

# Numerical comparison of Kanban mechanisms for production systems with time-dependent processing times 

Justus Arne Schwarz<br>joint work with Raik Stolletz

## Agenda

1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches

- Sectioning algorithm
- Simplifying heuristics

4 Numerical insights

5 Conclusions and future research

## Motivation - time-dependent flow lines



- Exponentially distributed times between demand arrivals: rate $\lambda(t)$.
- Exponentially distributed processing times at station $m$ : rate $\mu_{m}(t)$.

Time-dependent impacts

- Seasonal demand
- Replacement of machinery
- Learning effects
(Takahashi and Nakamura, 2002; Jaikumar and Bohn, 1992; Terwiesch and Bohn, 2001)


## Motivation - time-dependent flow lines



- Exponentially distributed times between demand arrivals: rate $\lambda(t)$.
- Exponentially distributed processing times at station $m$ : rate $\mu_{m}(t)$.

Time-dependent impacts

- Seasonal demand
- Replacement of machinery
- Learning effects
(Takahashi and Nakamura, 2002; Jaikumar and Bohn, 1992; Terwiesch and Bohn, 2001)


## Motivation - time-dependent flow lines



- Exponentially distributed times between demand arrivals: rate $\lambda(t)$.
- Exponentially distributed processing times at station $m$ : rate $\mu_{m}(t)$.

Time-dependent impacts

- Seasonal demand
- Replacement of machinery
- Learning effects
(Takahashi and Nakamura, 2002; Jaikumar and Bohn, 1992; Terwiesch and Bohn, 2001)


## Should buffer capacities be changed over time?

if, so
How should buffer capacities be changed over time?

## Related literature

- Flow lines correspond to Kanban systems (Berkley, 1991)
- Kanban cards allow for the flexible adaption of buffer capacities

- Existing approaches react to changes in demand Time-dependent processing times (Schwarz and Stolletz 2017)


## New approach to plan for changes of flow line parameters

## Agenda

## 1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches

- Sectioning algorithm
- Simplifying heuristics

4 Numerical insights

5 Conclusions and future research

## Proactive Kanban Systems

## Assumptions:

- Planing horizon is divided in / intervals
- Maximum of one change of the allocation per interval
- Allocation is changed at first demand arrival in each interval


## Indices

$$
\begin{aligned}
& m=1, \ldots, M \\
& i=1, \ldots, l
\end{aligned}
$$

Parameters
T
$b_{m}^{\max }$
$0=t_{1}^{*}, \ldots, t_{i}^{*}, \ldots, t_{l}^{*}$
Decision variables
B
$B_{m, i}$

Stages in the flow line Intervals

Length of planning horizon Maximum buffer capacity at stage ," Beginning of the $i$ th interval

Buffer allocation matrix, with
Planned buffer capacities at stage $m$ in interval $i$

## Proactive Kanban Card Setting Problem

$$
\begin{equation*}
\operatorname{minimize} \mathrm{E}[W(\mathbf{B})] \tag{1a}
\end{equation*}
$$

s.t.:

$$
\begin{align*}
S L^{\alpha}(\mathbf{B}) & \geq \alpha^{*}  \tag{1b}\\
0 \leq B_{m, i} & \leq b_{m}^{\max } \quad \forall m, \forall i \tag{1c}
\end{align*}
$$

with

## Dependent variables

W(B) Average WIP in the line over the planning horizon
SL ${ }^{\alpha}$ (B) Achieved $\alpha$-service level during the planning horizon

- Steady-state version of the problem addressed by Duri et al. (2000)

1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches

- Sectioning algorithm

■ Simplifying heuristics
4) Numerical insights
5) Conclusions and future research

## Solution approaches

Simulation-optimization approach, accounting for time-dependency

- Establish monotonicity properties
- Derive dominance criteria for allocations
- Derivation of upper and lower bounds
- Exploit dominance criteria and bounds in a sectioning algorithm

Simplifying heuristics, relaxing time-dependency

- Partially ignore time-dependency and application of steady-state methods


## Impact of t-d. buffer capacities on expected WIP and service level

## Theorem 1

For $M=1$ the cumulated no. of departed workpiece $D_{M}(t)$ from station $M$ and $D_{M+1}(t)$ from buffer $M$ are stochastically increasing in $B_{M, i} \forall i$.

## Corollary 1.2

$S L^{\alpha}(\mathbf{B})$ is increasing in $B_{M, i} \forall i$.

## Corollary 1.1

$\mathrm{E}[W(\mathbf{B})]$ is increasing in $B_{M, i} \forall i$.
Proof: Based on sample path arguments

- In contrast to steady-state $\mathrm{E}[W(\mathbf{B})]$ is not necessarily convex
- Numerical studies suggest: Corollary 1.1 and 1.2 hold for $M>1$ and generally distributed processing times and demand


## Dominance criteria

- Based on Corollary 1.1 and 1.2:

Every feasible allocation $\mathbf{B}$ with $S L^{\alpha}(\mathbf{B}) \geq \alpha^{*}$

- excludes all $\mathbf{B}^{\prime}$ with $B_{m, i}^{\prime} \geq B_{m, i} \forall m, \forall i$,
- and $\mathrm{E}[W(\mathbf{B})]$ provides an upper bound $\left(U B_{W}\right)$ on the objective value

Every infeasible allocation $\mathbf{B}$ with $S^{\alpha}(\mathbf{B})<\alpha^{*}$

- excludes all allocations $\mathbf{B}^{\prime}$ with $B_{m, i}^{\prime} \leq B_{m, i} \forall m, \forall i$
- if in addition $\mathrm{E}[W(\mathbf{B})]>U B_{W}$ holds
- exclude all allocations $\mathbf{B}^{\prime}$ with $B_{m, i}^{\prime} \geq B_{m, i} \forall m, \forall i$


## Derivation of (conditional) upper and lower bounds

For a given buffer $m^{\prime}$ in interval $i^{\prime}$ :

## Determine (conditional) lower bound on $B_{m^{\prime}, i^{\prime}}$

- Set all not considered buffers to upper bound (or fixed value)
- Search for smallest value of $B_{m^{\prime}, i^{\prime}}$ such that $S L^{\alpha} \geq \alpha^{*}$


## Determine (conditional) upper bound on $B_{m^{\prime}, i^{\prime}}$

- Set all not considered buffers to lower bound (or fixed value)
- Search for smallest value of $B_{m^{\prime}, i^{\prime}}$ such that $S L^{\alpha} \geq \alpha^{*}$


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

| fixed | $[?, ?, ?]$ |
| :--- | :--- |
| UB | $[9,10,7]$ |
| LB | $[0,0,5]$ |

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

$\square$ feasible allocations (unknown)


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer
feasible allocations (unknown)
- excluded by upper bound


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer
feasible allocations (unknown)excluded by upper bound
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

$\square$ feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\square$ excluded due to excessive WIP
$\longleftrightarrow$ bisection search
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

$\square$ feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to insufficient service level
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$
Schwarz \& Stolletz

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
- excluded by upper bound
$\longleftrightarrow$ bisection search
excluded due to insufficient service level
excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$
Schwarz \& Stolletz

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
- excluded by upper bound
$\square$ excluded due to excessive WIP
$\longleftrightarrow$ bisection search
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method

Example: $M=1, I=3, b_{M}^{\max }=10$

## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method


## Sectioning Algorithm - Key ideas

- while (no. of not fixed buffers $B_{m, i}>1$ )
- Determine conditional lower and upper bounds
- Determine buffer to be fixed and fix its capacity
- Search for smallest feasible value of unfixed buffer

- feasible allocations (unknown)
$\square$ excluded due to insufficient service level
$\square$ excluded by upper bound
$\longleftrightarrow$ bisection search
$\square$ excluded due to excessive WIP
- best feasible solution found by bisection method


## Sectioning Algorithm - Summary

## Recursive algorithm formulation

```
function ITERATEONALEVEL( fixedValues, fixedBuffer, bufferValue )
    fixedValues fixedBuffer }\leftarrow\mathrm{ bufferValue
    if noOfNotFixedBuffers > 1 then
        DETMINECONDITIONALLOWERBOUNDS(fixedValues)
        DetmineConditionalUpperBounds(fixedValues)
        nextFixedBuffer \leftarrow DetmineNextFixedBuffer(fixed)
        nextFixedBufferValue \leftarrow \lfloor(UB(fixedBuffer) - LB(fixedBuffer))})2
ItERATEONALEVEL(fixedValues, nextFixedBuffer, nextFixedBufferValue)
    else
        BiSEcSearchSmallestFeasibleAlloc(NotFixedBuffer)
    end if
    if LB(fixedBuffer) < bufferValue then
        ITERATEONALEVEL(fixedValues, fixedBuffer, smallerbufferValue)
    end if
    if UB(fixedBuffer) > bufferValue then
        IterateOnALEVEL(fixedValues, fixedBuffer, largerBufferValue)
    end if
end function
```

- The algorithm terminates (worst case: complete enumeration)
- If it exists, the algorithm returns the optimal solution


## Agenda

## 1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches

- Sectioning algorithm
- Simplifying heuristics

4 Numerical insights

5 Conclusions and future research

## Basic heuristic solutions: Relax time-dependency

Parameter generation


- SSA
- SIPP
- actual rate

Solving the problem
Simple Stationary Approximation (SSA)

- Solving a single steady-state problem
- Constant allocation

Stationary Independent Period by Period Approximation (SIPP)

- Solving $I+1$ steady-state problems
- Time-dependent allocation

Constant allocation (CONST)

- Solving a Proactive Kanban Card Setting Problem with $I=1$
- Constant allocation


## Agenda

1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches
= Sectioning algorithm

- Simplifying heuristics

4 Numerical insights

15 Conclusions and future research

## Comparison of solution approaches

## Parameters:

- $M=1, b_{M}^{\max }=20$
- $I=2, t_{2}^{*}=960$
- Poisson demand
- $\lambda=0.5, \alpha^{*}=0.98$
- Exponentially distributed processing times
- $\mu_{1}(t)=2 / 3, t \in[0 ; 960)$,
$\mu_{1}(t)=1, t \in[960 ; 1920)$

Results:

| Approach |  | $B_{1,0}$ | $B_{1,1}$ |  | $S L^{\alpha}(\mathbf{B})$ | $\mathrm{E}[W(\mathbf{B})]$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| SSA |  | 7 | 7 |  | 0.941 | 6.715 |
| SIPP |  | 13 | 5 |  | 0.973 | 8.440 |
| CONST |  | 12 | 12 |  | 0.980 | 12.395 |
| PKA |  | 15 | 6 |  | 0.980 | 9.850 |



- Evaluation in sectioning algorithm: 50,0000 replications


## Impact of magnitude in the rate change and its timing



- The greater the change in the rates, the greater the savings compared to constant allocations


$$
M=1, b_{M}^{\max }=20, I=2, t_{1}^{*}=960, \lambda=0.5, \alpha^{*}=0.95, \mu_{1}(t)=1, t \in\left[t_{2}^{*} ; 1920\right)
$$

## Heuristic solutions for multi-stage systems

## Parameters:

- $M=2 ; b_{m}^{\max }=20 \alpha^{*}=0.95$
- $I=1, t_{1}^{*}=960$

| t | $\mu_{1}(t)$ | $c v_{1}^{2}(t)$ | $\mu_{2}(t)$ | $c v_{2}^{2}(t)$ | $\mu_{3}(t)$ | $C v_{3}^{2}(t)$ | $\lambda(t)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[0,960)$ | 2/3 | 1 | 1 | 1 | 1 | 1 | 0.5 |
| [960, 1920] | 1 | 0.5 | 1 | 1 | 1 | 1 | 0.5 |

Results:

| M | Approach | $B_{1,1} \quad B_{1,2}$ |  | $\begin{array}{cc}B_{2,1} & B_{2,2}\end{array}$ |  | $B_{3,1}$ | $B_{3,2}$ | $S L^{\gamma}(\mathrm{B})$ | $\mathrm{E}[W(\mathrm{~B})]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | CONST | 4 | 4 | 6 | 6 | - | - | 0.950 | 10.15 |
|  | PKA | 3 | 1 | 9 | 5 | - | - | 0.951 | 8.984 (-11.4\%) |
| 3 | CONST | 3 | 3 | 2 | 2 | 8 | 8 | 0.952 | 13.365 |
|  | PKA | 4 | 1 | 2 | 1 | 8 | 7 | 0.951 | 11.812 (-11.6\%) |

## Performance of the seactioning algorithm

## Computational effort:

| No. of decision variables | 2 | 3 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| Average no. of evaluated alloc. | 18.88 | 165 | 508 | 44,444 |
| Average \% of all alloc. evaluated | 4.28 | 1.78 | 0.26 | 0.05 |

## Solution quality:

- Optimality of allocations is verified by complete enumeration for problems with $(I+1) \cdot M \leq 4$


## 1 Introduction \& Motivation

2 Problem formulation: Proactive Kanban Allocation Problem

3 Solution approaches

- Sectioning algorithm
- Simplifying heuristics

4 Numerical insights

5 Conclusions and future research

## Conclusions and future research

- New approach for Kanban allocation in stochastic and time-dependent flow lines
- Sectioning based simulation-optimization approach


## Managerial insights:

- Time-dependent allocation results in improvements compared to constant allocations
- Processing rate changes may require to change the structure of the buffer allocation


## Future research:

- Development of advanced heuristics
- Problem extensions
- Service level goals for subperiods of the planning horizon
- Extensions to other control policies such as CONWIP


## Thank you for your attention

## References I

Ammar, M. and S. Gershwin (1980, dec). Equivalence relations in queueing models of manufacturing networks. In 1980 19th IEEE Conference on Decision and Control including the Symposium on Adaptive Processes, pp. 715-721. IEEE.

Anantharam, V. and P. Tsoucas (1990). Stochastic concavity of throughput in series of queues with finite buffers. Advances in Applied Probability 22(3), 761-763.

Berkley, B. J. (1991). Tandem queues and kanban-controlled lines. International Journal of Production Research 29(10), 2057-2081.
Chao, X. and M. Pinedo (1992, dec). On reversibility of tandem queues with blocking. Naval Research Logistics 39(7), 957-974.
Chao, X., M. Pinedo, and K. Sigman (1989, apr). On the Interchangeability and Stochastic Ordering of Exponential Queues in Tandem with Blocking. Probability in the Engineering and Informational Sciences 3(02), 223.
Cheng, D. W. (1992). Second order properties in a tandem queue with general blocking. Operations Research Letters 12(3), 139-144.

Cheng, D. W. (1995, may). Line Reversibility of Tandem Queues with General Blocking. Management Science 41(5), 864-873.
Duri, C., Y. Frein, and M. Di Mascolo (2000). Comparison among three pull control policies: Kanban, base stock, and Generalized Kanban. Annals of Operations Research 93(1), 41-69.
Glasserman, P. and D. D. Yao (1996). Structured buffer-allocation problems. Discrete Event Dynamic Systems: Theory and Applications 6(1), 9-41.

Jaikumar, R. and R. E. Bohn (1992). A dynamic approach to operations management: An alternative to static optimization. International Journal of Production Economics 27(3), 265-282.

Lev, B. and D. I. Toof (1980, sep). The role of internal storage capacity in fixed cycle production systems. Naval Research Logistics Quarterly 27(3), 477-487.
Meester, L. E. and J. G. Shanthikumar (1990). Concavity of the Throughput of Tandem Queueing Systems with Finite Buffer Storage Space. Advances in Applied Probability 22(3), 764-767.

## References II

Schwarz, J. A. (2015). Analysis of buffer allocations in time-dependent and stochastic flow lines. Ph. D. thesis, University of Mannheim.

So, K. C. (1997). Optimal buffer allocation strategy for minimizing work-in-process inventory in unpaced production lines. IIE Transactions 29(1), 81-88.

Soyster, A. L. and D. I. Toof (1976). Some comparative \& design aspects of fixed cycle production systems. Naval Research Logistics Quarterly 23(3), 437-454.

Takahashi, K. and N. Nakamura (2002). Decentralized reactive Kanban system. European Journal of Operational Research 139(2), 262-276.
Tayur, S. R. (1992). Properties of serial kanban systems. Queueing Systems 12(3), 297-318.
Tayur, S. R. (1993). Structural Properties and a Heuristic for Kanban-Controlled Serial Lines. Management Science 39(11), 1347-1368.
Terwiesch, C. and R. E. Bohn (2001). Learning and process improvement during production ramp-up. International Journal of Production Economics 70(1), 1-19.

