POLITECNICO DI MILANO

Decomposition of asynchronous automated long lines

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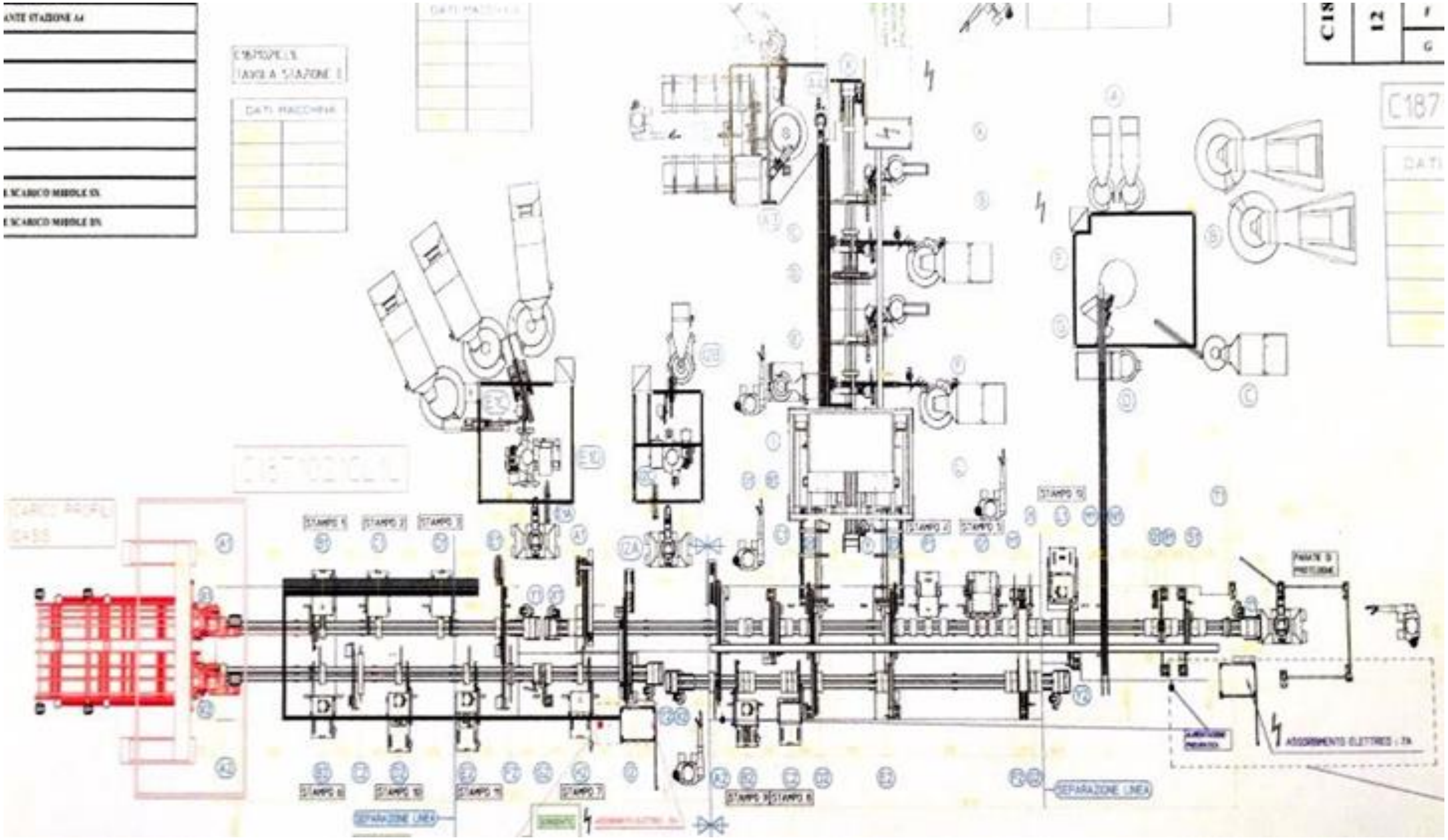
XI Conference on Stochastic Models of Manufacturing and Service Operations (Acaya 4-9 June 2017)

Outline

1. Motivation

- **Automated production line**

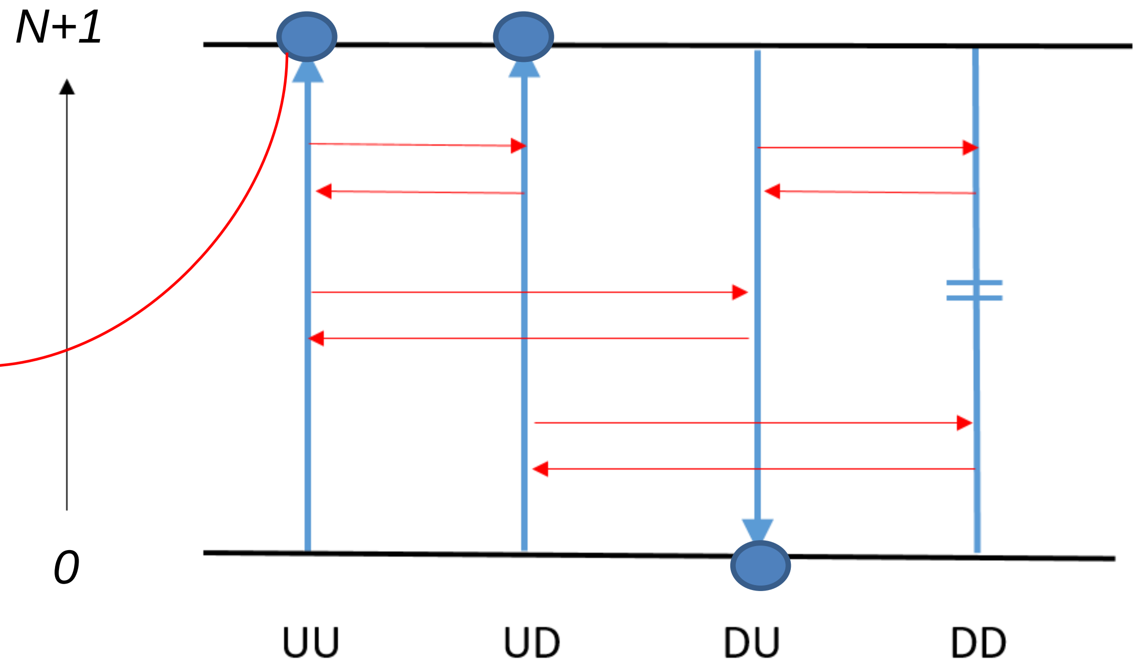


(Real system from Cosberg)

1. Motivation

	Discrete Deterministic Model	Discrete Exponential Model	Continuous Deterministic Model	Continuous approximation of Discrete Deterministic Model
Machine processing times	Deterministic; equal processing time for all the machines	Stochastic	Deterministic; each machine can have different processing times	Deterministic; each machine can have different processing times
Buffer	Finite and discrete capacity	Finite and discrete capacity	Finite and continuous capacity	Finite and continuous capacity
Flow of parts	Discrete flow	Discrete flow	Continuous flow	Continuous approximation of discrete flow
Machine states	Multiple up (with equal processing time); Multiple down	Multiple up (with different processing time); Multiple down	Multiple up (with different processing time); Multiple down	Multiple up (with different processing time); Multiple down
Modelling features	Automated synchronous system	Manual asynchronous system	Automated asynchronous system; Large buffer capacity	Automated asynchronous system

1. Motivation

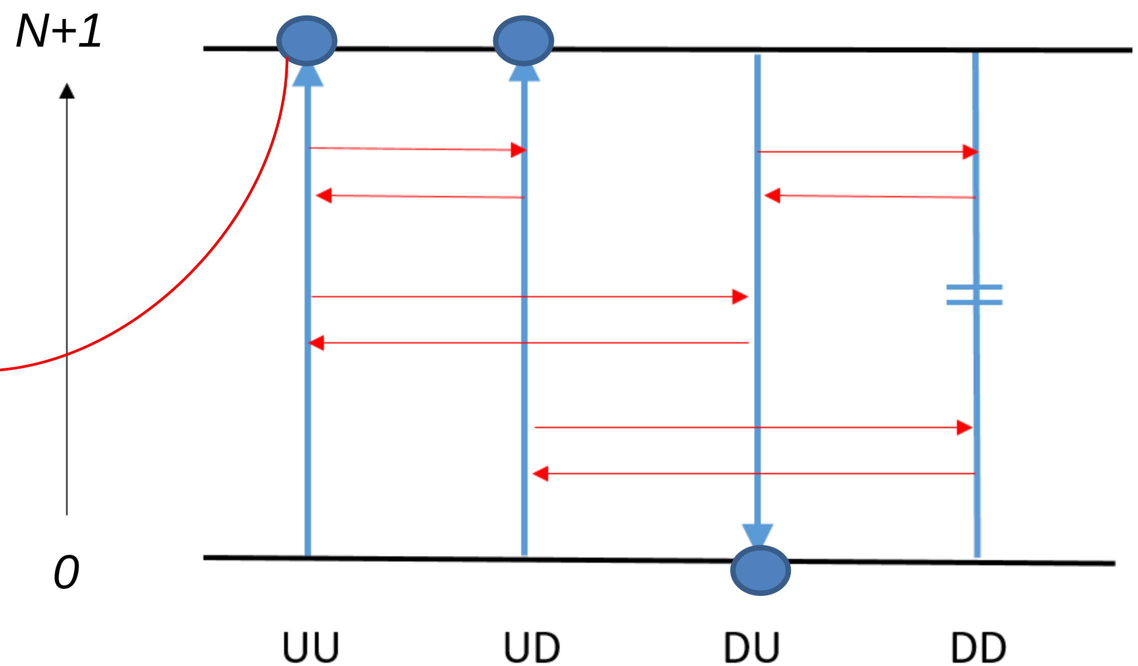


Slowdown state:

- ➢ How to treat it in a continuous decomposition method?
- ➢ Which dynamics are we capturing by propagating its effect?

- Single up model: production rate correction on the up state of machines;
- Multiple up model: combination of all possible slowdowns in each machine.

1. Motivation

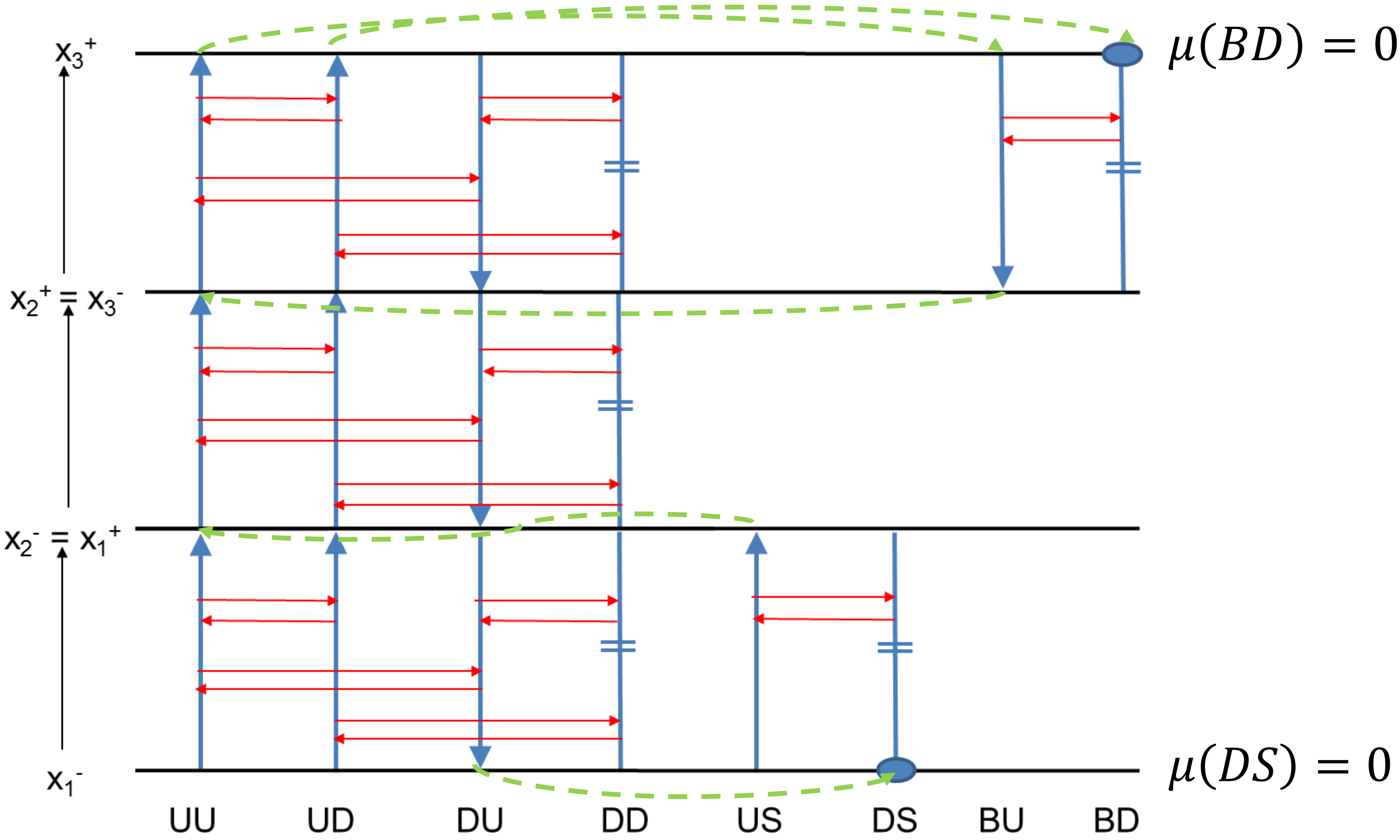


Probability of being in a (remote) slowdown state:

- What does it mean?

- Single up model: production rate correction on the up state of machines;
- Multiple up model: combination of all possible slowdowns in each machine.

1. Motivation



1. Motivation

Most of the existing automated production systems are asynchronous, and differences, even very small, among the deterministic production rates of the machines may result in impacts on overall system performance, that should be modelled explicitely.

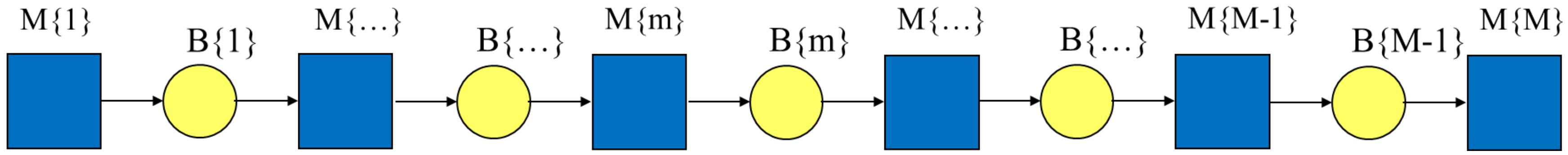
The **intent** of the present paper is to propose an approximate continuous model for deterministic asynchronous long machine lines with finite buffer capacity.

1. Motivation

- Approximation between continuous and discrete models:
  - Alvarez-Vargas et al. (1994) – Continuous models for production lines
  - De Koster and Wijngaard (1989) – Comparison of continuous and discrete
  - Suri and Fu (1994) – Continuous approximation of discrete production lines
- Continuous decomposition methods:
  - Gershwin and Schick (1980)
  - Burman (1995)
  - Le Bihan and Dallery (2000)
  - Gershwin and Burman (2000)
  - Levantesi et al. (2003)
  - Li, Meerkov (2009)
  - Colledani and Gershwin (2013)
- Some applications:
  - Helber (1999), Li and Huang (2005) – split/merge systems
  - Colledani and Tolio (2004), Zhao et al. (2014) – multiple part type
  - Chiang, Kuo, Meerkov (2001) – bottleneck identification

1. Motivation

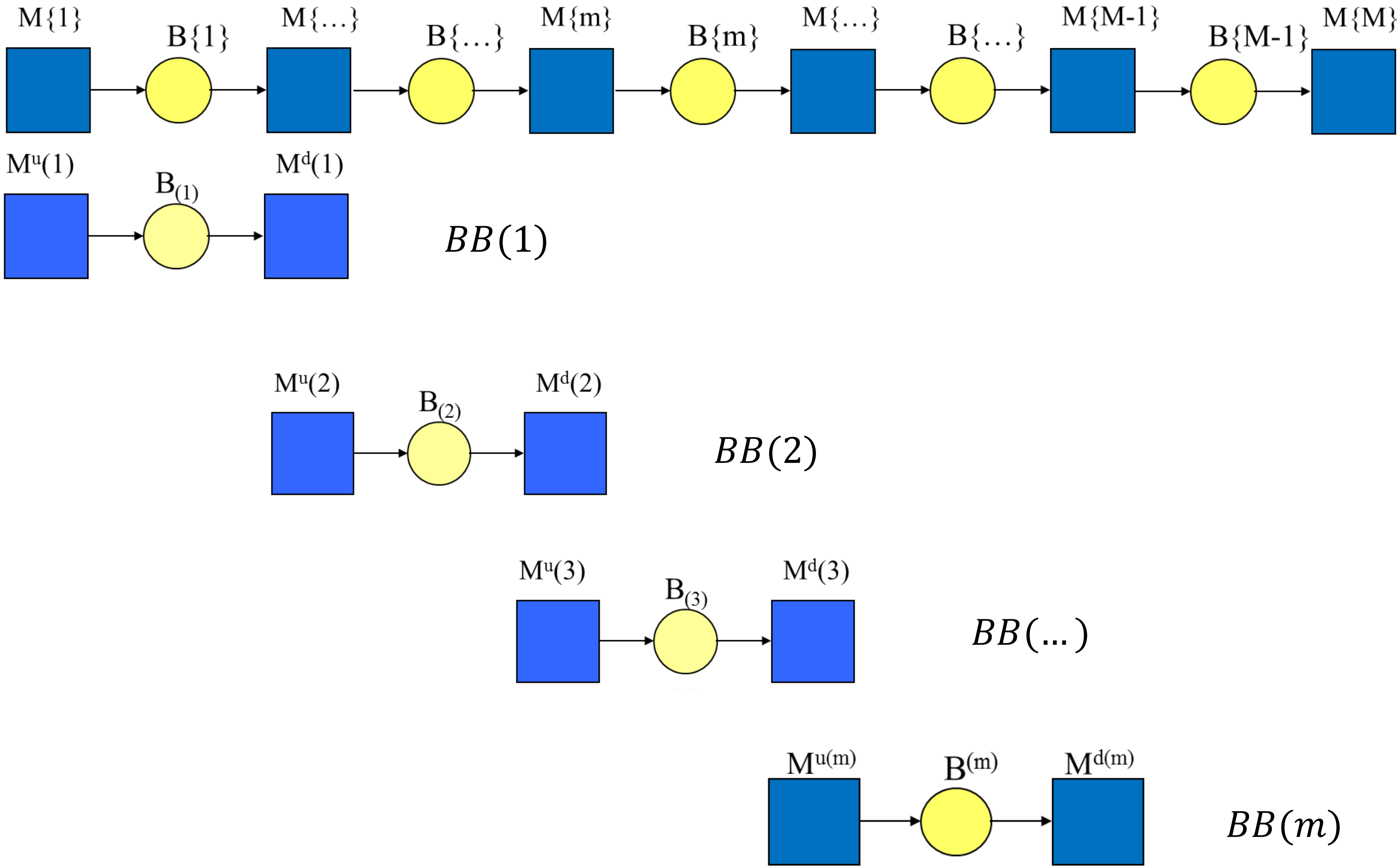
➢ Reference system



- parts are discrete and each machine processes one part at a time;
- the system is asynchronous i.e. each machine can start or finish a part at any time without synchronization with the other machine;
- processing times are deterministic and may be different between machines;
- the blocking discipline is Blocking After Service (BAS).
- each machine can be in one of the two states: up or down.
- Time To Failure and Time To Repair have exponential distributions.
- machine failures happen at the beginning of the operations on a part. Therefore when a failure happens there are no parts partially machined on the machine.

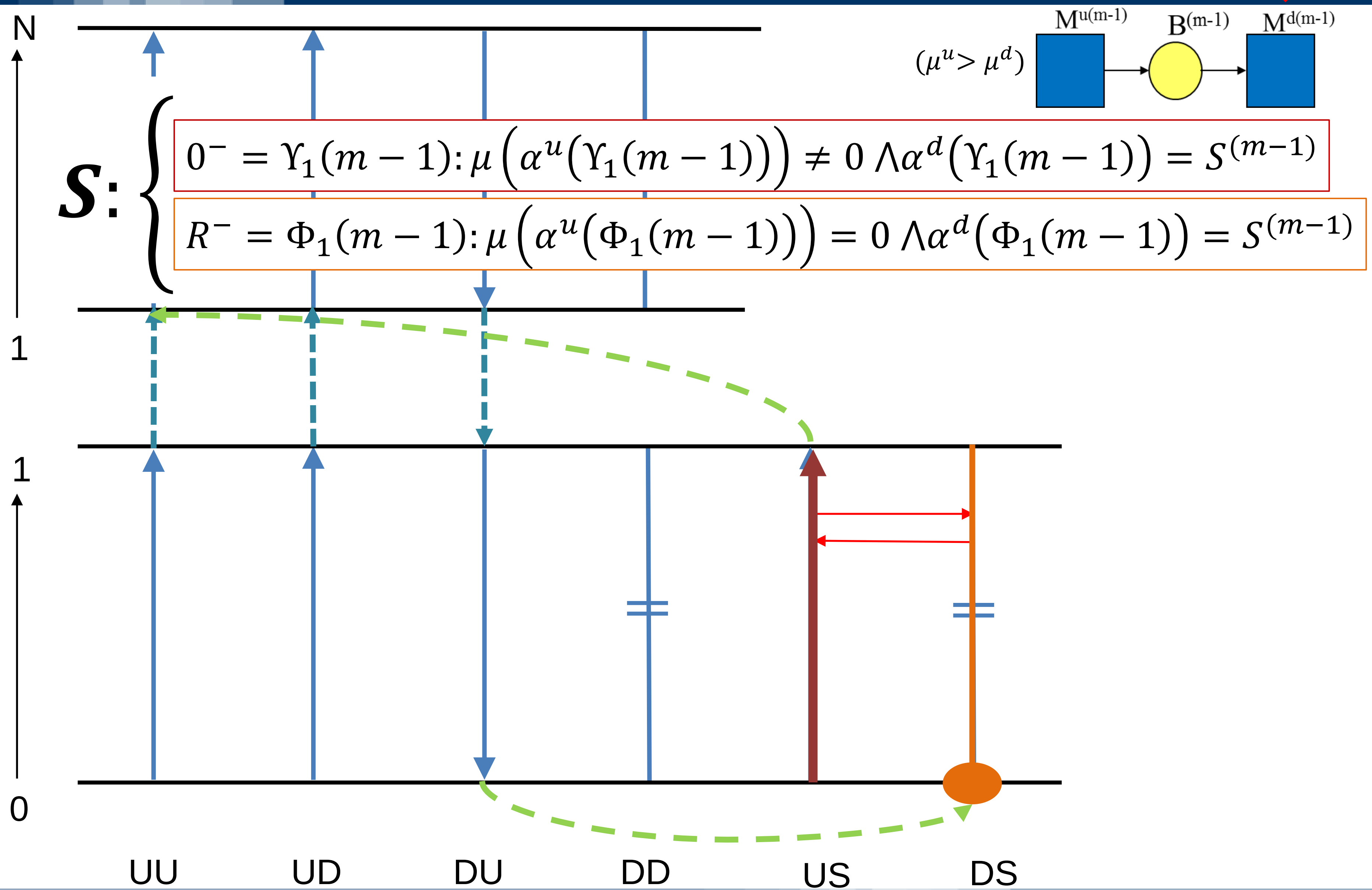
1. Motivation

➢ Decomposition of long line



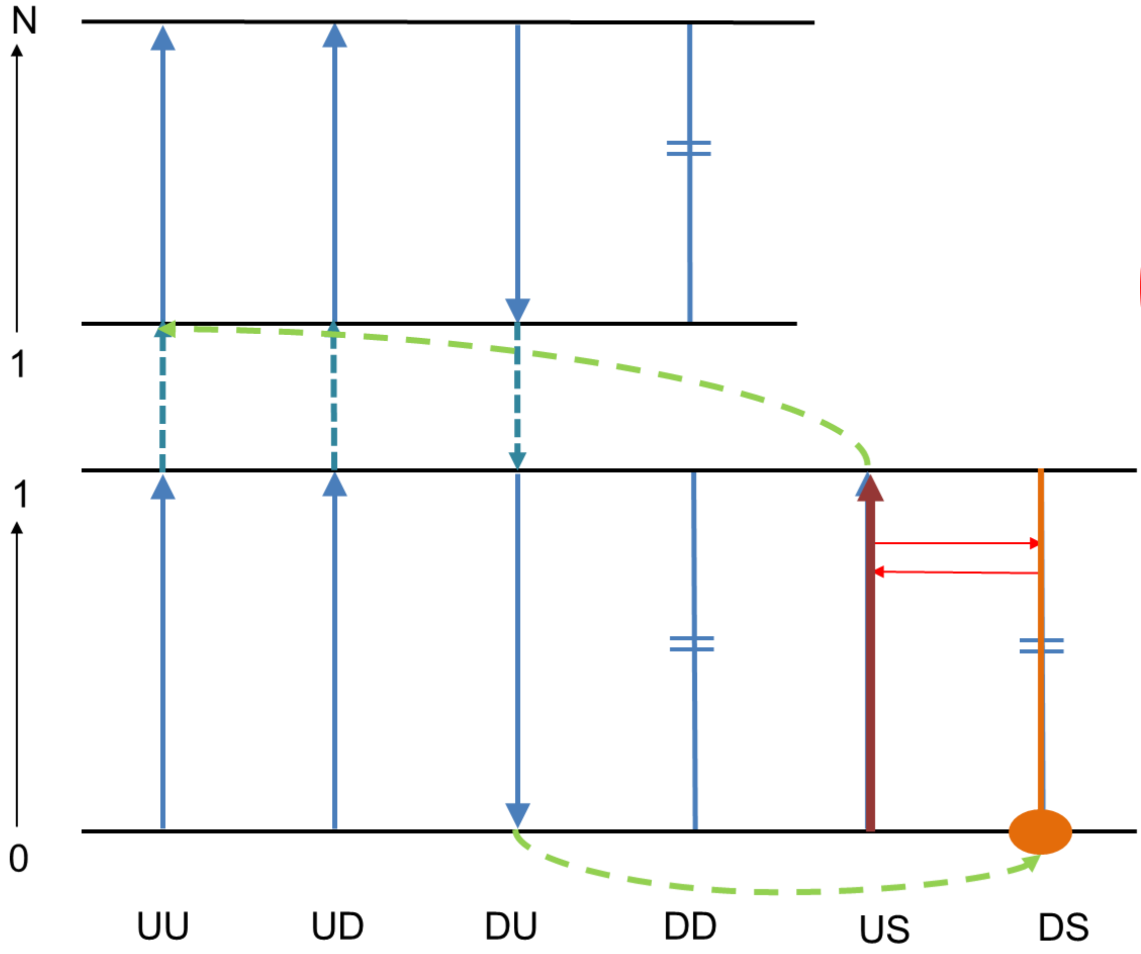
2. Outline of the approach

$$
S: \begin{cases} 0^{-} = \Upsilon_1(m-1): \mu\left(\alpha^{u}\left(\Upsilon_1(m-1)\right)\right) \neq 0 \wedge \alpha^{d}\left(\Upsilon_1(m-1)\right) = S^{(m-1)} \\ R^{-} = \Phi_1(m-1): \mu\left(\alpha^{u}\left(\Phi_1(m-1)\right)\right) = 0 \wedge \alpha^{d}\left(\Phi_1(m-1)\right) = S^{(m-1)} \end{cases}
$$

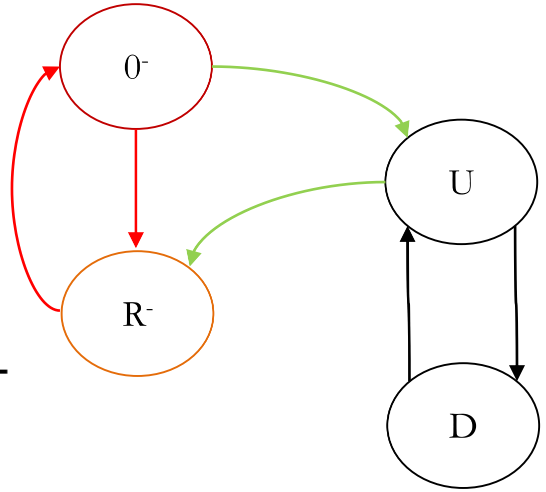


2. Outline of the approach

- Dynamics of ranges 1 and 2 of Building block BB(m-1)

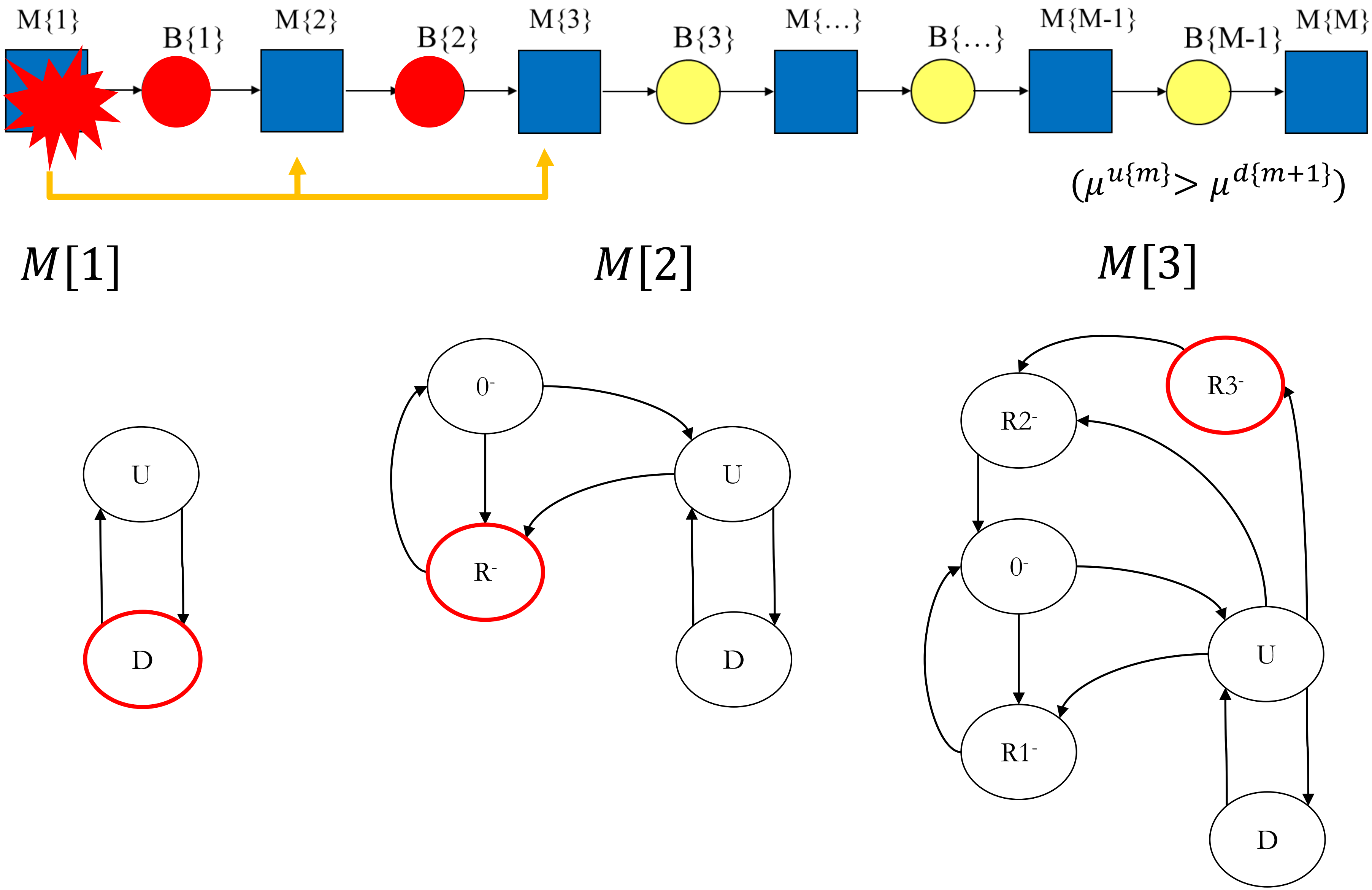


- Markov Chain of upstream pseudo-machine $M^{u(m)}$



Missing rates

2. Outline of the approach



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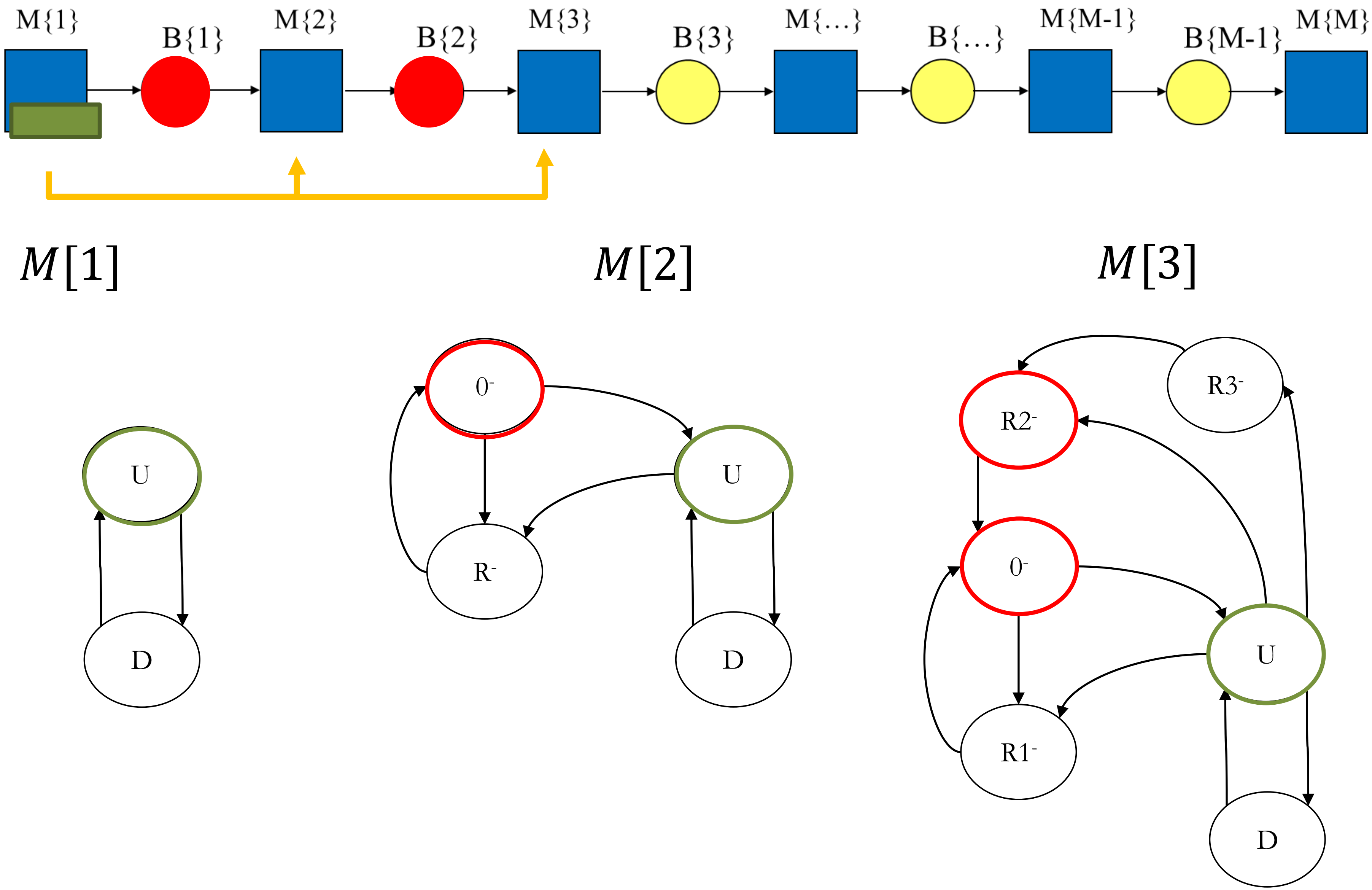
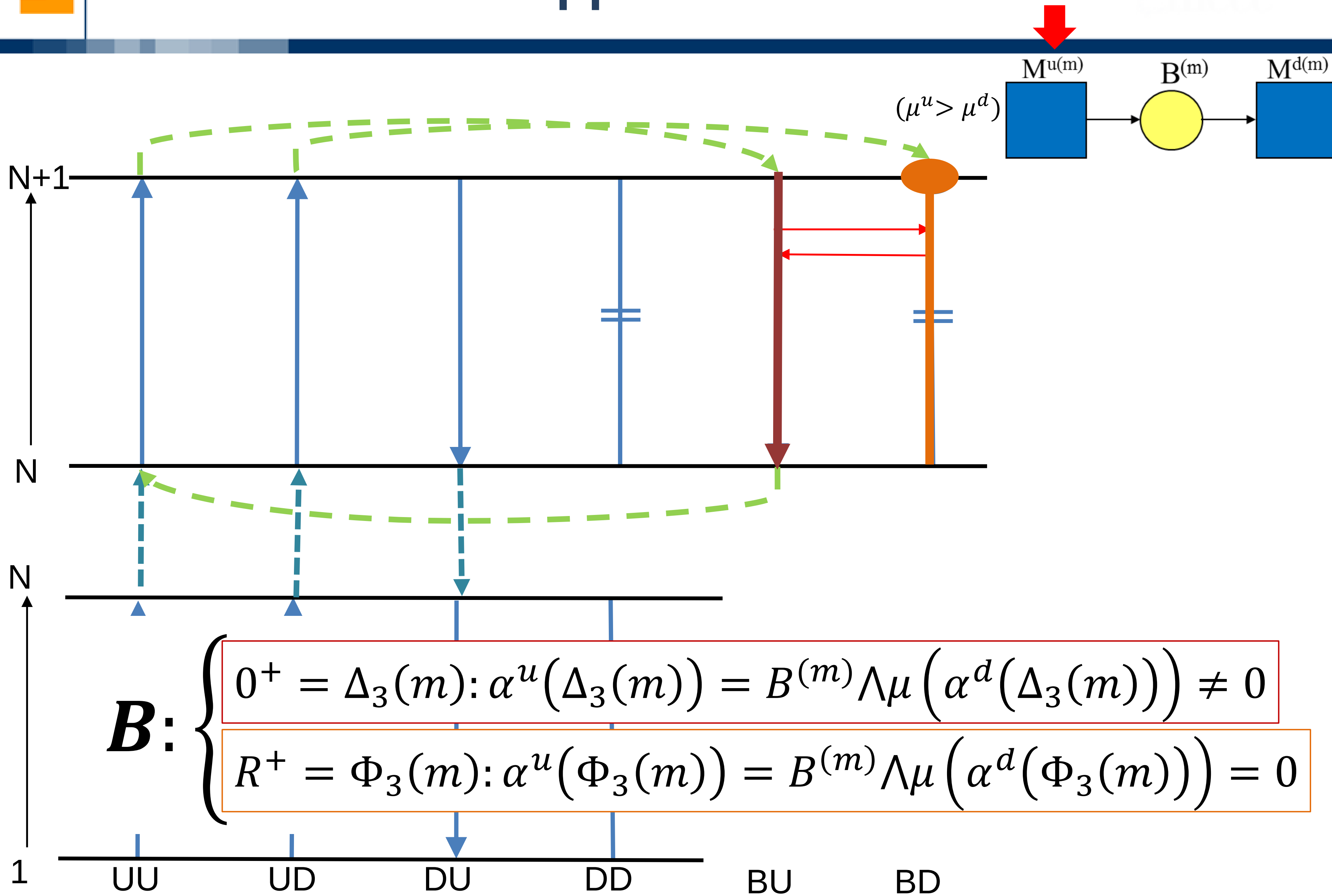


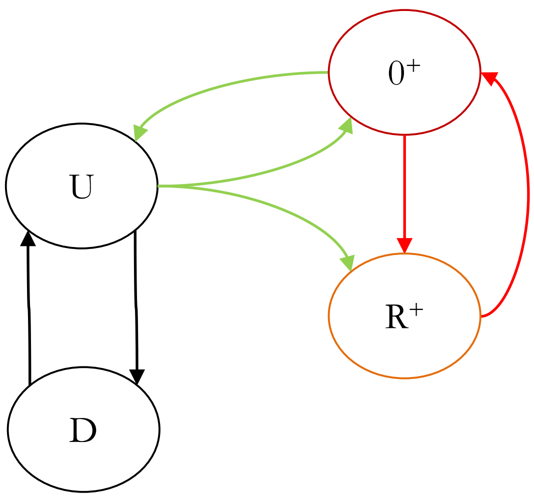
2. Outline of the approach



$$\boldsymbol{B}: \begin{cases} 0^+ = \Delta_3(m): \alpha^u\big(\Delta_3(m)\big) = B^{(m)} \Lambda \mu\Big(\alpha^d\big(\Delta_3(m)\big)\Big) \neq 0 \\ R^+ = \Phi_3(m): \alpha^u\big(\Phi_3(m)\big) = B^{(m)} \Lambda \mu\Big(\alpha^d\big(\Phi_3(m)\big)\Big) = 0 \end{cases}$$

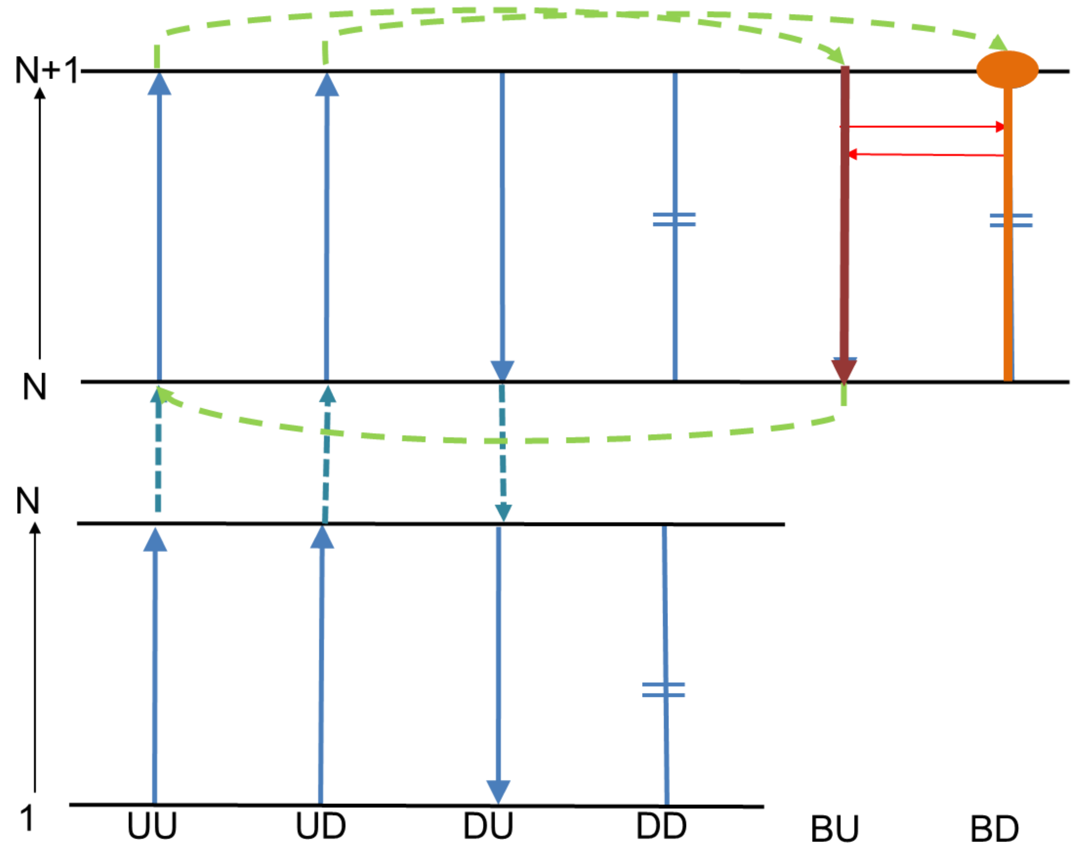
2. Outline of the approach

- Markov Chain of downstream pseudo-machine $M^{d(m-1)}$

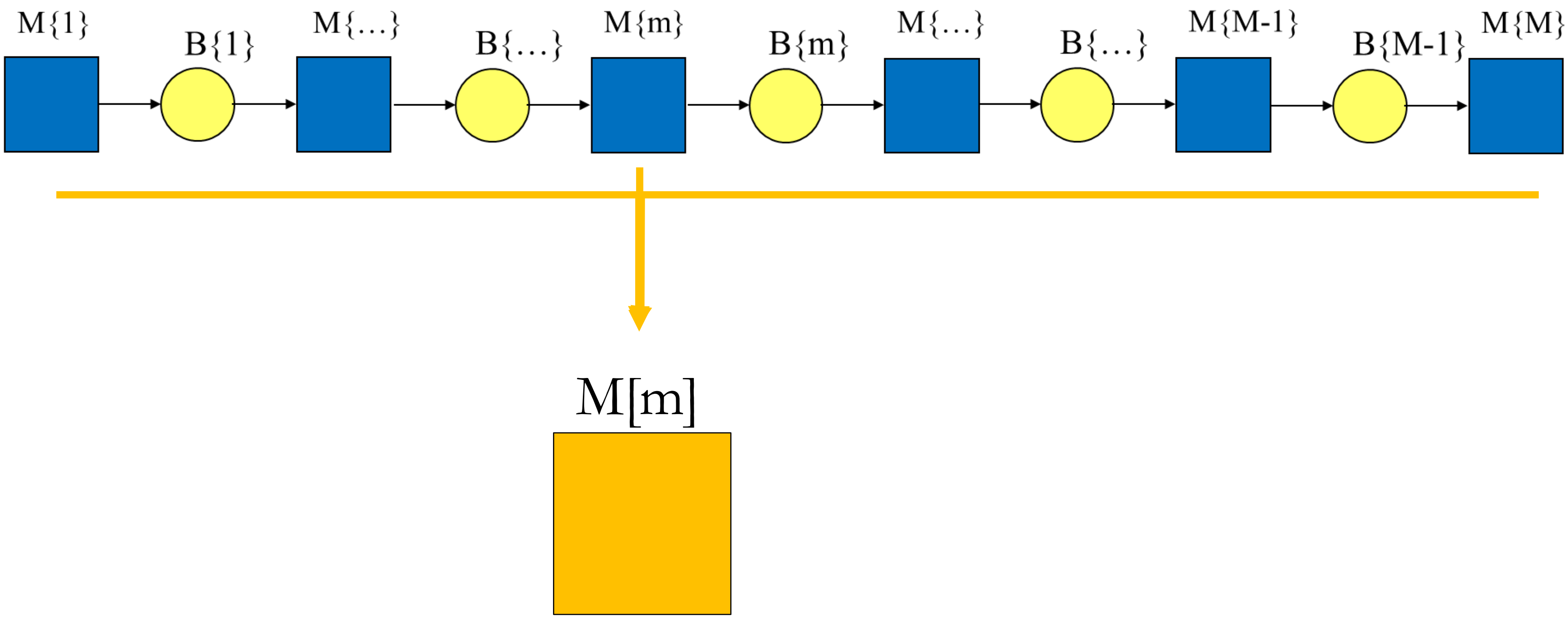


Missing rates

- Dynamics of ranges 2 and 3 of Building block BB(m)

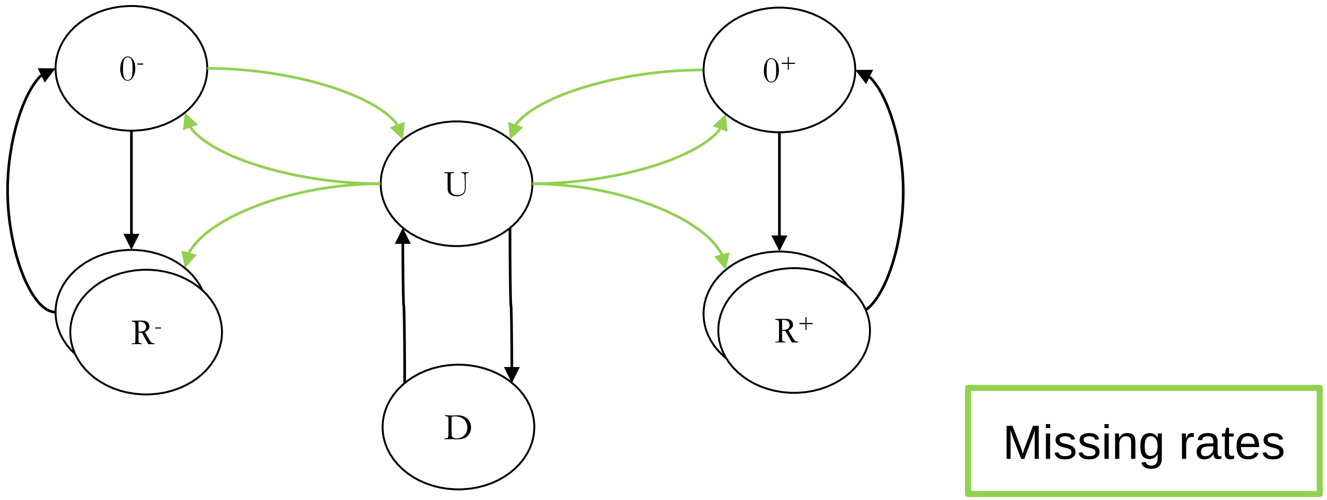


2. Outline of the approach



An **Integrated Machine** M[m] adds to the behavior of the corresponding machine M{m} of the original line (Local states) the starvation (*S*) and blocking states (*B*) which represent the interaction of the machine with the rest of the system.

- **Characterization of M[m]**



- Given the continuous nature of the model, a machine cannot be contemporarily starved and blocked therefore starvation and blocking states are indeed separate states.

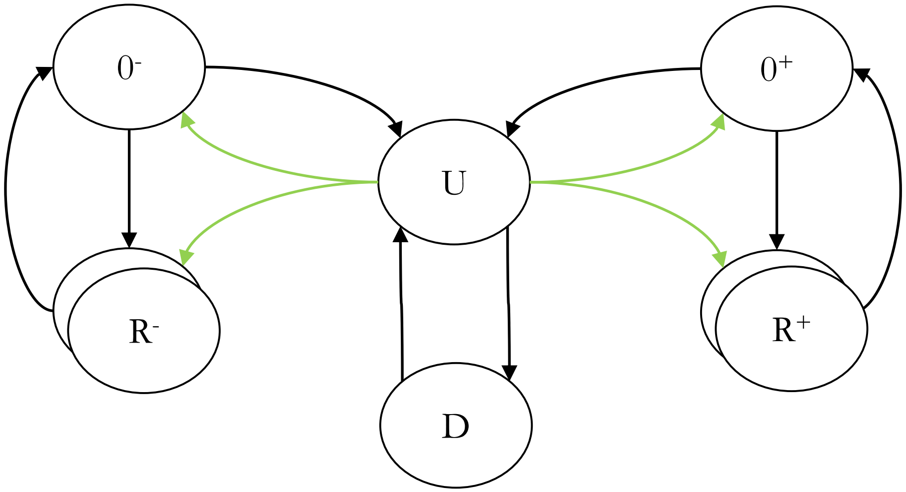
- It is impossible to go directly from an upstream limitation to a downstream limitation (or vice versa) without first being back in the (local) operational state, because starvation and blocking depend on the level of the neighboring buffers and the only way to get into blocking or starvation is that machine M[m] produces parts.

3. Decomposition equations

➢ **First set: entering the limiting states**

$$q_{Uj,j\in S} = \frac{g\left(x_1^{-(m-1)}, S_1^{u(m-1)} S_1^{d(m-1)}\right)}{\Pi_U^{m-1}} \qquad \Pi_U^{(m-1)} = \sum_{h=1}^{3}\left(\int_{x=x_h^-}^{x_h^+} f\left(x, \alpha^d\left(S_h^{(m-1)}\right) = U\right) dx\right)$$

$$q_{Uj,j\in B} = \frac{g\left(x_3^{+(m)}, S_3^{u(m)} S_3^{d(m)}\right)}{\Pi_U^{m}} \qquad \Pi_U^{(m)} = \sum_{h=1}^{3}\left(\int_{x=x_h^-}^{x_h^+} f\left(x, \alpha^u\left(S_h^{(m)}\right) = U\right) dx\right)$$

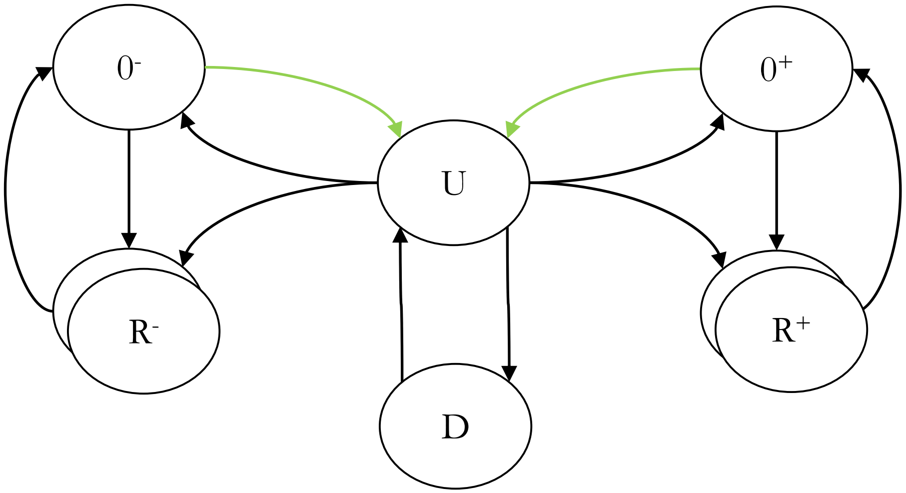


3. Decomposition equations

➢ Second set: exiting the limiting states

$$q_{jU,j\in\boldsymbol{S}} = \frac{g\left(x_1^{+(m-1)}, S_1^{u(m-1)} S_1^{d(m-1)}\right)}{\Pi_j^{(m-1)}}$$

$$q_{jU,j\in\boldsymbol{B}} = \frac{g\left(x_3^{-(m)}, S_3^{u(m)} S_3^{d(m)}\right)}{\Pi_j^{(m)}}$$



$$\Pi_j^{(m-1)} = \Pi\left(S_1^{(m-1)} = j\right) = \int_{x=x_1^-}^{x_1^+} f\left(x, S_1^{(m-1)} = j\right) dx + \pi\left(\Theta_1^{-(m-1)}(S_1^{(m-1)} = j)\right)$$

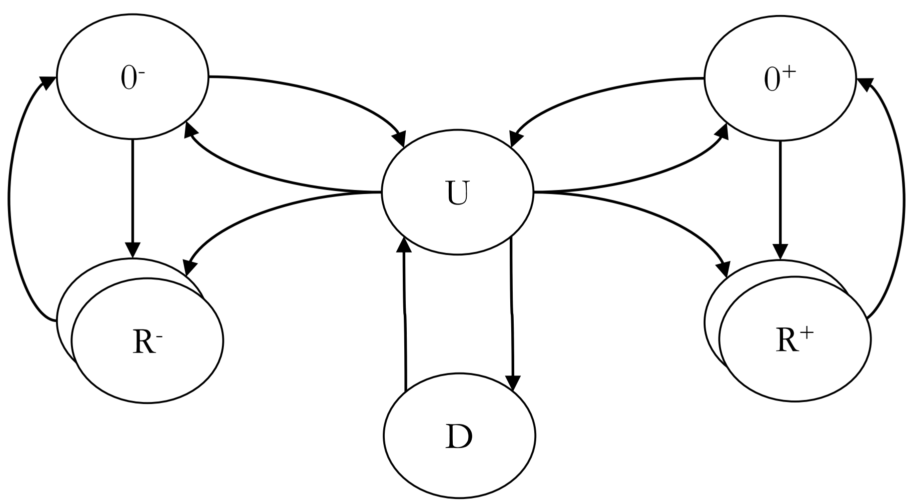
$$\Pi_j^{(m)} = \Pi\left(S_3^{(m)} = j\right) = \int_{x=x_3^-}^{x_3^+} f\left(x, S_3^{(m)} = j\right) dx + \pi\left(\Theta_3^{+(m)}(S_3^{(m)} = j)\right)$$

3. Decomposition equations

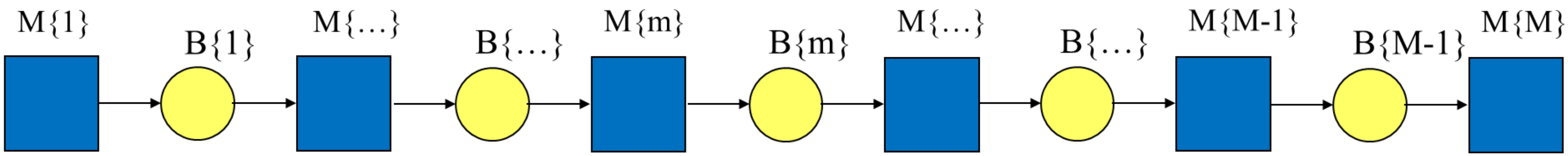
- **Third set: transitions among limiting states**

$$Q_{\boldsymbol{SS}} = Q^{u(m-1)}$$

$$Q_{\boldsymbol{BB}} = Q^{d(m)}$$

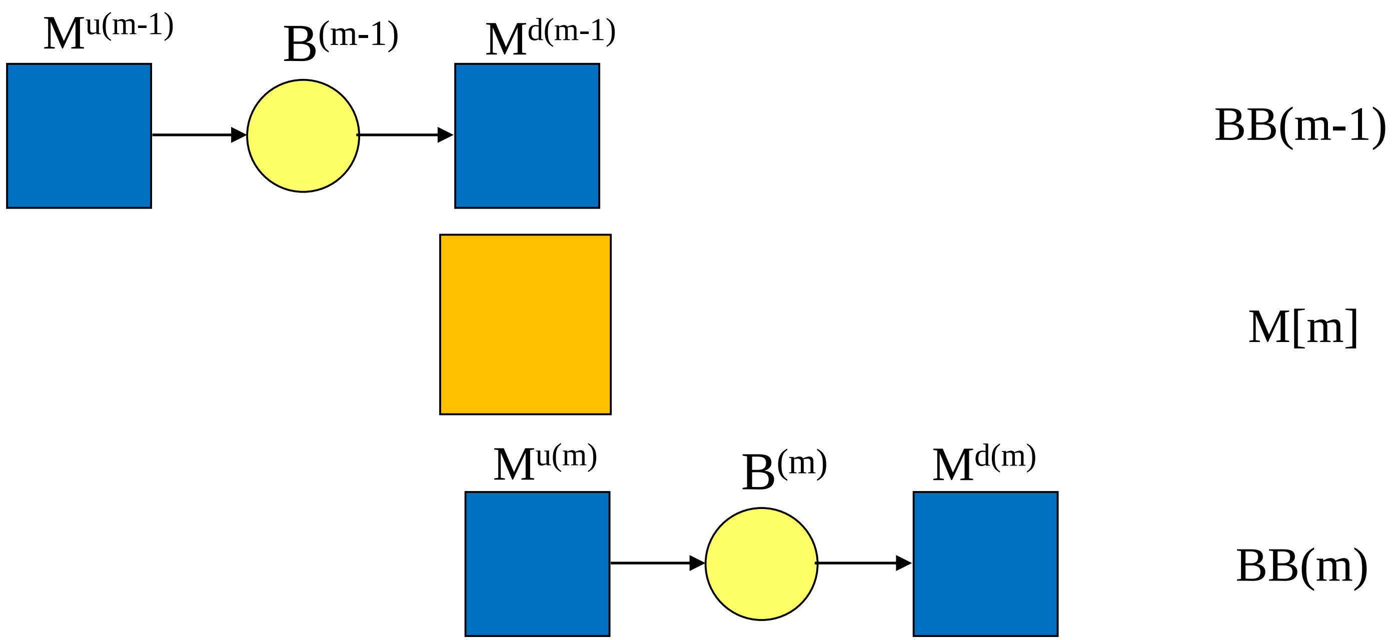


2. Outline of the approach



→ **Two-level decomposition**

- Buffer-level (*Building Blocks*)
- Machine-level (*Integrated Machines*)



2. Outline of the approach

The application of the proposed decomposition technique consists of three steps:

- STEP 1: Characterization of two-machine lines (*building blocks*) based on Tolio (2017).

- STEP 2: Characterization of machines (*integrated machines*) at system level, by means of decomposition equations.

- STEP 3: Application of an algorithm to solve decomposition equations efficiently.

2. Outline of the approach

- **1. Building block evaluation**: from T. Tolio (2017)

Performance:

- Probability density $f(x_h, S^u S^d)$ representing the probability density function of joint states within the various buffer ranges;
- Probability mass $\pi(\Theta(m))$: representing the probability masses of boundary states $\Theta(m)$;
- Probability flow $g(x_h, S^u S^d)$ representing the probability flow exiting the joint states $S^u S^d$.

2. Outline of the approach

The application of the decomposition technique consists of three fundamental steps:

• STEP 1: Characterization of two-machine lines (*building blocks*) based on Tolio (2017).

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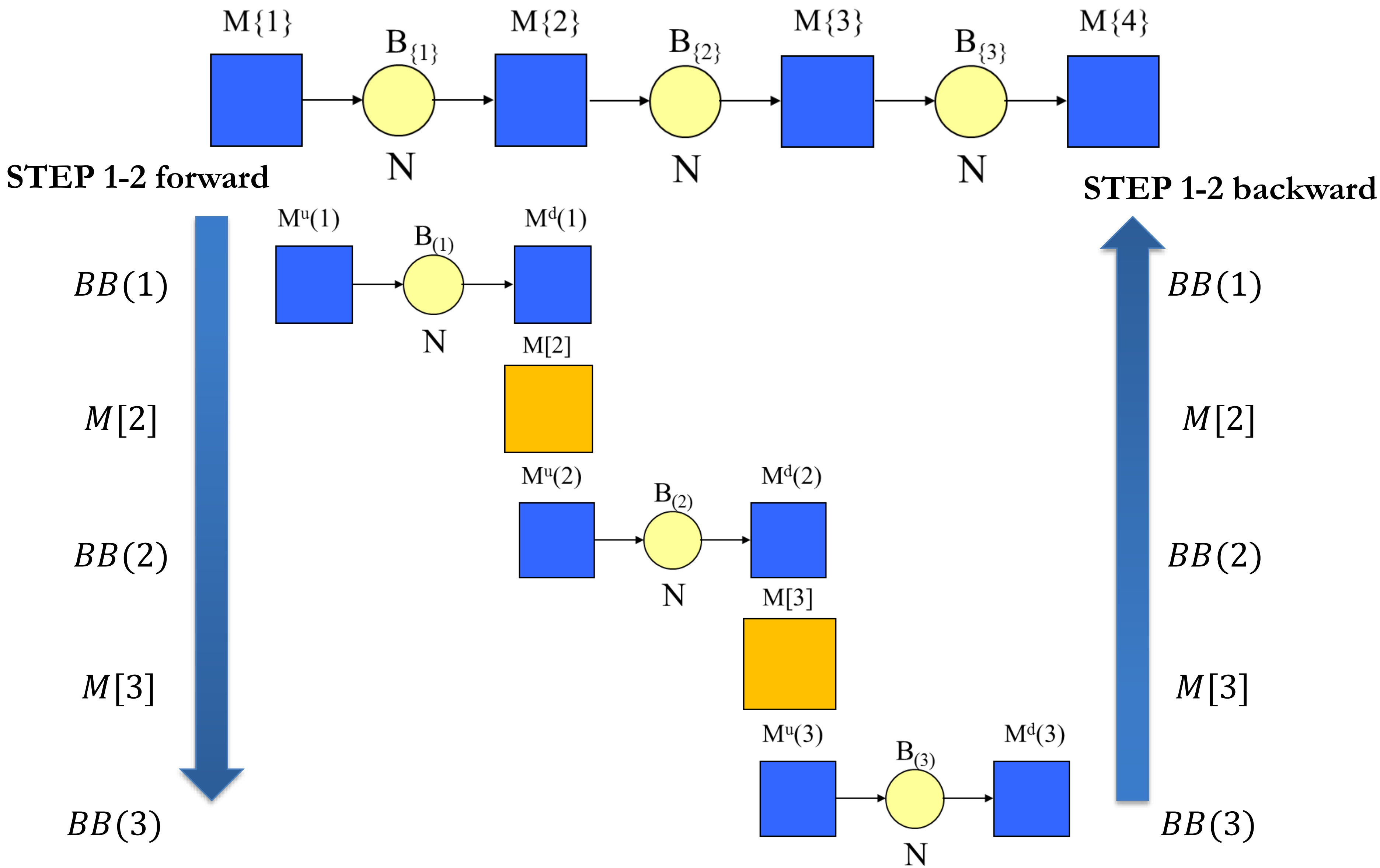
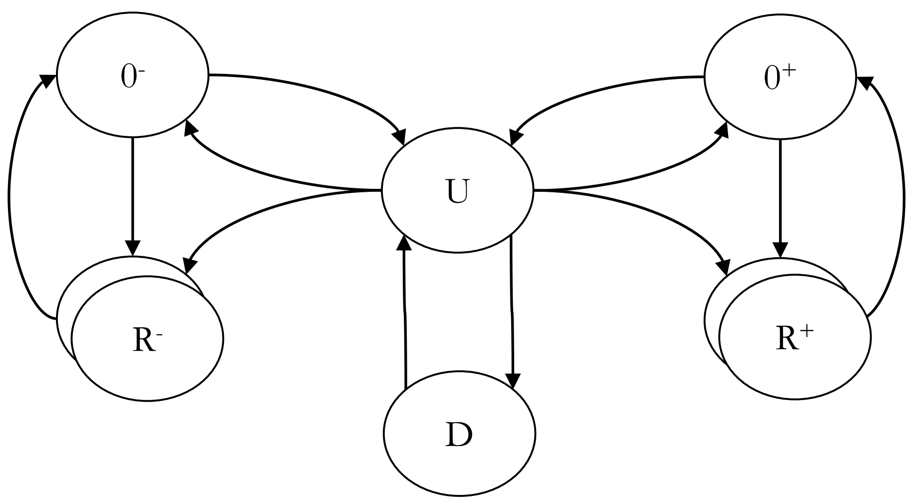


3. Decomposition equations

➢ **Transition rate matrix of Integrated Machine M[m]**

$$Q^{[m]} = \begin{bmatrix} Q_{\mathcal{L}\mathcal{L}} & Q_{\mathcal{L}\mathcal{S}} & Q_{\mathcal{L}\mathcal{B}} \\ Q_{\mathcal{S}\mathcal{L}} & Q_{\mathcal{S}\mathcal{S}} & Q_{\mathcal{S}\mathcal{B}} \\ Q_{\mathcal{B}\mathcal{L}} & Q_{\mathcal{B}\mathcal{S}} & Q_{\mathcal{B}\mathcal{B}} \end{bmatrix} = \begin{bmatrix} Q_{\mathcal{L}\mathcal{L}} & \begin{bmatrix} Q_{\mathcal{L}0^-} & Q_{\mathcal{L}R^-} \end{bmatrix} & \begin{bmatrix} Q_{\mathcal{L}0^+} & Q_{\mathcal{L}R^+} \end{bmatrix} \\ \begin{bmatrix} Q_{0^-\mathcal{L}} \\ 0 \end{bmatrix} & \begin{bmatrix} Q_{0^-0^-} & Q_{0^-R^-} \\ Q_{R^-0^-} & Q_{R^-R^-} \end{bmatrix} & 0 \\ \begin{bmatrix} Q_{0^+\mathcal{L}} \\ 0 \end{bmatrix} & 0 & \begin{bmatrix} Q_{0^+0^+} & Q_{0^+R^+} \\ Q_{R^+0^+} & Q_{R^+R^+} \end{bmatrix} \end{bmatrix}$$

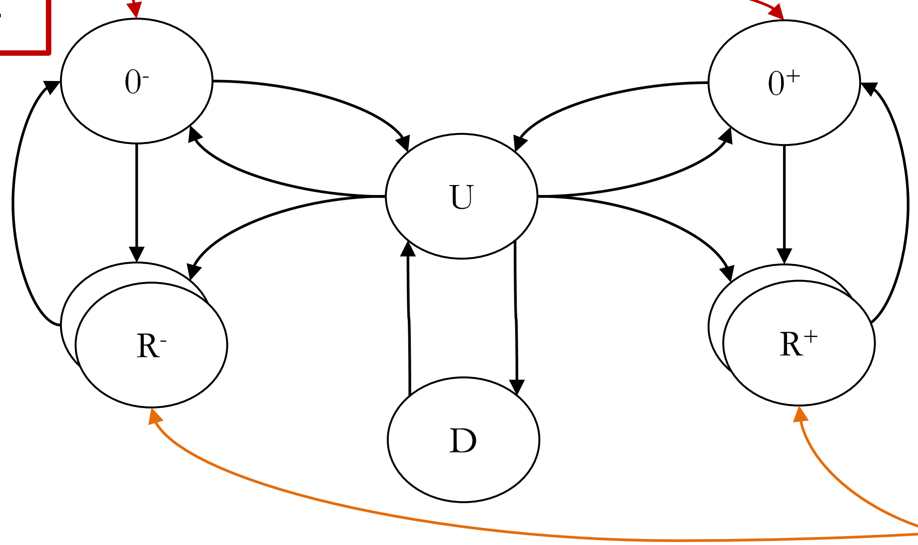


3. Decomposition equations

➢ **Transition rate matrix of Integrated Machine M[m]**

$$Q^{[m]} = \begin{bmatrix} Q_{\mathcal{LL}} & Q_{\mathcal{LS}} & Q_{\mathcal{LB}} \\ Q_{\mathcal{SL}} & Q_{\mathcal{SS}} & Q_{\mathcal{SB}} \\ Q_{\mathcal{BL}} & Q_{\mathcal{BS}} & Q_{\mathcal{BB}} \end{bmatrix} = \begin{bmatrix} Q_{\mathcal{LL}} & \begin{bmatrix} Q_{\mathcal{L}0^-} & Q_{\mathcal{L}\mathcal{R}^-} \end{bmatrix} & \begin{bmatrix} Q_{\mathcal{L}0^+} & Q_{\mathcal{L}\mathcal{R}^+} \end{bmatrix} \\ \begin{bmatrix} Q_{0^-\mathcal{L}} \\ 0 \end{bmatrix} & \begin{bmatrix} Q_{0^-0^-} & Q_{0^-\mathcal{R}^-} \\ Q_{\mathcal{R}^-0^-} & Q_{\mathcal{R}^-\mathcal{R}^-} \end{bmatrix} & 0 \\ \begin{bmatrix} Q_{0^+\mathcal{L}} \\ 0 \end{bmatrix} & 0 & \begin{bmatrix} Q_{0^+0^+} & Q_{0^+\mathcal{R}^+} \\ Q_{\mathcal{R}^+0^+} & Q_{\mathcal{R}^+\mathcal{R}^+} \end{bmatrix} \end{bmatrix}$$

Waiting time of first part's production from machines ±1

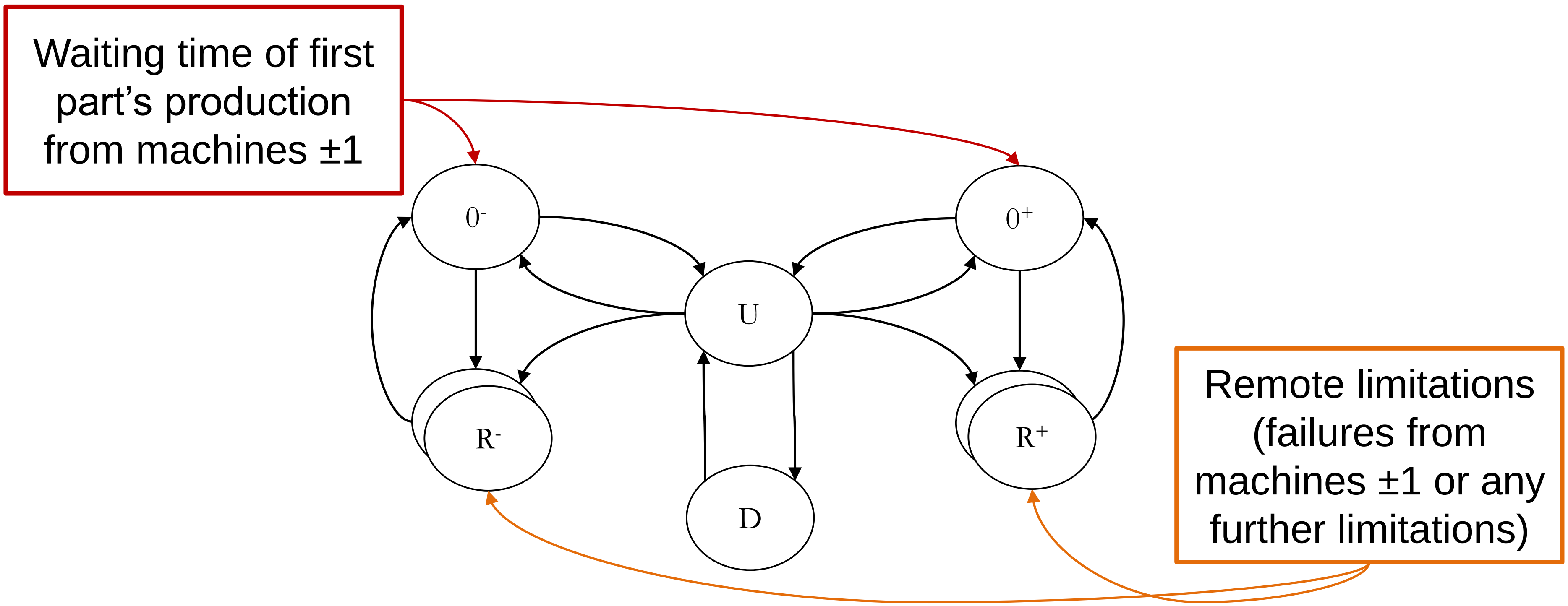


Remote limitations (failures from machines ±1 or any further limitations)

3. Decomposition equations

➢ **Transition rate matrix of Integrated Machine M[m]**

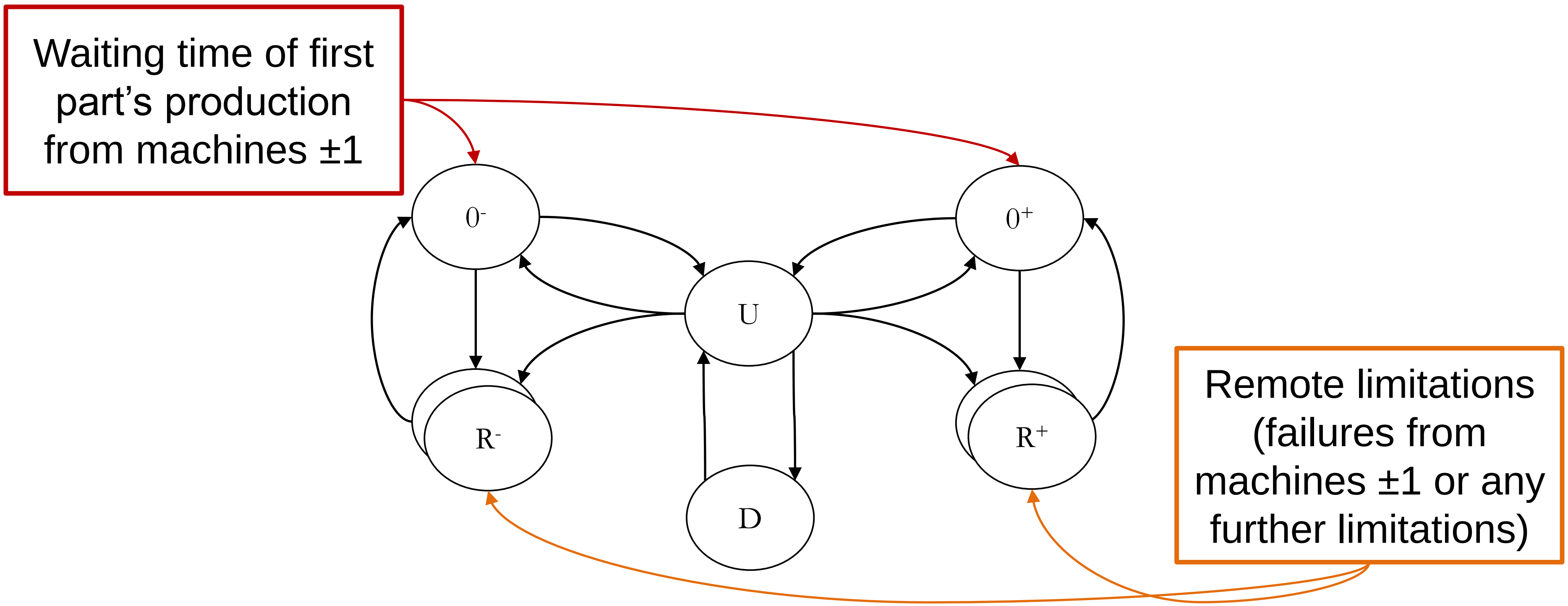
$$Q_{SS} = \begin{bmatrix} Q_{0^- 0^-} & Q_{0^- R^-} \\ Q_{R^- 0^-} & Q_{R^- R^-} \end{bmatrix} = \begin{bmatrix} Q_{\mathcal{LL}}^{[m-1]} & Q_{\mathcal{LS}}^{[m-2]} & Q_{\mathcal{LS}}^{[m-j]} & Q_{\mathcal{LS}}^{[1]} \\ Q_{\mathcal{SL}}^{[m-2]} & Q_{\mathcal{LL}}^{[m-2]} & Q_{\mathcal{LS}}^{[m-2]} & Q_{\mathcal{LS}}^{[m-2]} \\ 0 & Q_{\mathcal{SL}}^{[m-j]} & Q_{\mathcal{LL}}^{[m-j]} & Q_{\mathcal{LS}}^{[m-j]} \\ 0 & 0 & Q_{\mathcal{SL}}^{[1]} & Q_{\mathcal{LL}}^{[1]} \end{bmatrix}$$



3. Decomposition equations

- **Transition rate matrix of Integrated Machine M[m]**

$$Q_{\mathcal{BB}} = \begin{bmatrix} Q_{0^+0^+} & Q_{0^+R^+} \\ Q_{R^+0^+} & Q_{R^+R^+} \end{bmatrix} = \begin{bmatrix} Q_{\mathcal{LL}}^{[m+1]} & Q_{\mathcal{LB}}^{[m+2]} & Q_{\mathcal{LB}}^{[m+j]} & Q_{\mathcal{LB}}^{[M]} \\ Q_{\mathcal{BL}}^{[m+2]} & Q_{\mathcal{LL}}^{[m+2]} & Q_{\mathcal{LB}}^{[m+2]} & Q_{\mathcal{LB}}^{[m+2]} \\ 0 & Q_{\mathcal{BL}}^{[m+j]} & Q_{\mathcal{LL}}^{[m+j]} & Q_{\mathcal{LB}}^{[m+j]} \\ 0 & 0 & Q_{\mathcal{BL}}^{[M]} & Q_{\mathcal{LL}}^{[M]} \end{bmatrix}$$



4. Numerical results

Parameters

Case i	p_1	r_1	p_2	r_2	p_3	r_3	μ_1	μ_2	μ_3	$N_{\{1,2\}}$
Case 1	0.012	0.15	0.01	0.1	0.005	0.04	3	2.95	2.9	3
Case 2	0.012	0.15	0.01	0.1	0.005	0.04	3	2.95	2.9	10
Case 3	0.012	0.15	0.01	0.1	0.005	0.04	2.9	2.95	3	10

(DES model run on Arena)

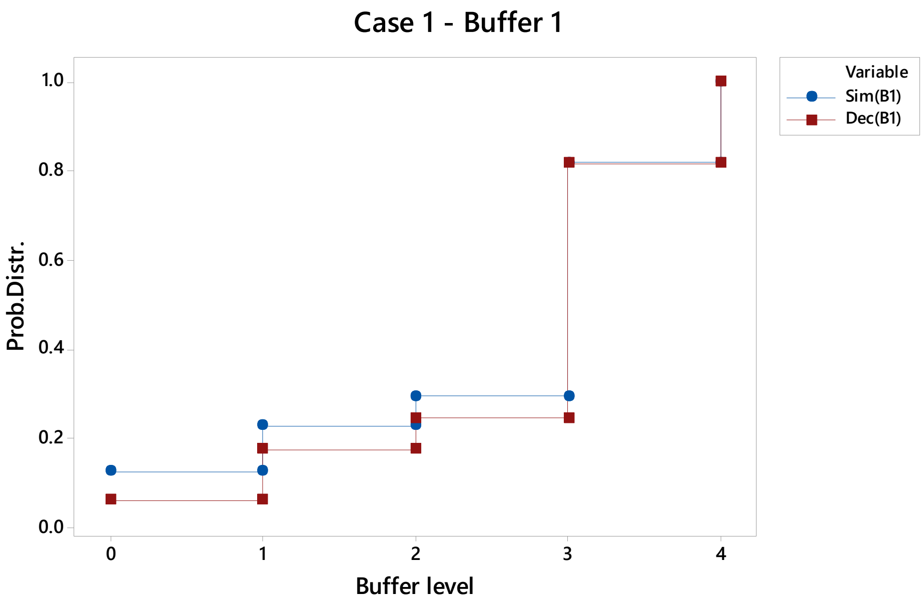
Throughput

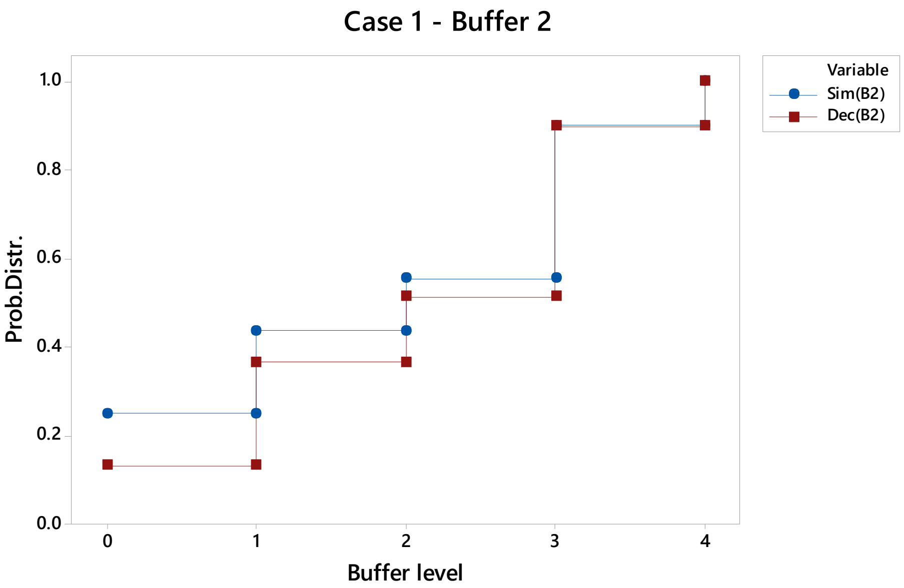
Case i	Sim	Dec	err%
Case 1	2.271	2.265	0.26
Case 2	2.312	2.312	¡0.01
Case 3	2.304	2.303	0.02

Av.Buffer Level

Case i	$N1$(Sim)	$N1$(Dec)	err	$N2$(Sim)	$N2$(Dec)	err
Case 1	2.57	2.71	-0.04	1.86	2.09	-0.23
Case 2	7.24	7.44	-0.02	4.48	4.72	-0.02
Case 3	5.03	4.85	0.02	2.33	2.07	0.02

4. Numerical results





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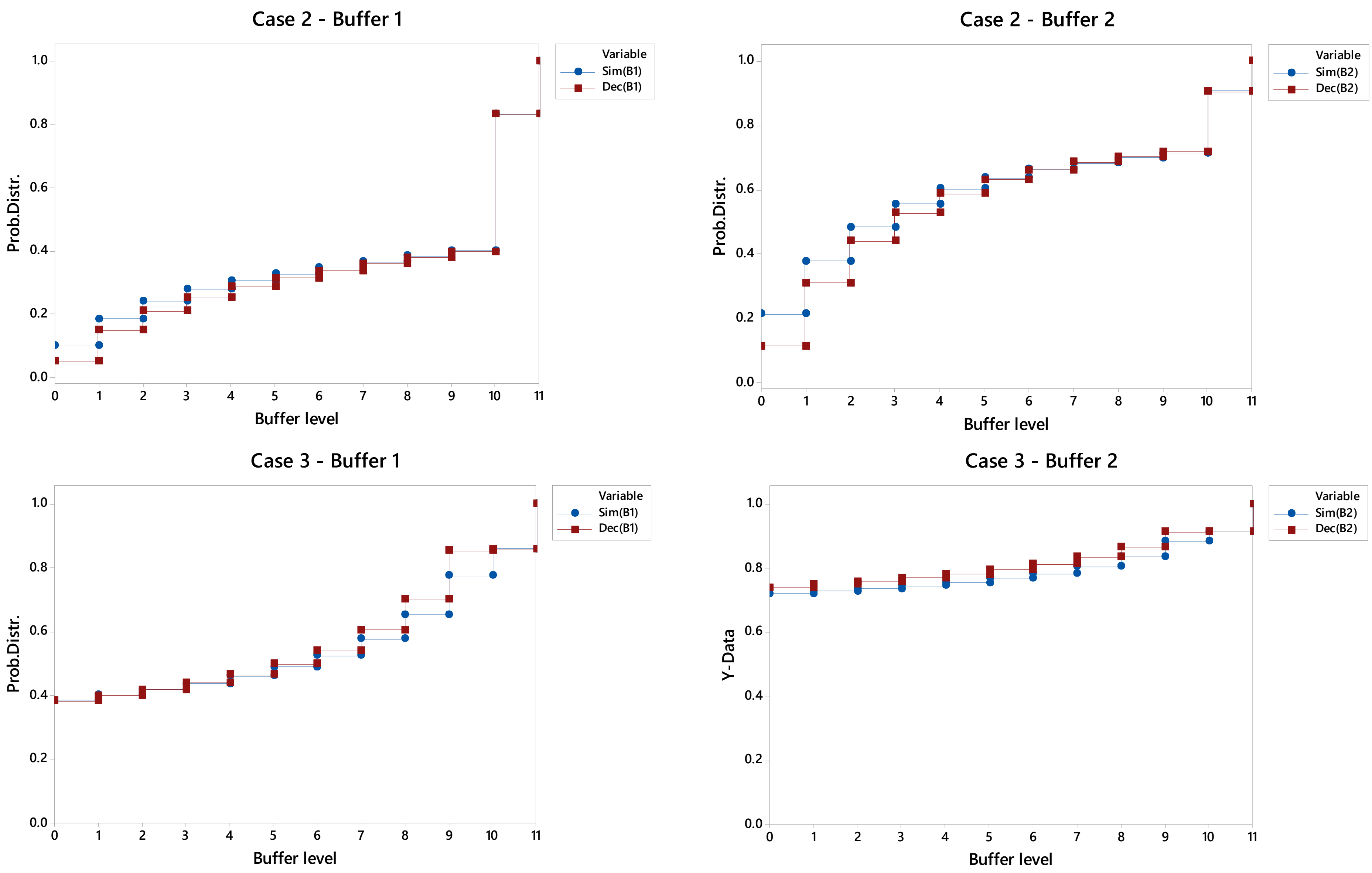

4. Numerical results

Parameters											Throughput			
Case i	p_1	r_1	p_2	r_2	p_3	r_3	μ_1	μ_2	μ_3	$N_{\{1,2\}}$	Case i	Sim	Dec	err%
Case 1	0.012	0.15	0.01	0.1	0.005	0.04	3	2.95	2.9	3	Case 1	2.271	2.265	0.26
Case 2	0.012	0.15	0.01	0.1	0.005	0.04	3	2.95	2.9	10	Case 2	2.312	2.312	¡0.01
Case 3	0.012	0.15	0.01	0.1	0.005	0.04	2.9	2.95	3	10	Case 3	2.304	2.303	0.02

(simulation model run on Arena)

- **Case 1: steady state probabilities of M[m], m=1,2,3**

M[1]				M[2]				M[3]			
	cause	state	M1		cause	state	prob.		cause	state	prob.
		up	0.7551		*local*	*up*	0.7679		*local*	*up*	0.7811
		down	0.0604		*local*	*down*	0.0768		*local*	*down*	0.0976
downstream limitation	*M2-up*	*0+*	0.0129	upstream limitation	*M1-up*	*0-*	0.0025	upstream limitation	*M2-up*	*0-*	0.0047
	M2-down	*R+*	0.0740		*M1-down*	*R-*	0.0511		*M2-down*	*R-*	0.0698
	M3-up	*R+*	0.0038	downstream limitation	*M3-up*	*0+*	0.0064		*M1-up*	*R-*	0.0023
	M3-down	*R+*	0.0938		*M3-down*	*R+*	0.0953		*M1-down*	*R-*	0.0444

5. Conclusions and future research

- **Conclusion:**
  The approach seems to guarantee fairly precise approximation of the performance of asynchronous automated lines producing discrete parts.
  - ✓ Extensive testing and validation needed.

- **Future research:**

- Integrated Machine structure gives an intuition about the behavior of the real machine (time to starvation, time to blocking, resumption of flow…).
- Performance evaluation with production and transportation batches, including supply chains;
- System topology: assembly/disassembly systems, split/merge…

THANK YOU

…questions?