SMMSO 2017
Stochastic Models for Manufacturing and Service Operations

The Decomposition Equations of Serial Flow Lines with Multiple Exponential Unreliable Non-identical Parallel-machine Workstations

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PRESENTATION OUTLINE

- PROBLEM DEFINITION
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- THE DECOMPOSITION ALGORITHM
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PROBLEM DEFINITION

- The objective of this study is the analysis of a large production line with parallel machines at each work station like the one depicted in Figure 1, using the decomposition method.

- The difference of the proposed approach from other approaches, is that each parallel-machine workstation is not replaced by an equivalent workstation. That is, the decomposition approach is applied directly to each one of the parallel machines for each workstation without using replacement techniques.

- The exact analysis of a production line with parallel machines at each workstation is impossible due to the enormous state space which leads to huge equation systems that are difficult to be solved.

A PRODUCTION LINE WITH PARALLEL MACHINES AT EACH WORK STATION

- In the system depicted in Figure 1 each workstation M_i consists of S_i unreliable non identical parallel servers.
- Each one of the parallel servers at each workstation performs an operation and the produced part is placed into the downstream buffer

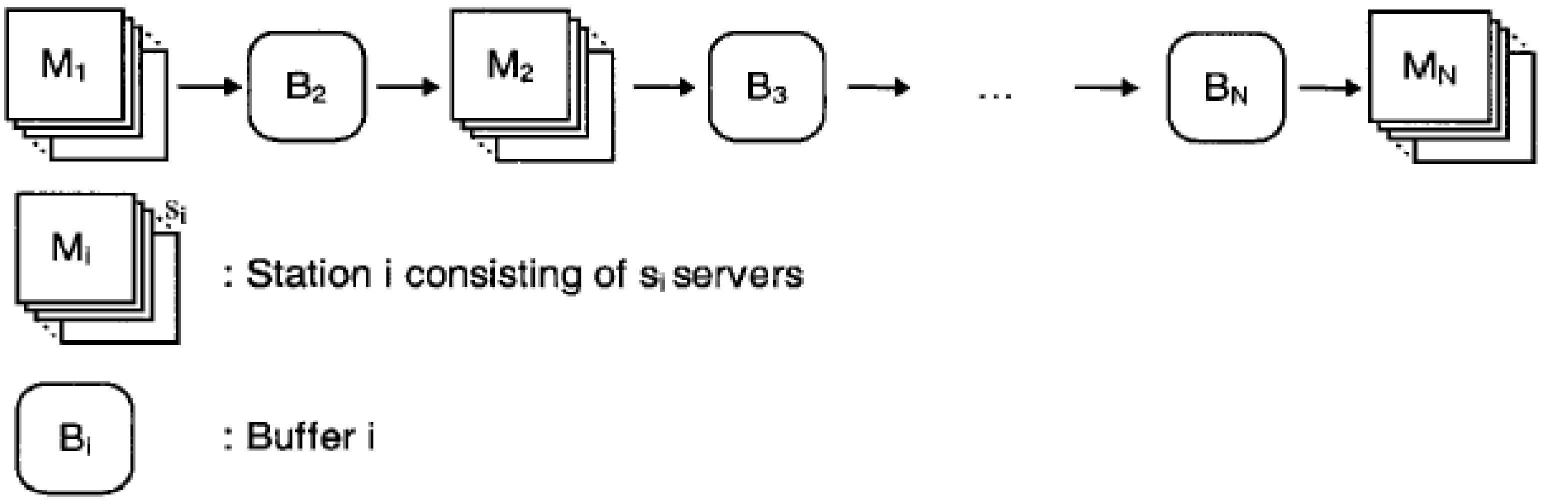


Figure 1: A production line with N workstations where each workstation i consists of S_i parallel machines.

- In this work, a flow line consisting of N parallel-machine workstations, $M_1,\ldots, M_N$ and N-1 intermediate buffers $B_1,\ldots, B_{N-1}$ of finite capacities, like the one depicted in Figure 1, is analyzed by decomposition. Each workstation M_i consists of S_i, i=1,...,N, unreliable and non identical parallel machines. The S_i parallel machines at workstation M_i, i=1,2,...,N, have exponentially distributed service times, times to failure and times to repair with means $\frac{1}{\mu_{i,j}}, \frac{1}{p_{i,j}}$ *and* $\frac{1}{r_{i,j}}$

  ,i=1,2,...,N and j=1,2,…,S_i, respectively.

LITERATURE REVIEW

- The original decomposition method proposed by Gershwin (1987 & 1994) and Dallery et al (1998) has been extensively applied to systems under various assumptions with a common characteristic that each workstation consists of only one server.

- There is a vast literature on the analysis of flow lines with *single machine* workstations (Dallery and Gershwin (1992), Gershwin (1994) and Papadopoulos et al. (1993), Li and Meerkov (2009), among others).

- Considering the literature on the analysis of flow lines with *multiple parallel-machine* workstations, Burman (1995) presents a method that replaces each parallel server workstation by a single equivalent workstation for the case of continuous flow of material. He assumes that the equivalent workstation has a maximum processing rate which equals the sum of the processing rates of the parallel machines.

LITERATURE REVIEW

- Assembly/Disassembly (A/D) systems with parallel machines at each workstation and discrete flow of material were also analyzed using another replacement technique by Jeong and Kim (1999). Their method transformed each workstation consisting of multiple parallel machines into an equivalent single machine workstation

- For the case of serial flow lines with multiple parallel machines at each workstation, Patchong and Willaeys (2001) presented a replacement technique and the sets of equations that are necessary for this replacement. A parallel machine aggregating method was also introduced by Li and Meerkov (2009).

LITERATURE REVIEW

- Diamantidis et al. (2007), examined serial long production lines with identical reliable parallel servers at each workstation without using any replacement techinque. The decomposition approach was applied directly to each one of the parallel machines for each workstation using a two-workstation one-buffer decomposition block. The performance measures for long production lines were also estimated.

- Similarly, serial flow lines with identical unreliable parallel machines at each workstation were analyzed using decomposition without any replacement technique by Shin and Moon (2016). The decomposition block that was used in this paper was a three-machine two-buffer sub-system.

THE DECOMPOSITION BLOCK

To analyze the system depicted in Figure 1 via the decomposition approach, first a sub-system like the one depicted in Figure 2 should be analyzed first. The system depicted in Figure 2 has two workstations each one consisting of S1 and S2 unreliable non identical parallel servers.

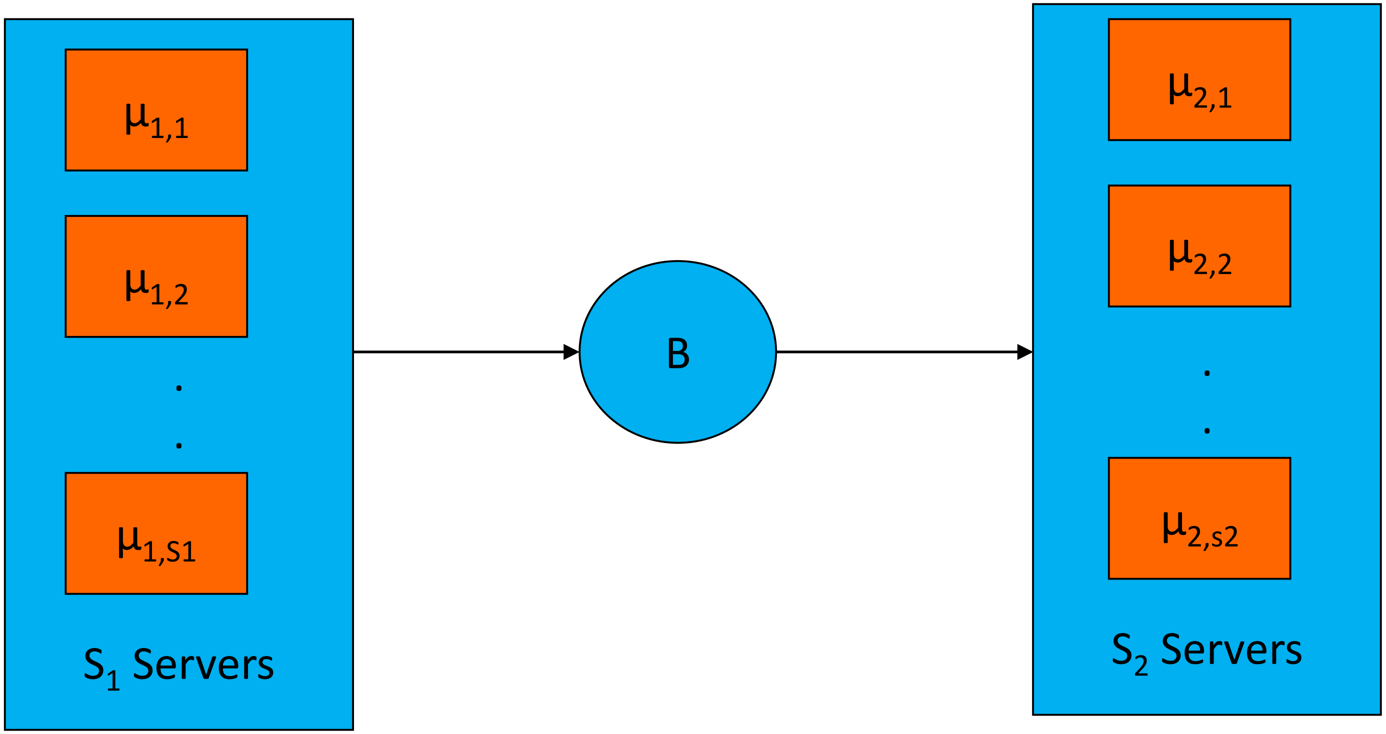
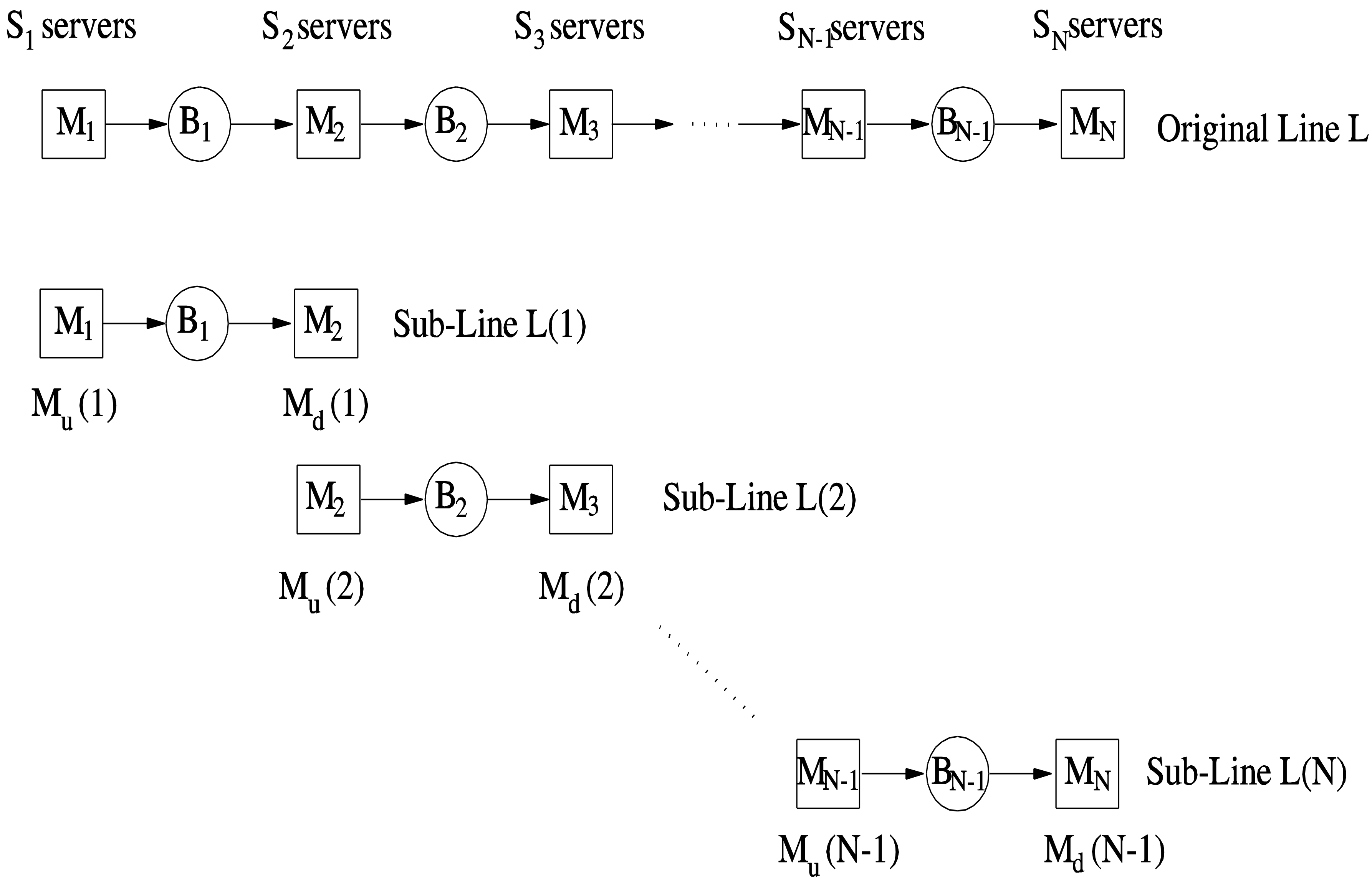


Figure 2: A system with two workstations, parallel servers and an intermediate buffer

- The solution of the two-workstations, one-buffer decomposition block is presented in Diamantidis and Papadopoulos (2009).

- To apply the decomposition method, the production line depicted in Figure 1 must be decomposed into N-1 two-workstations one-buffer subsystems (decomposition blocks), like the one presented in Figure 2. Each workstation of the system depicted in Figure 2 may have parallel non identical unreliable machines.


S1 servers
S2 servers
S3 servers
SN-1 servers
SN servers
M1
B1
M2
B2
M3
MN-1
BN-1
MN
Original Line L
M1
B1
M2
Sub-Line L(1)
Mu (1)
Md (1)
M2
B2
M3
Sub-Line L(2)
Mu (2)
Md (2)
MN-1
BN-1
MN
Sub-Line L(N)
Mu (N-1)
Md (N-1)

The service, repair and failure times of each one the S_i pseudo parallel machines of pseudo workstation $M_u(i)$ are exponentially distributed with mean $\frac{1}{\mu_{u_{j(i)}}}$, $\frac{1}{r_{u_{j(i)}}}$ and $\frac{1}{p_{u_{j(i)}}}$ $, j = 1, \ldots, S_i$

Similarly the service, repair and failure times of the S_{i+1} pseudo parallel machines of pseudo workstation $M_d(i)$ are also exponentially distributed with mean $\frac{1}{\mu_{d_{k(i)}}}$, $\frac{1}{r_{d_{k(i)}}}$ and $\frac{1}{p_{d_{k(i)}}}$, respectively, $k = 1, \ldots, , S_{i+1.}$

The target is to evaluate the service, failure and repair parameters of all upstream and downstream pseudo machines, respectively. The evaluation must be done in such a way that the flow into and out of each buffer in all sub-lines equals the flow into and out of the corresponding buffers of the original production line, L. In order to evaluate all these parameters, four sets of equations are necessary to be derived.

THE SETS OF THE DECOMPOSITION EQUATIONS

- THE CONSERVATION OF FLOW EQUATIONS

  (Describe the fact that the flow among the workstations of the line is conserved)

- THE FLOW RATE IDLE TIME EQUATIONS

  (are used to evaluate the processing rates of the virtual pseudo upstream and downstream machines)

- THE INTERRUPTION OF FLOW EQUATIONS

  (are used to evaluate the failure rates of the virtual upstream and downstream pseudo machines)

- THE RESUMPTION OF FLOW EQUATIONS

  (are used to evaluate the repair rates of the virtual upstream and downstream pseudo machines)

THE CONSERVATION OF FLOW EQUATIONS

Is expressed with the following equations

$$P_u(i) = P_d(i) = P(i) = P(i+1)$$
$$for\ all\ i = 1, \ldots, N-1$$

where $P_u(i) and\ P_d(i)$ are the mean production rates of the upstream and downstream workstations of sub-line L(i), respectively. $P(i)$ is the mean production rate of line sub-L(i), i=1,...,N-1.

THE FLOW RATE IDLE TIME EQUATIONS

The processing rates of the virtual upstream and downstream machines can be calculated using the following equations:

$$\frac{1}{e_{i,j}\mu_{i,j}} + \frac{1}{P_{i,j}} = \frac{1}{e_{u_j}(i)\mu_{u_j}(i)} + \frac{1}{e_{d_j}(i-1)\mu_{d_j}(i-1)}$$

$$i = 2, \dots, N-1 \; and \; j = 1, \dots, S_i$$

THE DECOMPOSITION ALGORITHM

- In order to solve simultaneously all the decomposition equations an algorithm is required that consists of three steps which are the following:

Step1 : Initialization

For each two-workstation line L(i), the initial guesses for its parameters are

$$\mu_{u_j}(i) = \mu_{i,j}\,,$$

$$r_{u_j}(i) = r_{i,j}\,,$$

$$p_{u_j}(i) = p_{i,j}\quad,$$

$$j = 1, \dots, S_i,\ \mathrm{i=1,\dots,N\text{-}1}$$

$$\mu_{d_j}(i) = \mu_{i+1,j}\,,$$

$$r_{d_j}(i) = r_{i+1,j}\,,$$

$$p_{d_j}(i) = p_{i+1,j}\,,$$

$$j = 1, \dots, S_{i+1}, i = 1, \dots, N-1$$

Step 2 : Perform Step 2(a) and Step 2(b) alternately until the Termination Condition is satisfied

Step 2(a): Let i range over values from 2 to i=N-1. Evaluate all the quantities (using the decomposition equations)

$$\mu_{u_j}(i)\,,\, r_{u_j}(i),\;\; p_{u_j}(i)$$
$$j = 1, \ldots, S_i\,,$$

using the most recent values of all the involved parameters for the calculation of these parameters.

Step 2(b): Let i range over values from N-2 to i=1. Evaluate all the quantities

$$\mu_{d_j}(i), r_{d_j}(i)\;,\;\; p_{d_j}(i)$$
$$j = 1, \ldots, S_{i+1}$$

using the most recent values of all the involved parameters for the calculation of these parameters.

Step 3: Termination condition

Terminate the algorithm when for i=2,...,N-1 where ε is a prespecified small positive number (i.e. ε=0.001 or ε=0.0001 or less). Where P(i) is the throughput of the two workstation line L(i).

FURTHER RESEARCH

- Implement the proposed algortihm using a computer programming language to obtain numerical results
- Use this algorithm as an evaluative tool along with search techniques to solve design/optimization problems (such as the buffer allocation problem, the server allocation problem and the workload allocation problem).

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