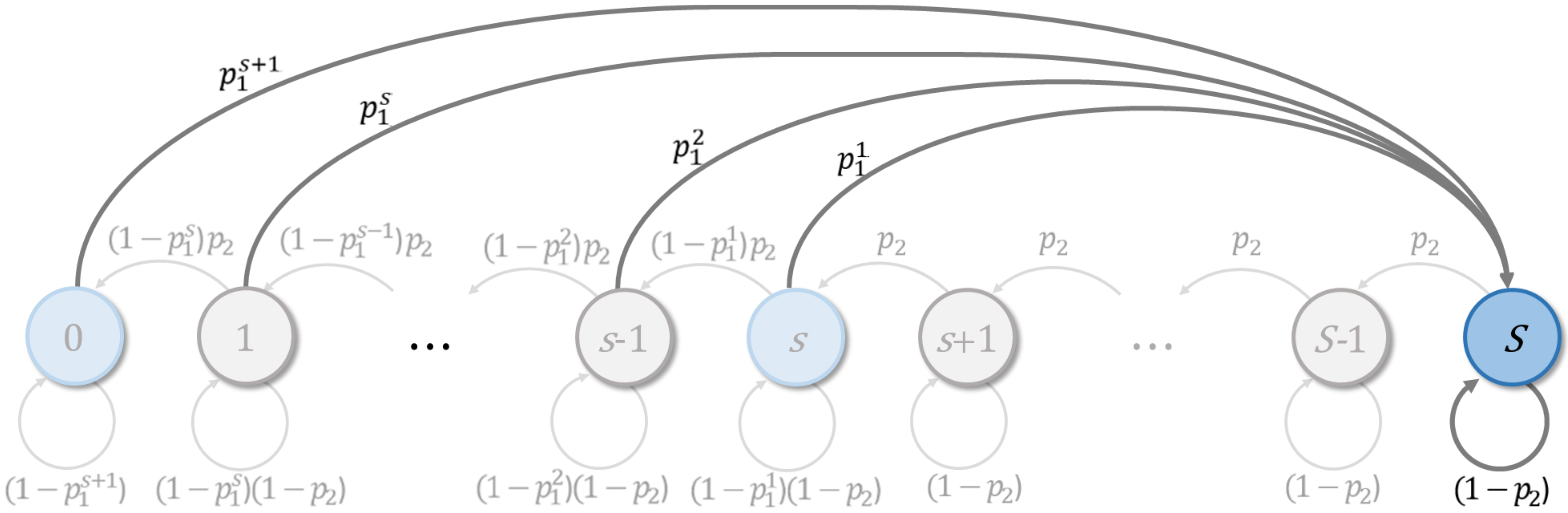

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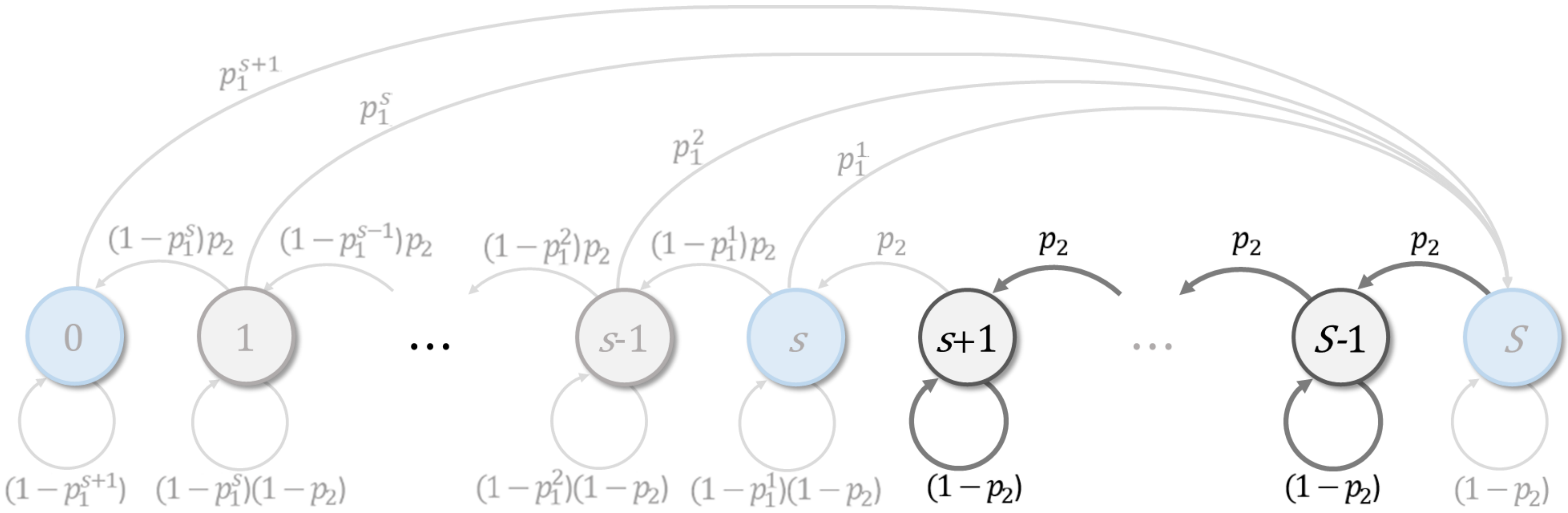

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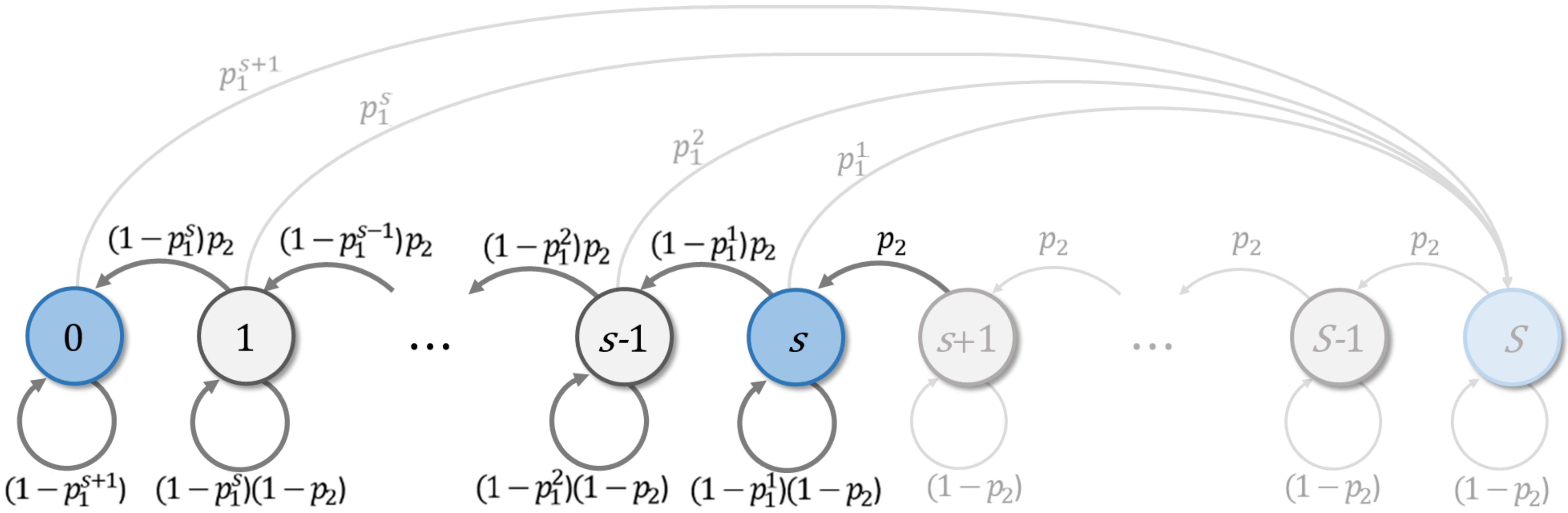

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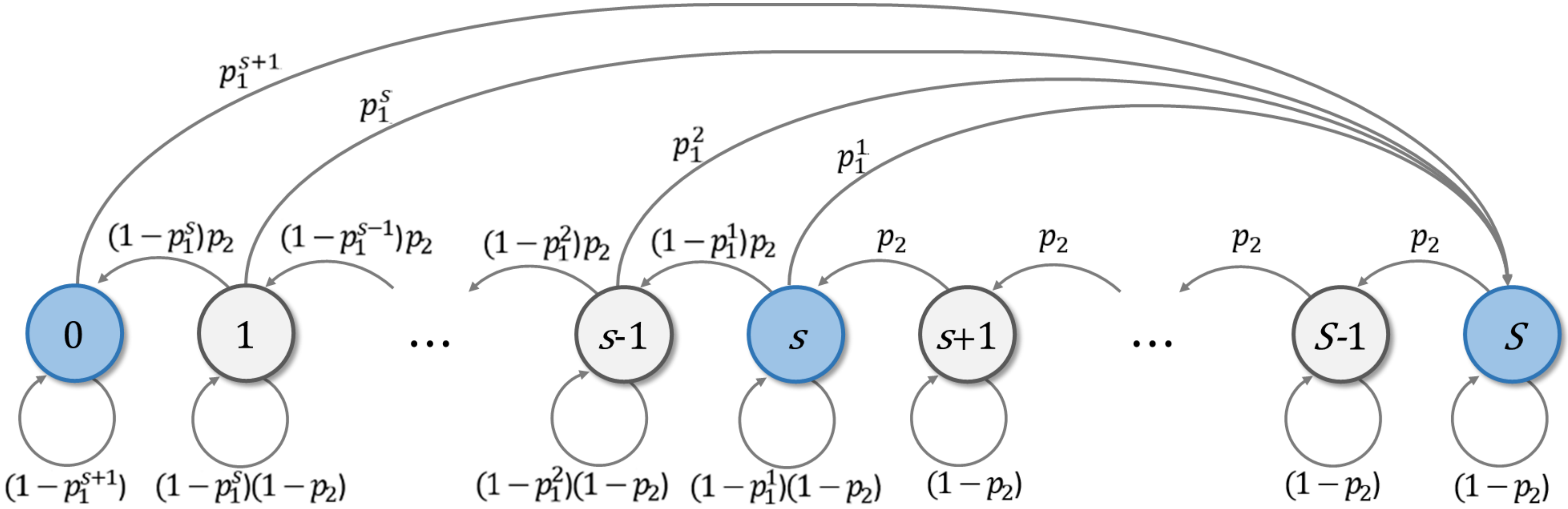
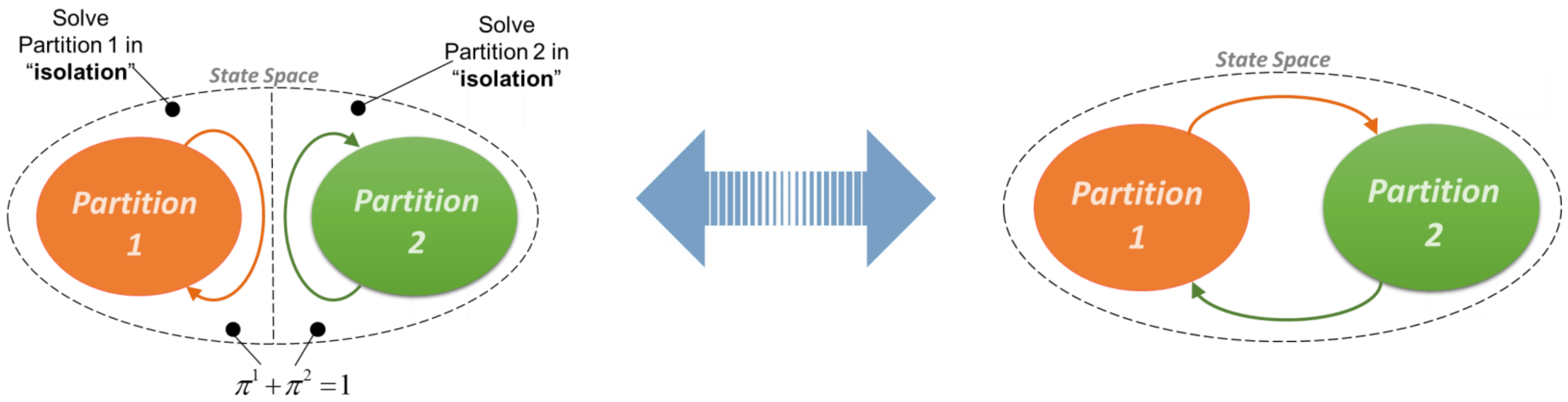


Model solution: Partioning procedure



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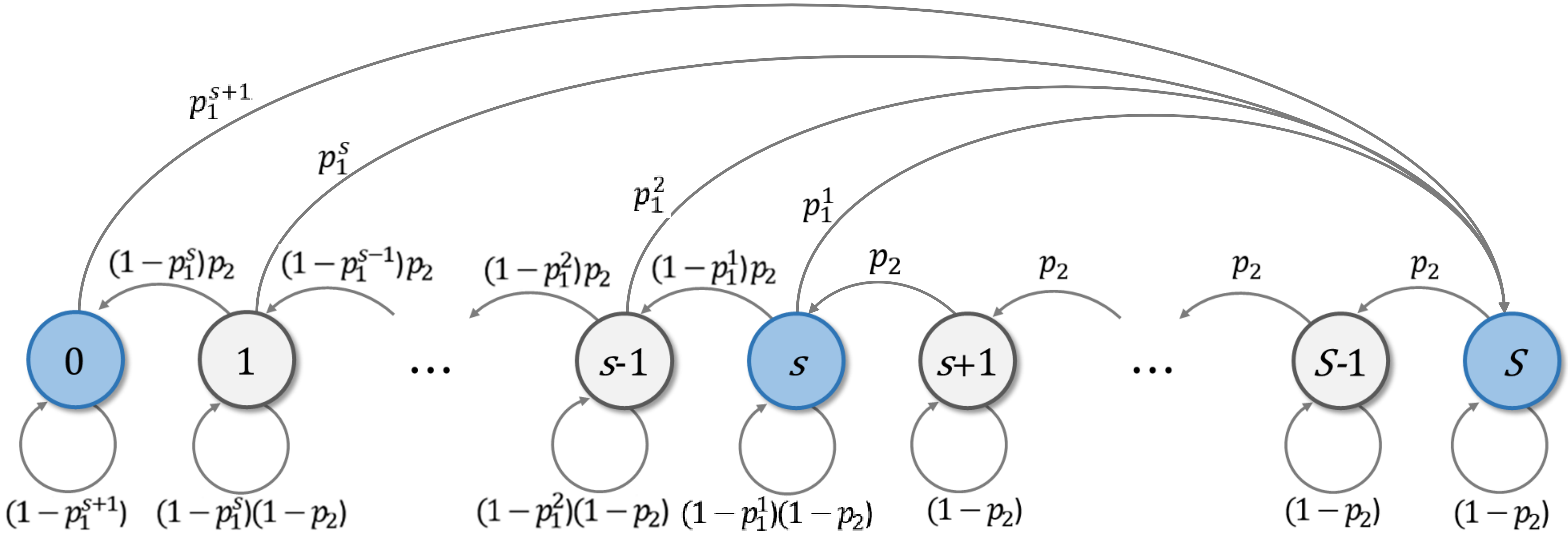

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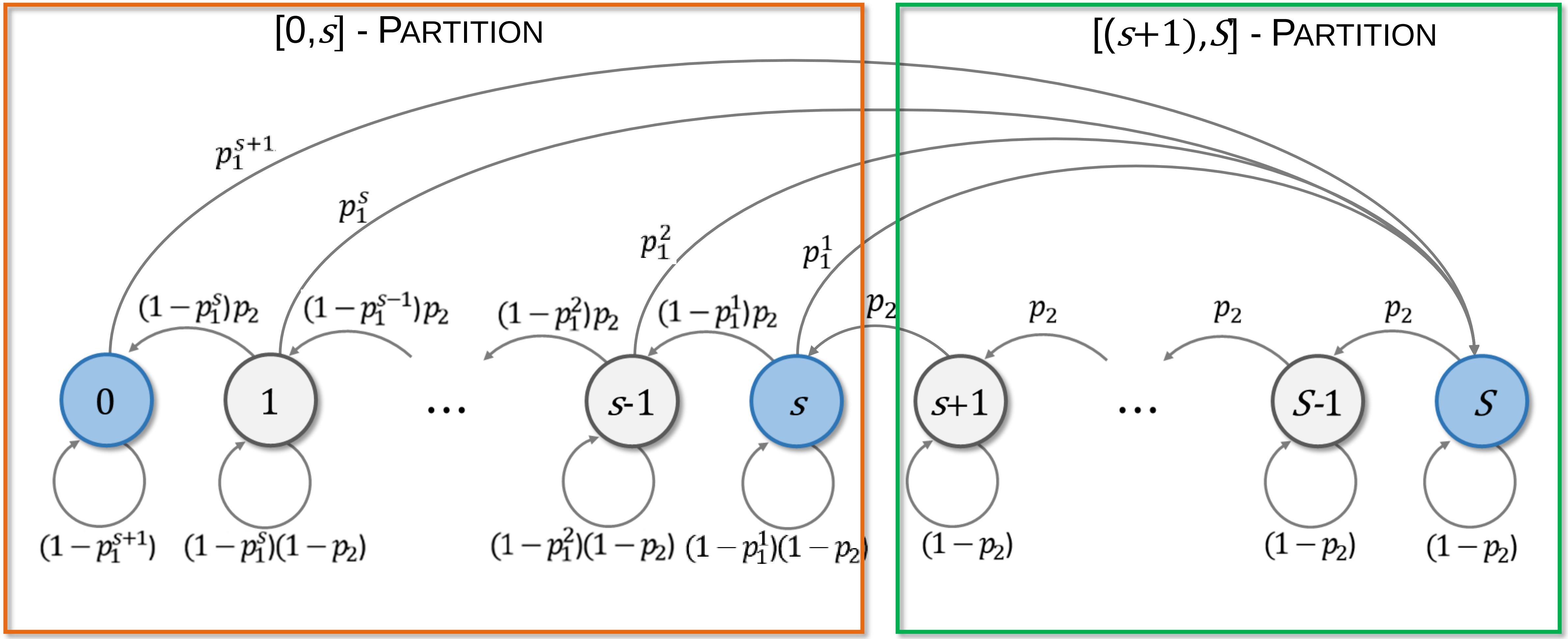

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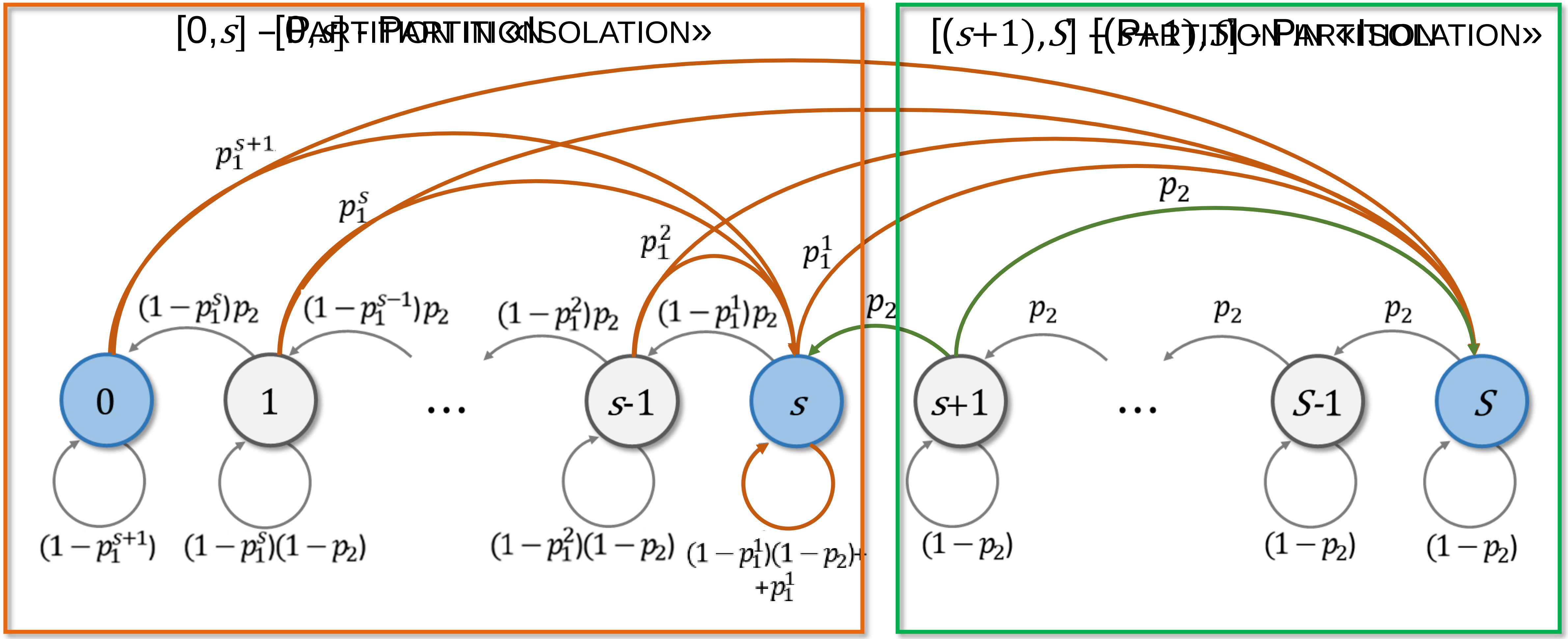
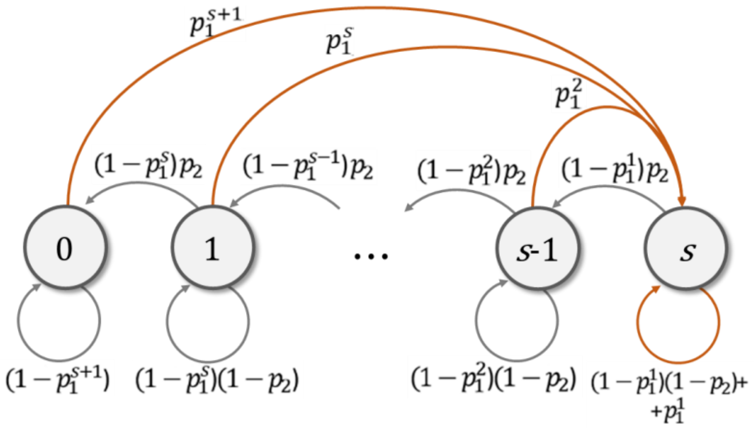


Bernoulli Model of a SV-SB System with [s, S] Inventory Control Policy

[0,s] – Partition in «Isolation»



Balance equations

$$\mathbf{p}^{[0,s]}(s) = (1 - p_2 + p_1^1 p_2)\mathbf{p}^{[0,s]}(s) + \sum_{h=0}^{s-1} p_1^{s-h+1}\mathbf{p}^{[0,s]}(h)\,,$$

$$\mathbf{p}^{[0,s]}(n) = (1 - p_1^{s-n+1})(1 - p_2)\mathbf{p}^{[0,s]}(n) + (1 - p_1^{s-n})p_2\mathbf{p}^{[0,s]}(n+1)\,,$$

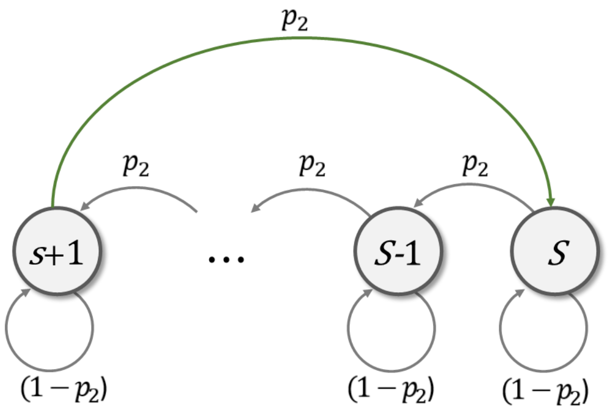
$$\text{for } n = 1, \ldots, s-1\,,$$

$$\mathbf{p}^{[0,s]}(0) = (1 - p_1^{s+1})\mathbf{p}^{[0,s]}(0) + p_2(1 - p_1^{s})\mathbf{p}^{[0,s]}(1)\,.$$

Normalization equation

$$\sum_{n=0}^{s} \mathbf{p}^{[0,s]}(n) = 1\,.$$

[(s+1),S] – Partition in «Isolation»



Balance equations

$$\mathbf{p}^{[s+1,S]}(n) = (1 - p_2)\mathbf{p}^{[s+1,S]}(n) + p_2\mathbf{p}^{[s+1,S]}(n+1)\,,$$

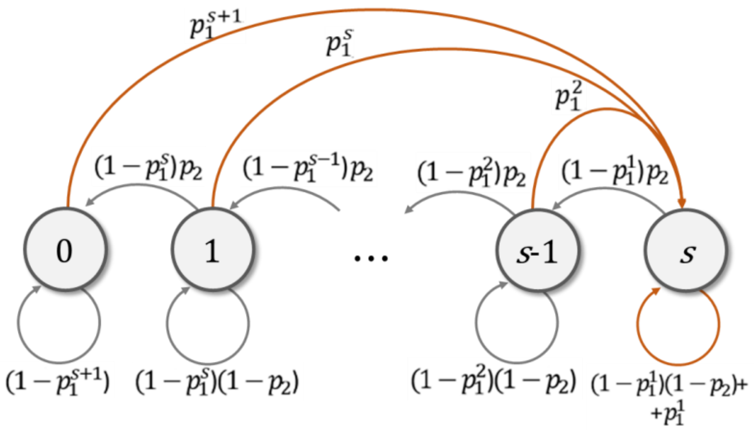
$$\text{for } n = s+1, \ldots, S-1\,,$$

Normalization equation

$$\sum_{n=s+1}^{S} \mathbf{p}^{[s+1,S]}(n) = 1\,.$$

Bernoulli Model of a SV-SB System with [s, S] Inventory Control Policy

[0,s] – Partition in «Isolation»



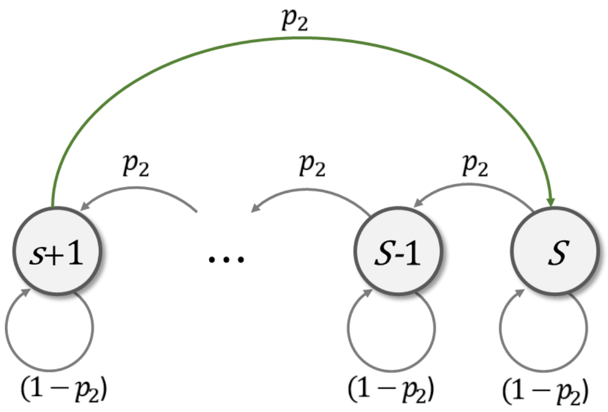
Solution in «isolation»

$$\mathbf{p}^{[0,s]}(n) = \left(\prod_{i=0}^{n-1} \alpha(i)\right) \mathbf{p}^{[0,s]}(0), \quad \text{for } n = 1, \ldots, s,$$

$$\alpha(i) = \frac{p_1^{s-i+1} + \Phi(i)\, p_2\, (1 - p_1^{s-i+1})}{p_2 (1 - p_1^{s-i})} \qquad \Phi(i) = \begin{cases} 1 & \text{if } i > 0 \\ 0 & \text{if } i = 0 \end{cases}$$

$$\mathbf{p}^{[0,s]}(0) = \frac{1}{1 + \sum_{n=1}^{s} \left(\prod_{i=0}^{n-1} \alpha(i)\right)}$$

[(s+1),S] – Partition in «Isolation»

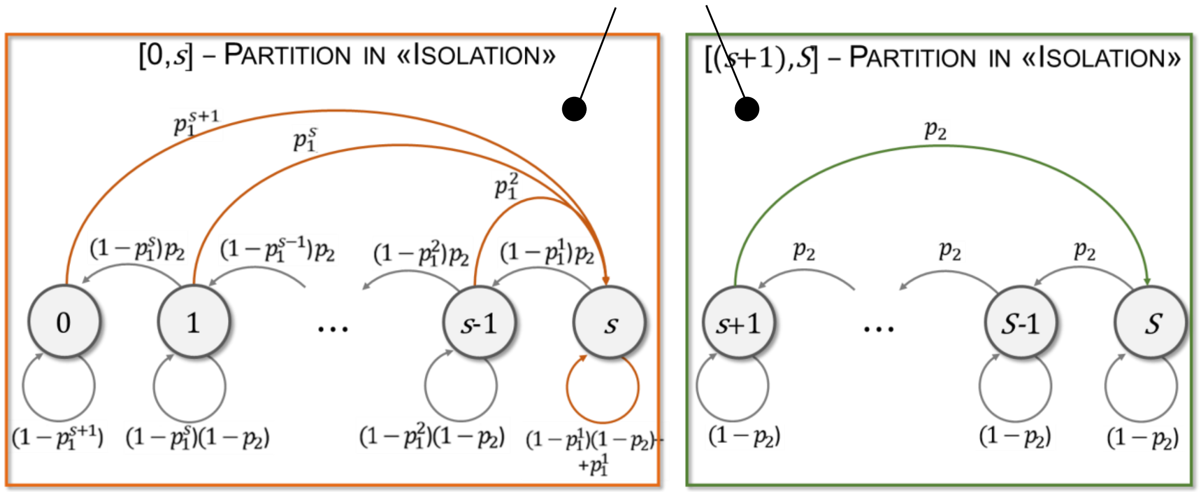


Solution in «isolation»

$$\mathbf{p}^{[s+1,S]}(n) = \frac{1}{S - s}, \qquad \text{for } n = s+1, \ldots, S.$$

Bernoulli Model of a SV-SB System with [s, S] Inventory Control Policy

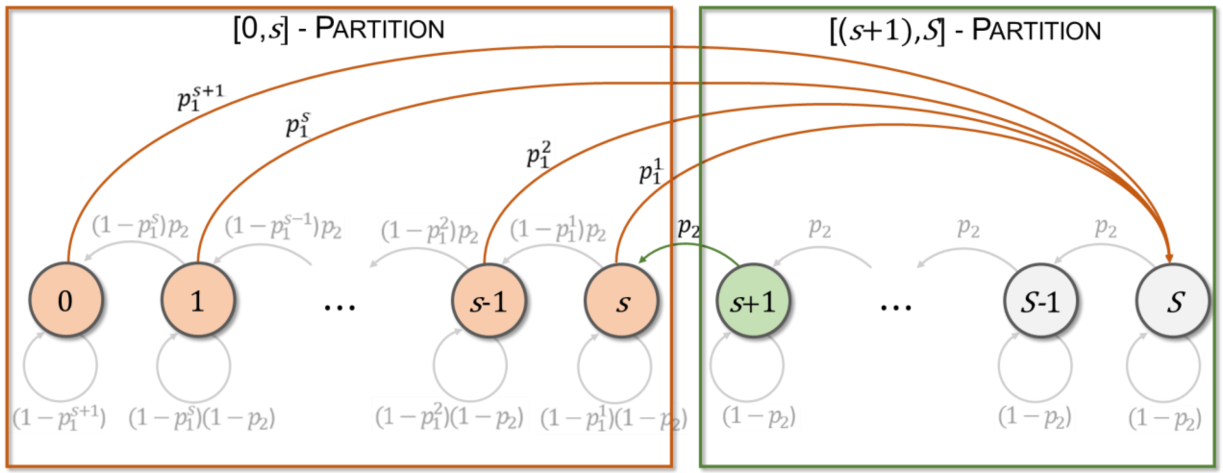
$$\pi^{[0,s]} + \pi^{[s+1,S]} = 1$$



✓ Partition probabilities

$$\pi^{[0,s]} \sum_{n=0}^{s} p_1^{s-n+1} \mathbf{p}^{[0,s]}(n) = \pi^{[s+1,S]} p_2 \mathbf{p}^{[s+1,S]}(s+1),$$

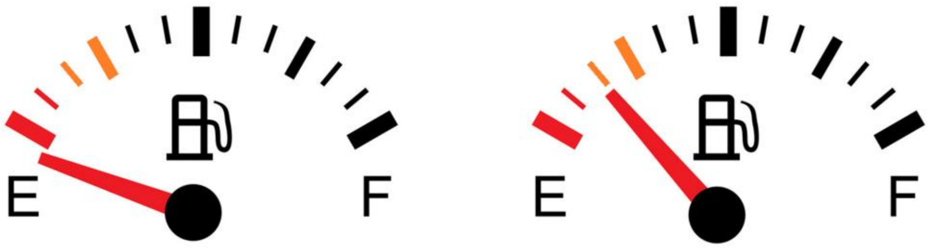
$$\pi^{[0,s]} + \pi^{[s+1,S]} = 1$$



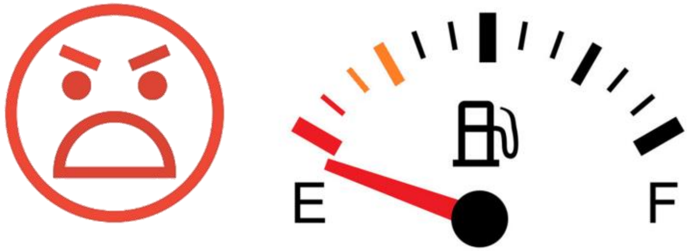
System solution

$$\mathbf{p}(n) = \begin{cases} \pi^{[0,s]}\,\mathbf{p}^{[0,s]}(n), & \text{if } n = 0, 1, \ldots, s \\ \pi^{[s+1,S]}\,\mathbf{p}^{[s+1,S]}(n), & \text{if } n = s+1, \ldots, S \end{cases}$$

Performance measures

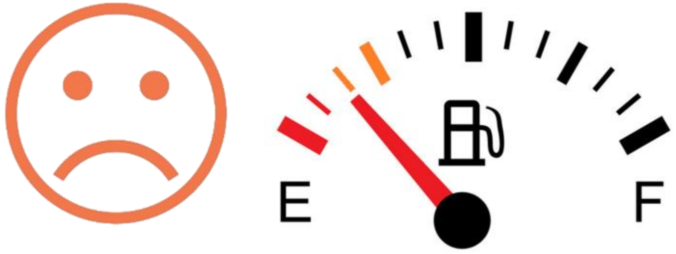


Performance measures



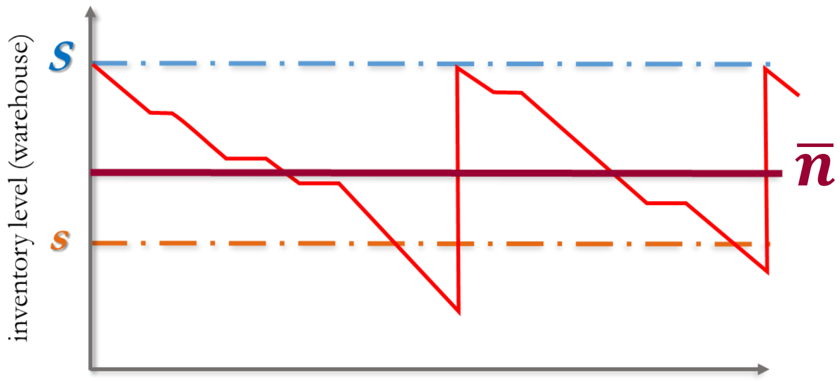
✓ Stock-out probability

$$p^{\text{stock-out}} = \mathbf{p}(0) = \frac{\pi^{[0,s]}}{1 + \sum_{n=1}^{s} \left(\prod_{i=0}^{n-1} \alpha(i) \right)}$$



✓ Probability of being below a threshold $h \leq s$ (safety stock)

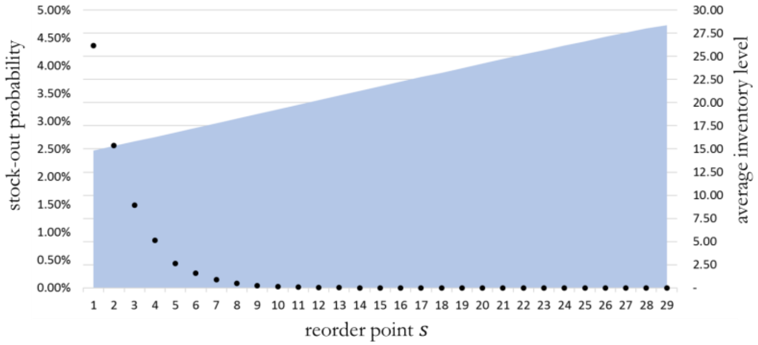
$$p^{\text{under-}h} = \sum_{n=0}^{h} \mathbf{p}(n)$$

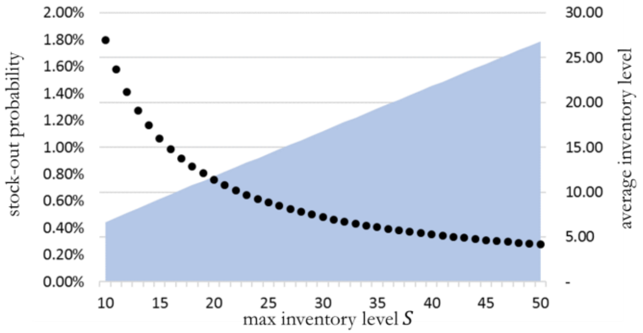


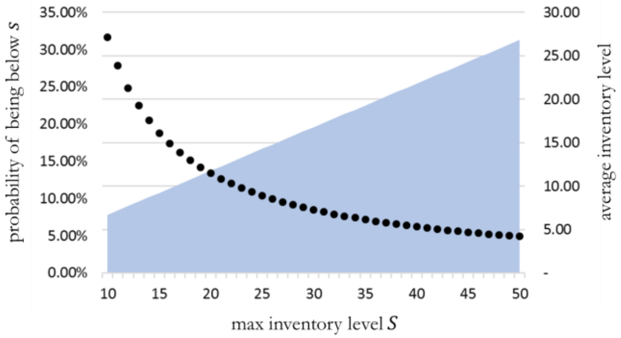
✓ Average inventory level

$$\bar{n} = \sum_{n=0}^{S} \left(n \, \mathbf{p}(n) \right)$$

Numerical results







Numerical results

VENDOR

WAREHOUSE

BUYER

SUPPLY PROBABILITY ($n \leq s$)
Increases by 2% each time th the inventory level decreases by 1 item

$p_1^1 = 0.4$ Supply probability at s

$p_1^2 = 0.408$ Supply probability at $s-1$

$p_1^3 = 0.416$ Supply probability at $s-2$

... ...

CONSUMPTION PROBABILITY
Probabity that the buyer consumes an item in a time unit

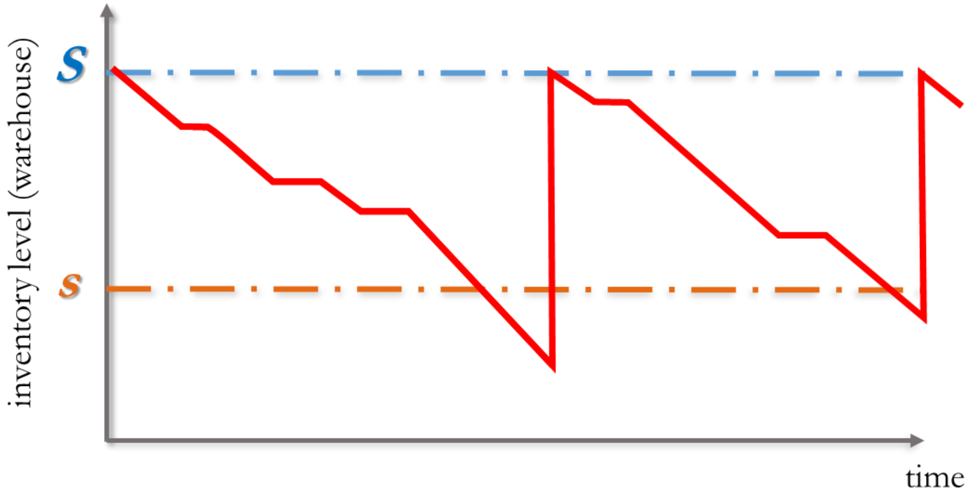
$p_2 = 0.95$

Numerical results

VENDOR

WAREHOUSE

BUYER



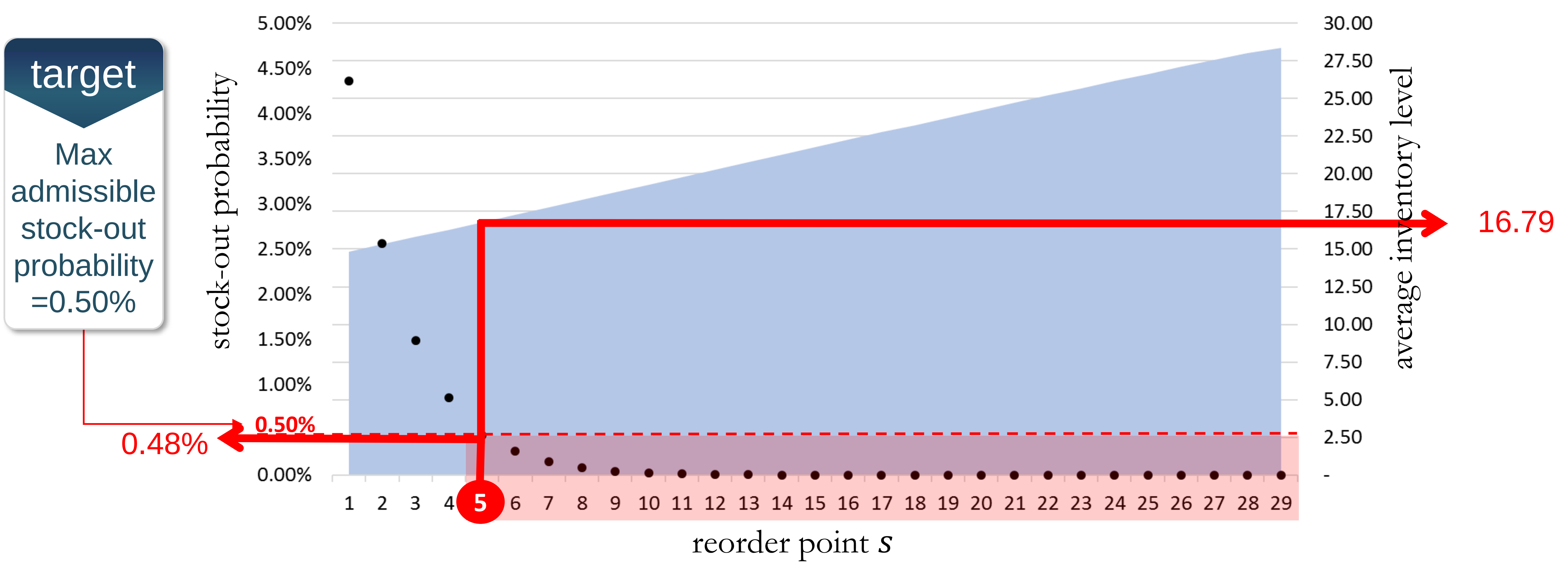
Given that the maximum inventory capacity S is fixed, which is the best reorder point s ?

Given that the reorder point s is fixed, which is the best level for the S ?

Numerical results

✓ Fixed maximum inventory capacity: $S = 30$

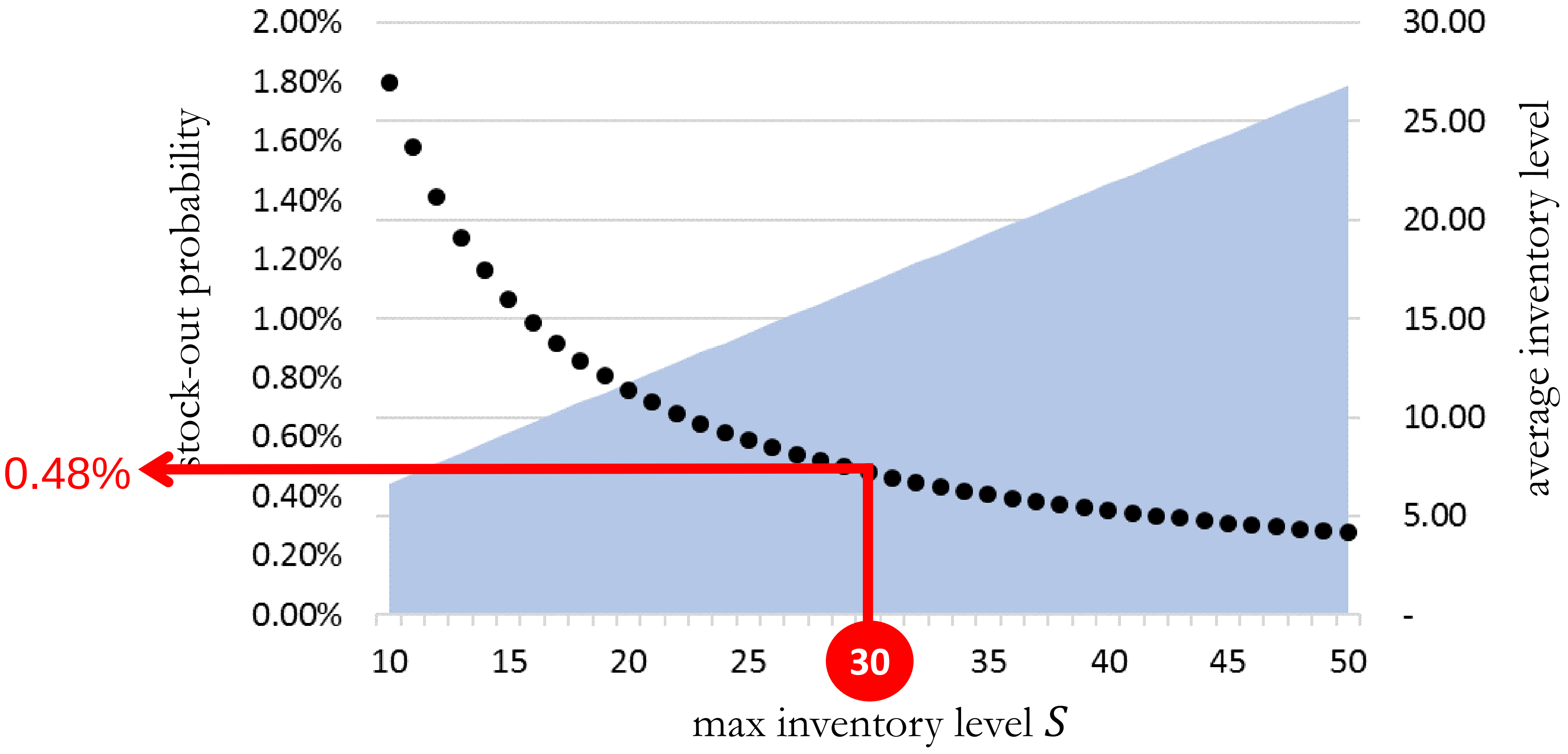
Performance measures as the reorder point s varies from 1 item to 29 items



Numerical results

✓ Fixed reorder point fixed: $s = 5$

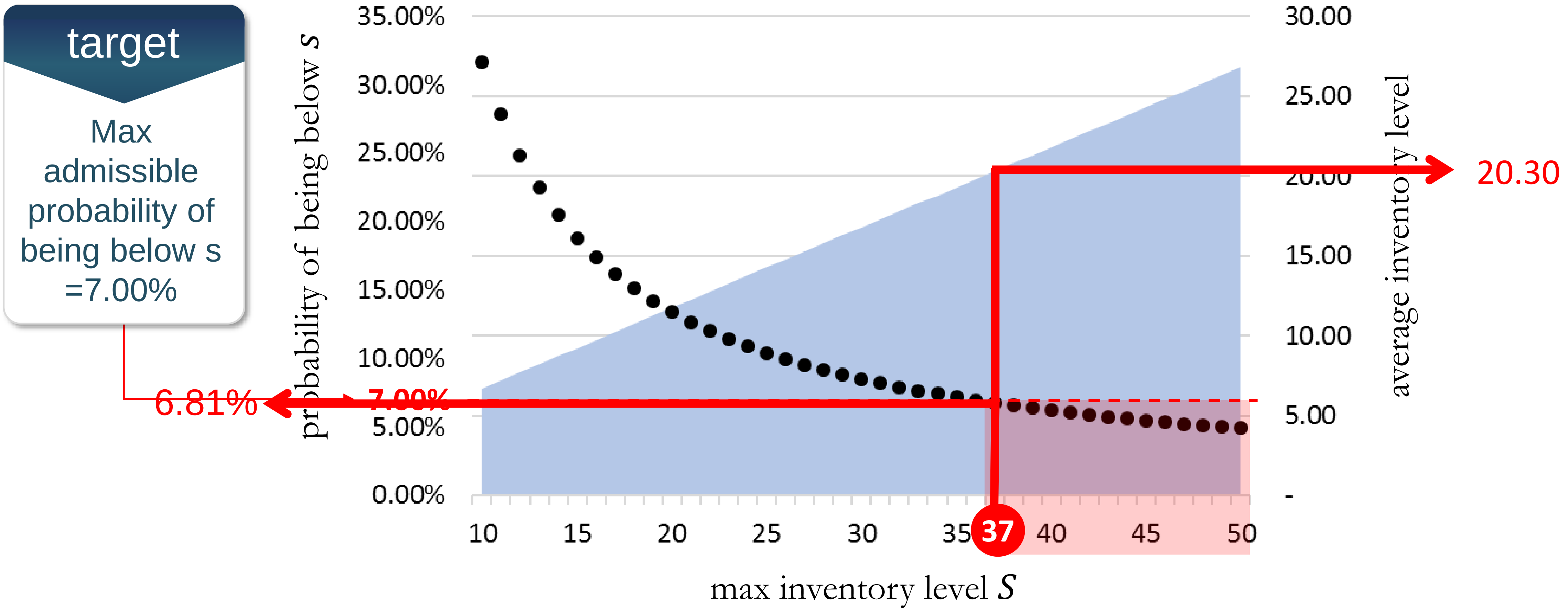
Performance measures as the max inventory capacity S varies from 10 to 50 items



Numerical results

✓ Reorder point fixed to 5 items: $s = 5$

Performance measures as the max inventory capacity S varies from 10 to 50 items



Conclusions

Conclusions

✓ Stochastic modeling of supply chains would allow

- evaluation of the system performance measures (i.e., stock-out probability, probability of being below the safety stock and average inventory level)
- transparent and rational decision-making process (e.g., decision about the inventory control policy between the parties)

✓ Early Bernoulli model of a single-vendor single-buyer supply systems with [s,S]-inventory policy.

✓ Future works should investigate

- Different inventory control policies (such as fixed order quantity, lot-for-lot, etc.)
- more complex network structures (such as multi-vendor and/or multi-buyer supply chains)

Thanks for your attention

A Bernoulli model for the single-vendor single-buyer supply chain

elisa.gebennini@unimore.it