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Numerical comparison of Kanban mechanisms for production systems with time-dependent processing times

Justus Arne Schwarz joint work with Raik Stolletz



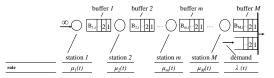
SMMSO 2017, June 2017, Lecce Italy

Agenda

- 1 Introduction & Motivation
- 2 Problem formulation: Proactive Kanban Allocation Problem
- 3 Solution approaches
 - Sectioning algorithm
 - Simplifying heuristics
- 4 Numerical insights
- 5 Conclusions and future research

Motivation - time-dependent flow lines

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- Exponentially distributed times between demand arrivals: rate λ(t).
- Exponentially distributed processing times at station *m*: rate $\mu_m(t)$.

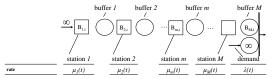
Time-dependent impacts

- Seasonal demand
- Replacement of machinery
- Learning effects

(Takahashi and Nakamura, 2002; Jaikumar and Bohn, 1992; Terwiesch and Bohn, 2001)

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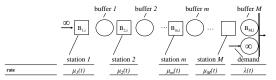
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Should buffer capacities be changed over time?

if, so

How should buffer capacities be changed over time?

Schwarz & Stolletz

Related literature

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- Flow lines correspond to Kanban systems (Berkley, 1991)
- Kanban cards allow for the flexible adaption of buffer capacities

Used information to trigger reconfiguration		
Past data	Current system status	Time-dependent distribution parameters
Reactive Kanban Takahashi&Nakamura 2002 Takahashi 2003 Takahashi et al. 2004	Adaptive Kanban Tardif & Maaseidvaag 2001	Proactive Kanban Today's talk

 Existing approaches *react* to changes in demand Time-dependent processing times (Schwarz and Stolletz 2017)

New approach to *plan* for changes of flow line parameters

Schwarz & Stolletz

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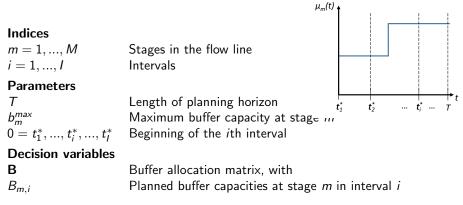
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Proactive Kanban Systems

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Assumptions:

- Planing horizon is divided in *I* intervals
- Maximum of one change of the allocation per interval
- Allocation is changed at first demand arrival in each interval





Proactive Kanban Card Setting Problem

minimize
$$E[W(B)]$$
 (1a)
s.t.:
 $SL^{\alpha}(B) \ge \alpha^{*}$ (1b)
 $0 \le B_{m,i} \le b_{m}^{max}$ $\forall m, \forall i$ (1c)

with

Dependent variables

 $W(\mathbf{B})$ Average WIP in the line over the planning horizon $SL^{\alpha}(\mathbf{B})$ Achieved α -service level during the planning horizon

• Steady-state version of the problem addressed by Duri et al. (2000)

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Solution approaches

Simulation-optimization approach, accounting for time-dependency

- Establish monotonicity properties
- Derive dominance criteria for allocations
- Derivation of upper and lower bounds
- Exploit dominance criteria and bounds in a sectioning algorithm

Simplifying heuristics, relaxing time-dependency

• Partially ignore time-dependency and application of steady-state methods

Impact of t-d. buffer capacities on expected WIP and service level

Theorem 1

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For M = 1 the cumulated no. of departed workpiece $D_M(t)$ from station M and $D_{M+1}(t)$ from buffer M are stochastically increasing in $B_{M,i}$ $\forall i$.

Corollary 1.2

 $SL^{\alpha}(\mathbf{B})$ is increasing in $B_{M,i}$ $\forall i$.

Corollary 1.1

 $E[W(\mathbf{B})]$ is increasing in $B_{M,i}$ $\forall i$.

Proof: Based on sample path arguments

- In contrast to steady-state E[W(B)] is not necessarily convex
- Numerical studies suggest: Corollary 1.1 and 1.2 hold for M > 1 and generally distributed processing times and demand

Schwarz & Stolletz

Dominance criteria

• Based on Corollary 1.1 and 1.2:

Every feasible allocation **B** with $SL^{\alpha}(\mathbf{B}) \geq \alpha^*$

- excludes all **B**' with $B'_{m,i} \ge B_{m,i} \ \forall m, \forall i$,
- and $E[W(\mathbf{B})]$ provides an upper bound (UB_W) on the objective value

Every infeasible allocation **B** with $SL^{\alpha}(\mathbf{B}) < \alpha^*$

- excludes all allocations **B**' with $B'_{m,i} \leq B_{m,i} \forall m, \forall i$
- if in addition $E[W(\mathbf{B})] > UB_W$ holds
 - exclude all allocations \mathbf{B}' with $B'_{m,i} \geq B_{m,i} \ \forall m, \forall i$

Derivation of (conditional) upper and lower bounds

For a given buffer m' in interval i':

Determine (conditional) lower bound on $B_{m',i'}$

- Set all not considered buffers to upper bound (or fixed value)
- Search for smallest value of $B_{m',i'}$ such that $SL^{lpha} \geq lpha^*$

Determine (conditional) upper bound on $B_{m',i'}$

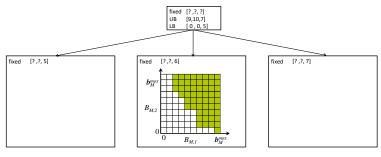
- Set all not considered buffers to lower bound (or fixed value)
- Search for smallest value of $B_{m',i'}$ such that $SL^{lpha} \geq lpha^*$

- while (no. of not fixed buffers $B_{m,i} > 1$)
 - Determine lower and upper bounds
 - Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

fixed	[?,?,?]
UB	[9,10,7]
LB	[0,0,5]

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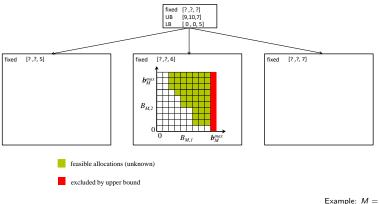


feasible allocations (unknown)

Example: $M = 1, I = 3, b_M^{max} = 10$ June 2017, Lecce Italy 13

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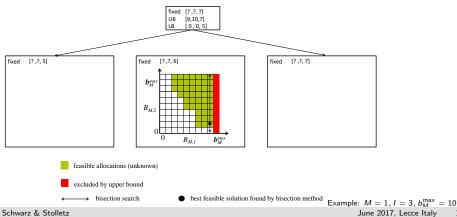
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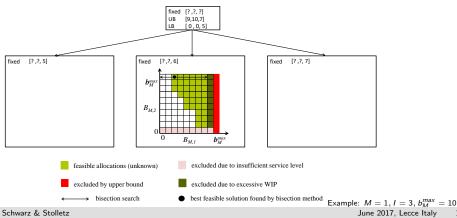
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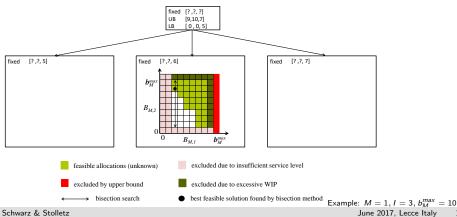
Sectioning Algorithm - Key ideas

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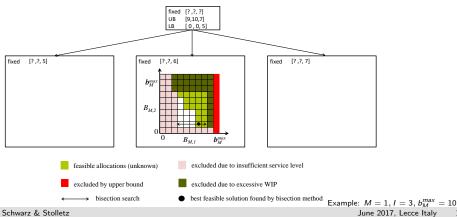
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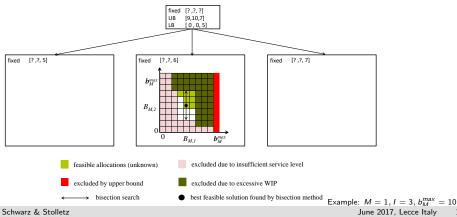
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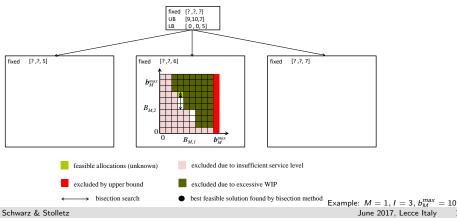
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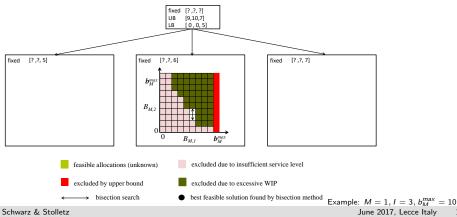
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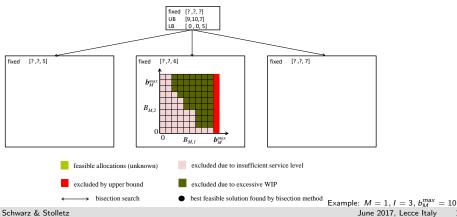
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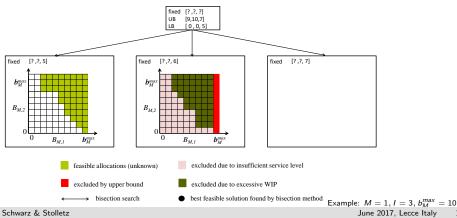


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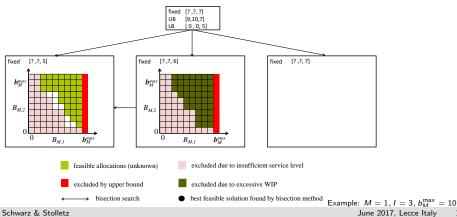


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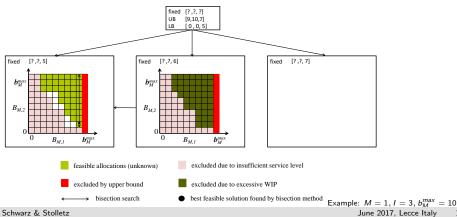


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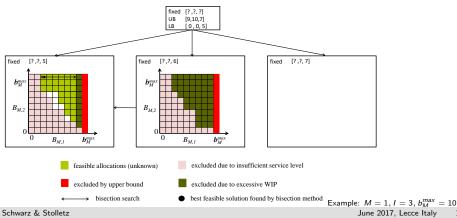


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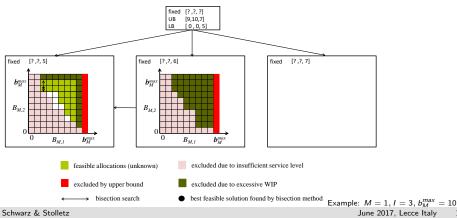
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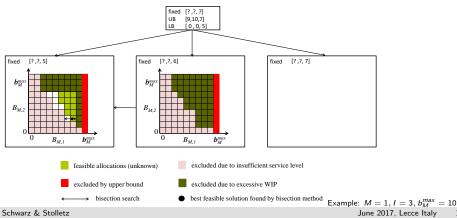
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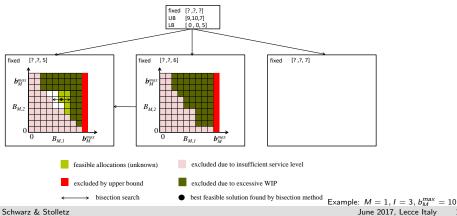
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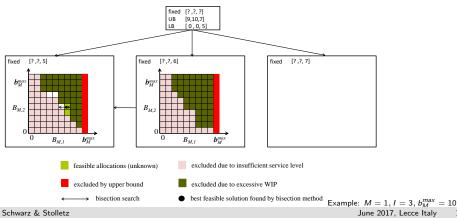
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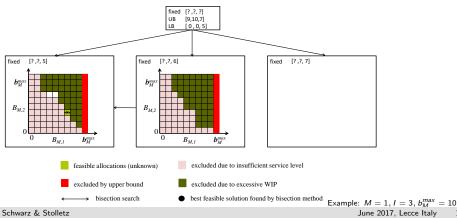
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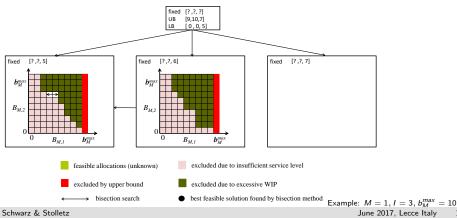
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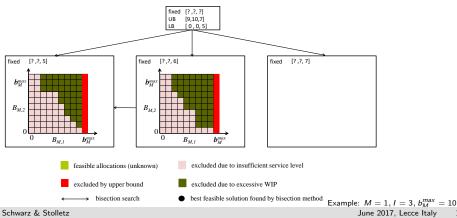
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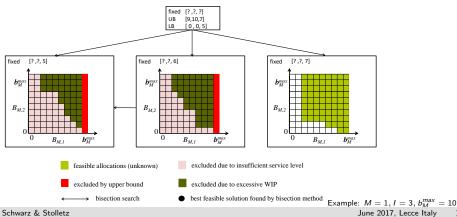
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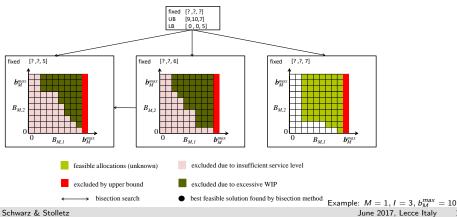
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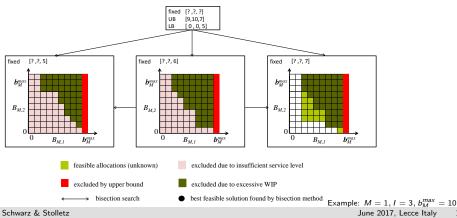
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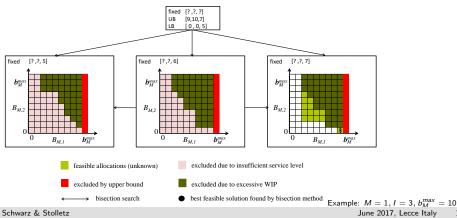
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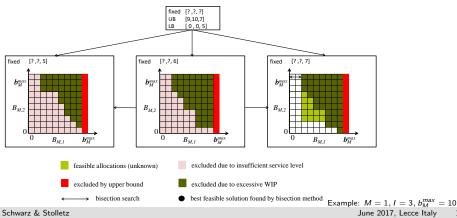
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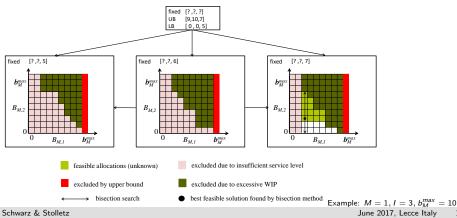
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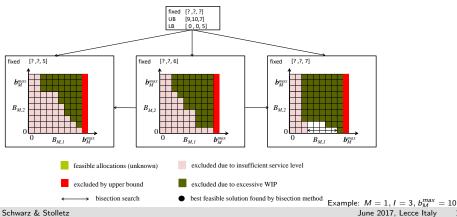
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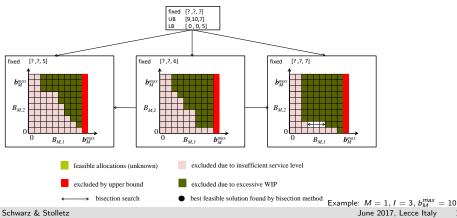
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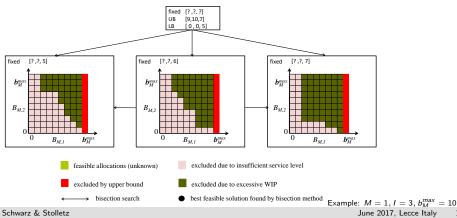
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Sectioning Algorithm - Summary

Recursive algorithm formulation

```
function ITERATEONALEVEL( fixedValues, fixedBuffer, bufferValue )
   fixedValues<sub>fixedBuffer</sub> ← bufferValue
   if noOfNotFixedBuffers > 1 then
      DETMINECONDITIONALLOWERBOUNDS(fixedValues)
      DETMINECONDITIONAL UPPERBOUNDS (fixed Values)
      nextFixedBufferValue \leftarrow |(UB(fixedBuffer) - LB(fixedBuffer))/2|
ITERATEONALEVEL(fixedValues, nextFixedBuffer, nextFixedBufferValue)
   else
      BISECSEARCHSMALLESTFEASIBLEALLOC(NotFixedBuffer)
   end if
   if LB(fixedBuffer) < bufferValue then
      ITERATEONALEVEL(fixedValues, fixedBuffer, smallerbufferValue)
   end if
   if UB(fixedBuffer) > bufferValue then
      ITERATEONALEVEL(fixedValues, fixedBuffer, largerBufferValue)
   end if
end function
```

- The algorithm terminates (worst case: complete enumeration)
- If it exists, the algorithm returns the optimal solution



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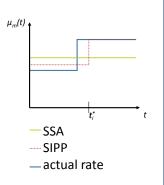
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Basic heuristic solutions: Relax time-dependency

Parameter generation

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Solving the problem

Simple Stationary Approximation (SSA)

- Solving a single steady-state problem
- Constant allocation

Stationary Independent Period by Period Approximation (SIPP)

- Solving *I* + 1 steady-state problems
- Time-dependent allocation

Constant allocation (CONST)

- Solving a Proactive Kanban Card Setting Problem with *I* = 1
- Constant allocation

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Comparison of solution approaches

Parameters:

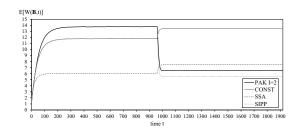
• M = 1, $b_M^{max} = 20$

• $I = 2, t_2^* = 960$

- Poisson demand
- λ = 0.5, α^{*} = 0.98
- Exponentially distributed processing times
- $\mu_1(t) = 2/3, t \in [0;960), \ \mu_1(t) = 1, t \in [960;1920)$

Results:

Approach	<i>B</i> _{1,0}	$B_{1,1}$	_	$\mathit{SL}^{lpha}(\mathbf{B})$	E[W(B)]
SSA	7	7		0.941	6.715
SIPP	13	5		0.973	8.440
CONST	12	12		0.980	12.395
PKA	15	6		0.980	9.850

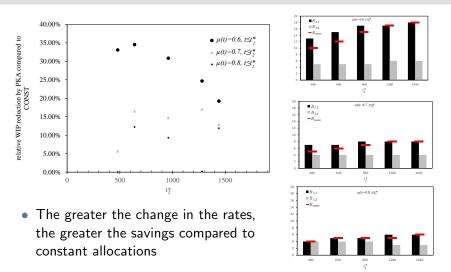


 Evaluation in sectioning algorithm: 50,0000 replications

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Impact of magnitude in the rate change and its timing

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 $M = 1, \ b_M^{max} = 20, I = 2, \ t_1^* = 960, \ \lambda = 0.5, \ \alpha^* = 0.95, \ \mu_1(t) = 1, \ t \in [t_2^*; 1920)$

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Heuristic solutions for multi-stage systems

Parameters:

•
$$M = 2$$
 ; $b_m^{max} = 20 \ \alpha^* = 0.95$

•
$$I = 1, t_1^* = 960$$

t	$\mu_1(t)$	$cv_1^2(t)$	$\mu_2(t)$	$cv_2^2(t)$	$\mu_3(t)$	$cv_3^2(t)$	$\lambda(t)$
[0,960)	2/3	1	1	1	1	1	0.5
[960, 1920]	1	0.5	1	1	1	1	0.5

Results:

М	Approach	$B_{1,1}$	$B_{1,2}$	$B_{2,1}$	B _{2,2}	$B_{3,1}$	B _{3,2}	$SL^{\gamma}(\mathbf{B})$	$E[W(\mathbf{B})]$
2	CONST PKA	4 3	4 1	6 9		-		0.950 0.951	10.15 8.984 (-11.4%)
	CONST PKA	3 4	3 1	2 2	2 1	8 8	-		13.365 11.812 (-11.6%)



Performance of the seactioning algorithm

Computational effort:

No. of decision variables	2	3	4	6
Average no. of evaluated alloc.	18.88	165	508	44,444
Average $\%$ of all alloc. evaluated	4.28	1.78	0.26	0.05

Solution quality:

• Optimality of allocations is verified by complete enumeration for problems with $(I + 1) \cdot M \le 4$

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Conclusions and future research

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- New approach for Kanban allocation in stochastic and time-dependent flow lines
- Sectioning based simulation-optimization approach

Managerial insights:

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- Time-dependent allocation results in improvements compared to constant allocations
- Processing rate changes may require to change the structure of the buffer allocation

Future research:

- Development of advanced heuristics
- Problem extensions
 - Service level goals for subperiods of the planning horizon
 - Extensions to other control policies such as CONWIP

Thank you for your attention

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