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The Impact of Orbit Dependent Return Rate on the Control Policies of a Hybrid Production System

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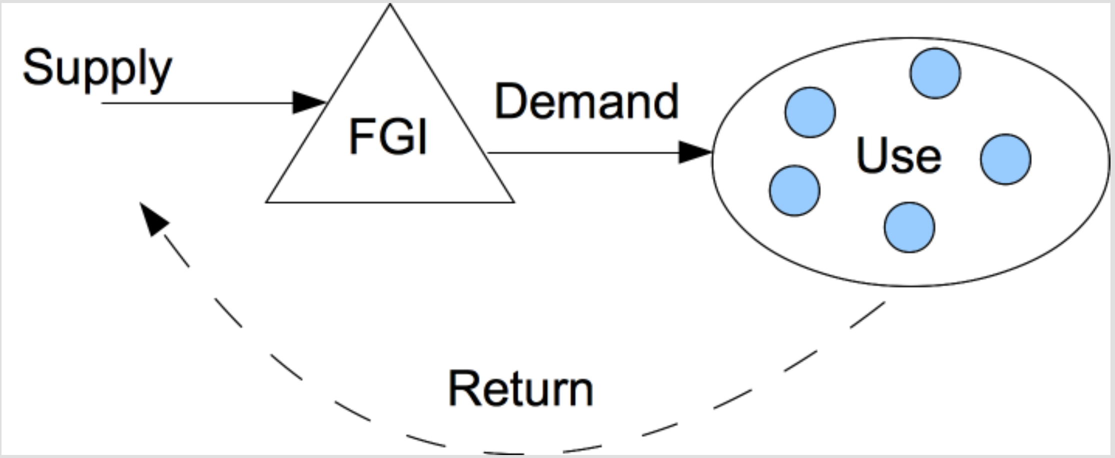
Production Control and Return Flows



Objective: Production control under uncertain return flows.

- Returns may affect inventory levels.
- Return flow estimates are required.
- Returns and supplied demand are dependent through products in use

Production Control with Return Flows

- Literature assumes demand and return flows to be independent in most cases.
- Demand generates new users when the sales is not for replacement.
- The more new products are sold to new users the higher the number of products in use (in orbit).
- The size of the orbit determines the return rate.
- When some of the products in orbit are disposed off rather than being returned an unobservable decrease occurs in the products in use.
- The return rate only gives a partial information about this fact.

Literature on Optimal Control Models

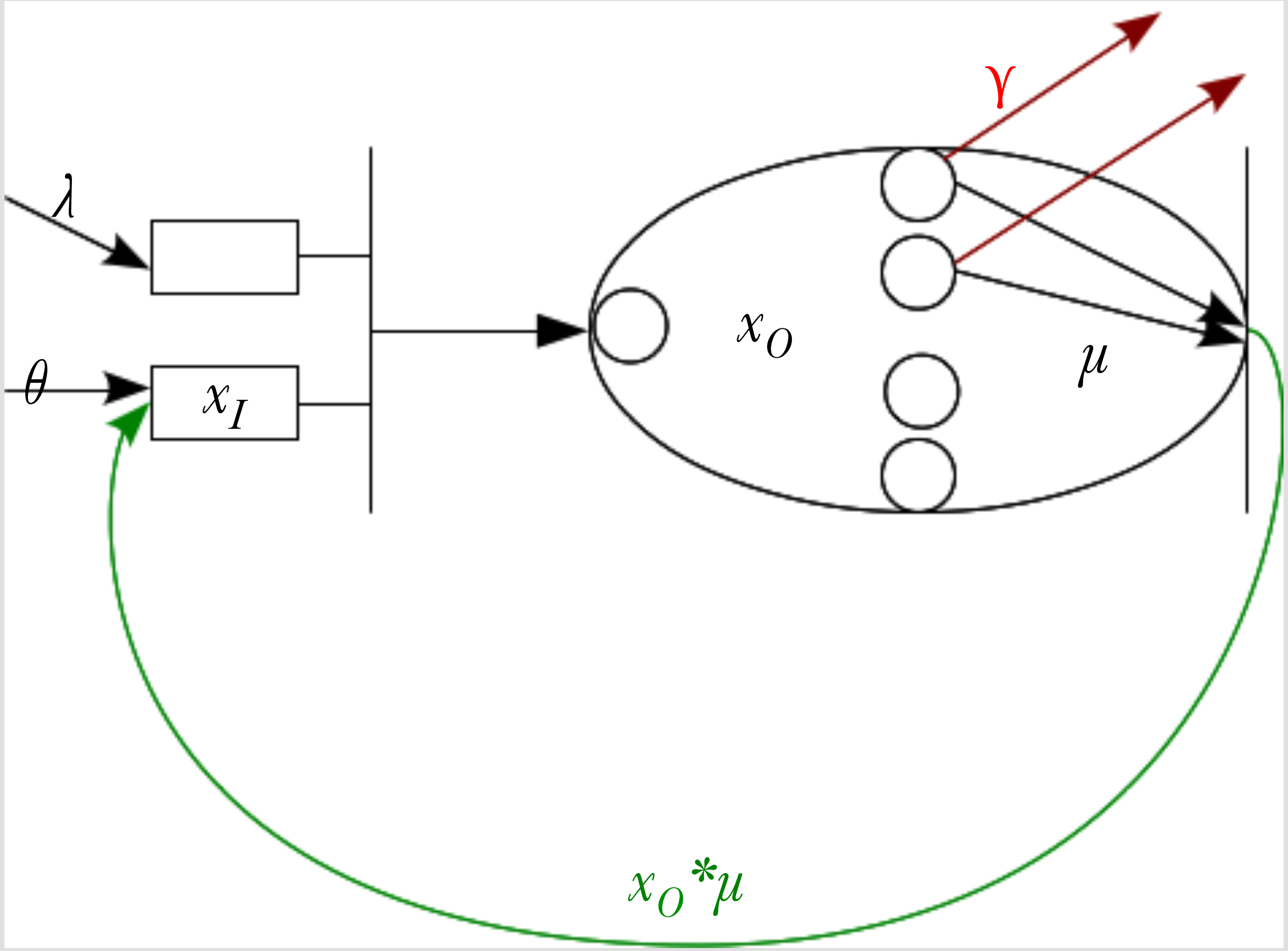
How would the information about orbit affect the production system control?

- Toktay et. *al* (2000)
  - CLSC model using QN with ./M/∞ queue representing users
- Flapper et. *al* (2012)
  - Optimal production control with advanced return info.
- Zerhouni et. *al* (2013)
  - Comparison of dependent vs. independent returns under base stock control.
  - Continuous review base stock optimal for independent returns
  - Instantaneous usage rate assumption for dependent returns

Objectives of the Study

- How does the production decision change under return flows dependent on the size of the orbit?
- What is the value added of using return flows in manufacturing?
- What is the cost of ignoring/inaccurately estimating the information of orbit size?

The Model



λ :Demand rate

μ :Usage rate

γ :Disposal rate

θ :Production rate

$p = \mu/(\gamma+\mu)$

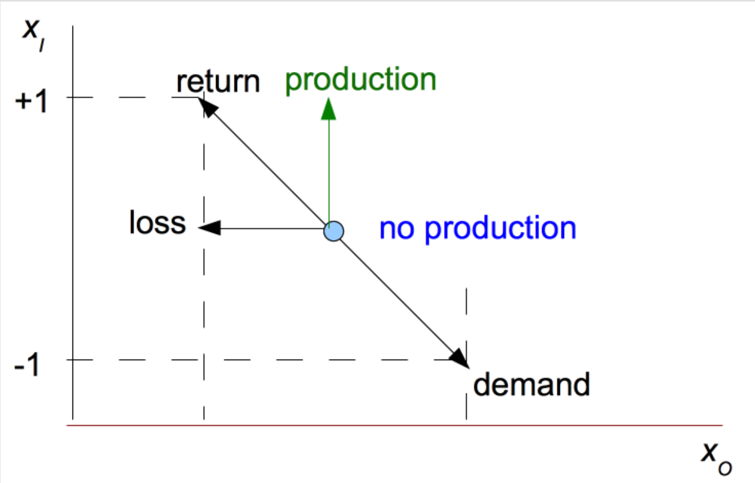
x_0 :Orbit size

x_I :Inv. level

Assumptions

- Each product in orbit can be disposed of with a positive probability.
- No need for remanufacturing (or instant remanufacturing to as good as new).
- All returns are accepted.
- The system is continuously observed.
- x_O is fully observable.

Value Function



$$v(x)+g=\Lambda_2^{-1}\Big[hx_I^{+}+\mu x_O v(x_O-1,x_I+1)$$
$$+\gamma x_O v(x_O-1,x_I)$$
$$+\lambda\begin{cases} v(x_O+1,x_I-1),\ x_I>0 \\ v(x_O,x_I)+c_L,\ x_I=0 \end{cases}$$
$$+\min\theta\{v(x_O,x_I+1),v(x_O,x_I)\}\Big]$$

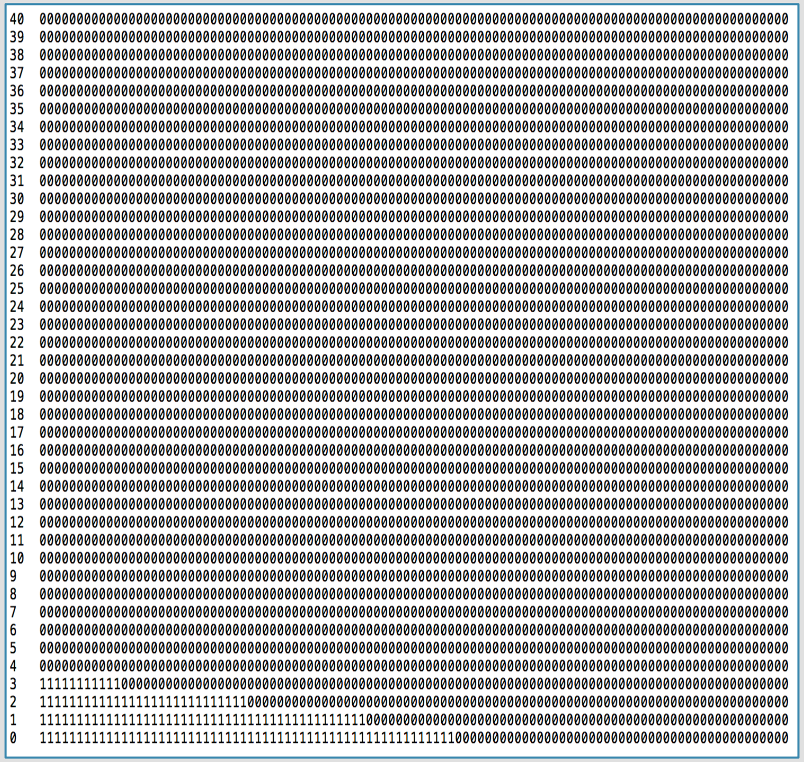
$$\Lambda_1=(\mu+\gamma)x_O^{\max}+\lambda+\theta$$

h : holding cost

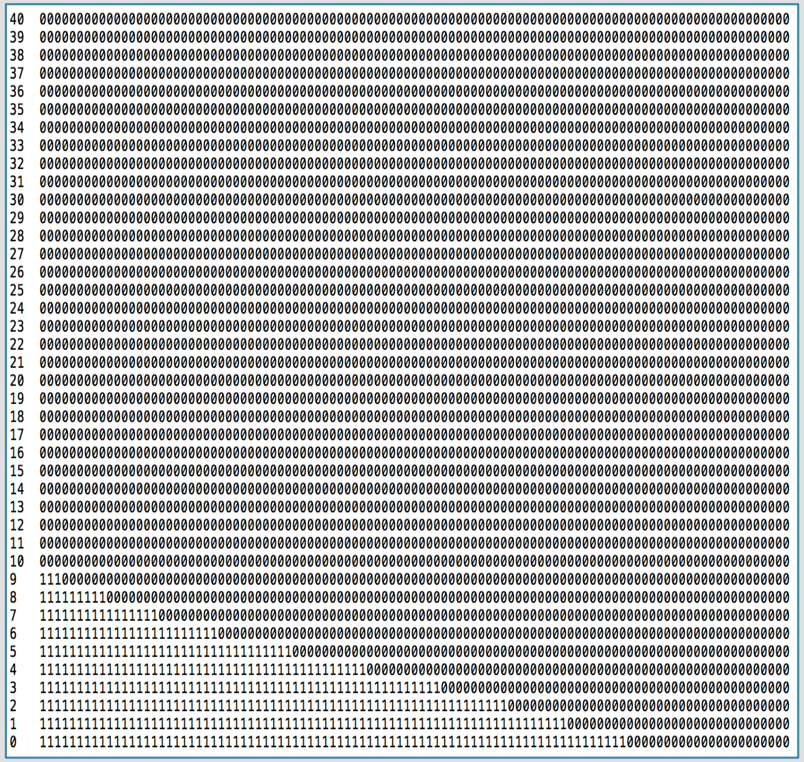
c_L : lost sales cost

Optimal Production Control w. Returns

$$\theta = 1, \lambda = 1.2, \gamma = 0.002, \mu = 0.018, (p = 0.9)$$



c_L/h = 10, S_{max} = 3,



c_L/h = 50, S_{max}=9,

A Basis for Comparison

- Zerhouni et. *al* (2013) show for a state independent stationary return flow the optimal control policy is base stock.
- State independent return rate estimates are needed.
- An estimate can be given as $\hat{\mu} = \alpha \cdot \bar{\mu}, \alpha \in [0, \infty)$

where $\bar{\mu} = E[x_0] \cdot \mu$.

- Thus the average orbit size is required.

LP Model

- The LP equivalent of the value function calculates the stationary distribution $P(x_I, x_0)$ (Büyükdağlı and Fadıloğlu, 2016).

$$\min Z = \sum_{x\in S}\left(c_{x,0}\cdot y_{x,0} + c_{x,1}\cdot y_{x,1}\right)$$

$$s.t.$$

$$\sum_{x\in S}\left(y_{x,0} + y_{x,1}\right) = 1$$

$$\begin{aligned} y_{x,0} + y_{x,1} - \Lambda^{-1}\Big\{&\theta\left(y_{x,0} + y_{x-e_I,1}\right) + \lambda\left(y_{x-e_0+e_I,0} + y_{x-e_0+e_I,1}\right) \\ &+ x_0\mu\left(y_{x+e_0-e_I,0} + y_{x+e_0-e_I,1}\right) + x_0\gamma\left(y_{x-e_I,0} + y_{x-e_I,1}\right) \\ &+ (M - x_0)(\gamma + \mu)\left(y_{x,0} + y_{x,1}\right)\Big\} = 0 \end{aligned}$$

$$\forall\, y_{x,0} \geq 0,\ y_{x,1} \geq 0$$

$$\forall x \in S$$

$$x = (x_0, x_I), e_0 = (1,0), e_I = (0,1)$$

$$c_{x,d} = h\cdot x_I + (1 - x_I)^+ \cdot \lambda \cdot c_L, d = \{0,1\}$$

The Procedure

$\hat{\mu} = \alpha \cdot E[x_0] \cdot \mu$ from LP

↓

$S_{\hat{\mu}}$ from $v(x_I)$

↓

$S_{\hat{\mu}}$ to $v(x) = v_{S_{\hat{\mu}}}(x)$

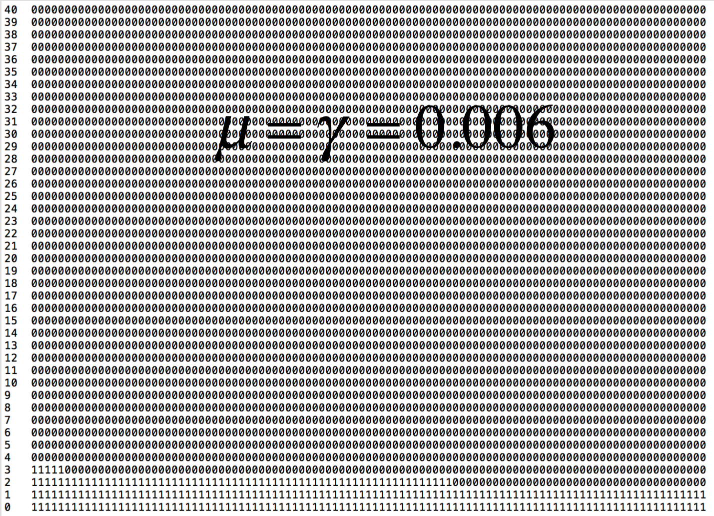
↓

Compare $v(x)$ & $v_{S_{\hat{\mu}}}(x)$

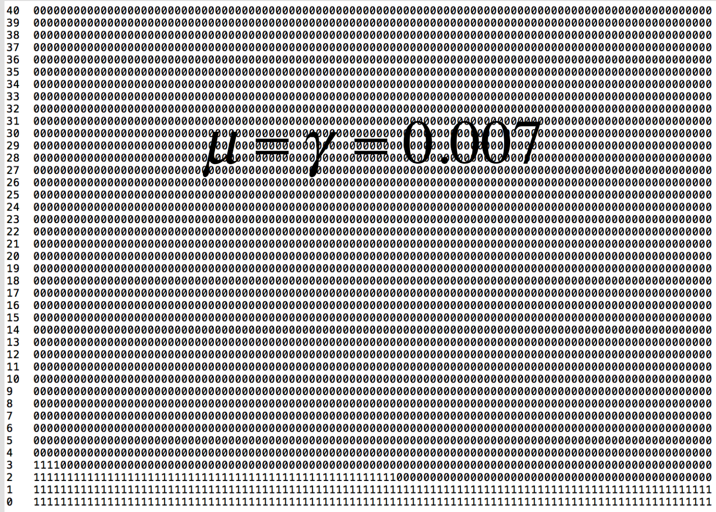
$v(x_I)$ is the value function for optimizing the FGI without using the orbit information.

$S_{\hat{\mu}}$ is the optimal base-stock level for $v(x_I)$.

Decision Matrix from Value Iteration

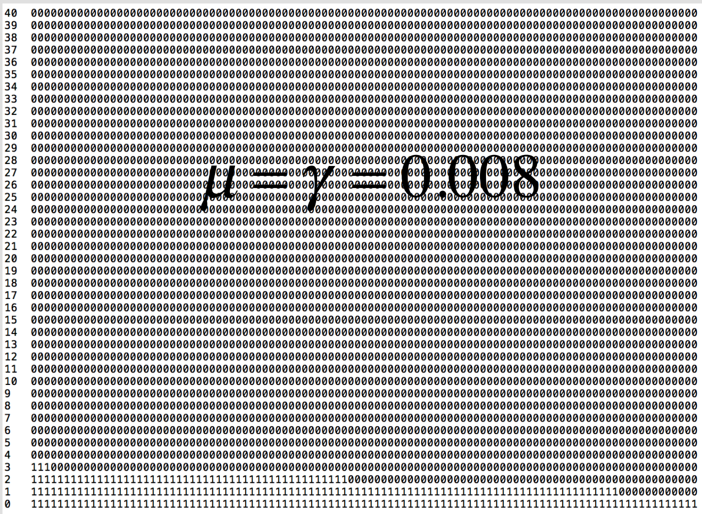


$\mu = \gamma = 0.006$

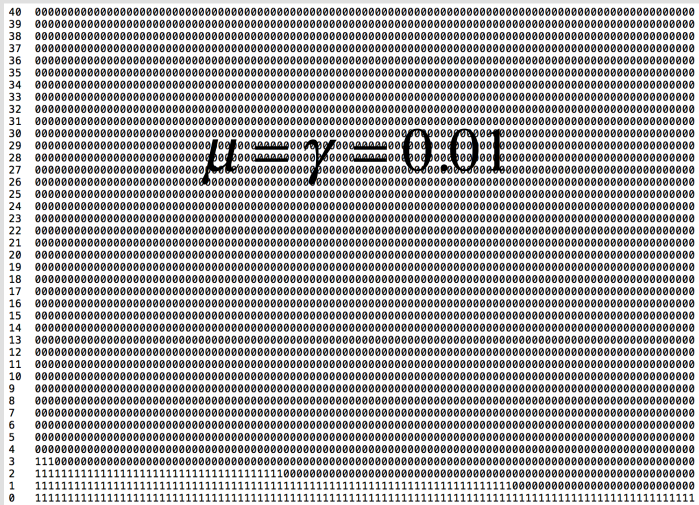


$\mu = \gamma = 0.007$

$$\lambda = 1,\ \theta = 1,\ \mu/\gamma = 1, c_L/h = 50$$



$\mu = \gamma = 0.008$



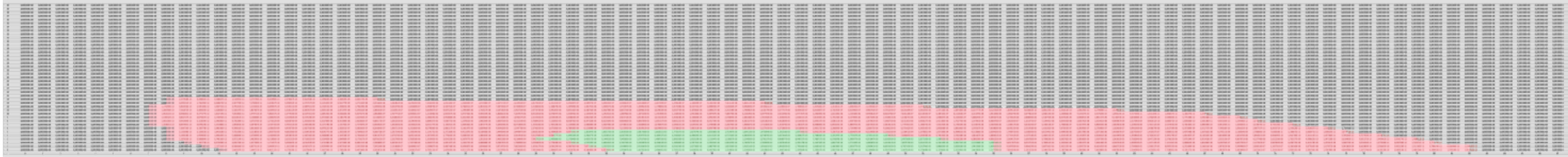
$\mu = \gamma = 0.01$

Stationary Distribution from LP

$\mu = \gamma = 0.006$

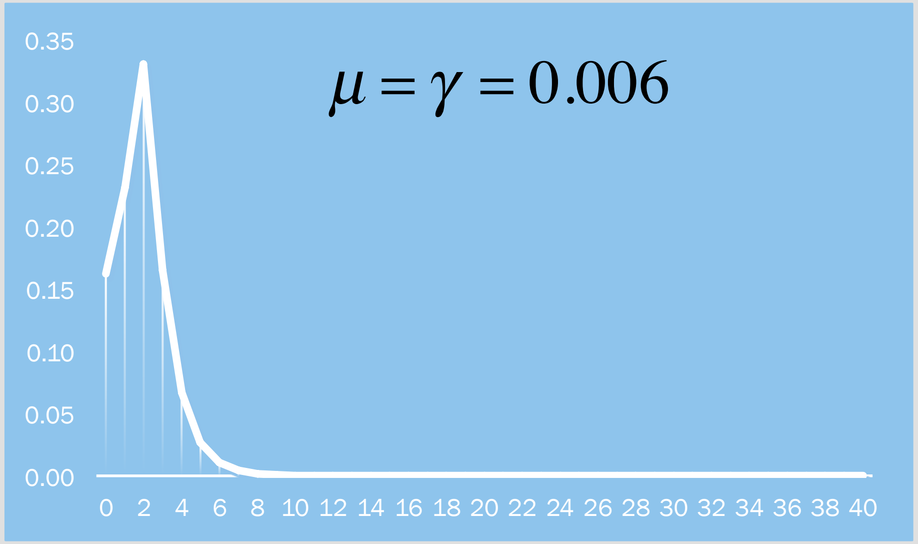
$\mu = \gamma = 0.007$

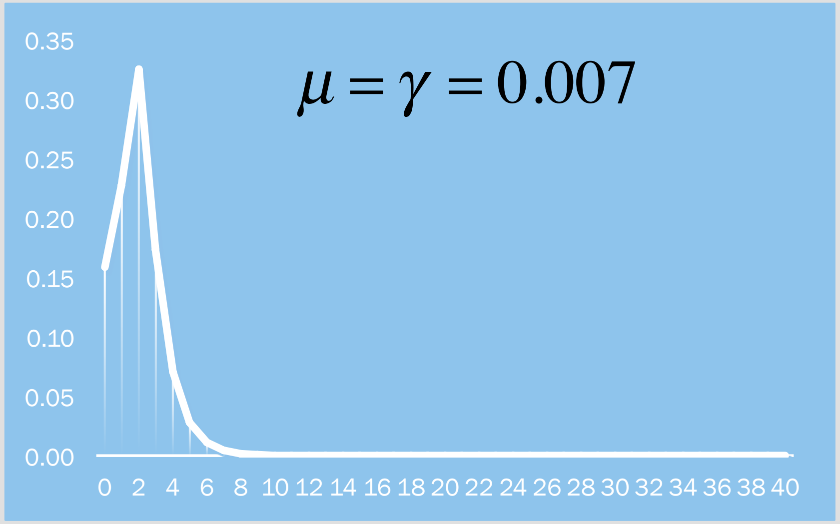
$\mu = \gamma = 0.008$

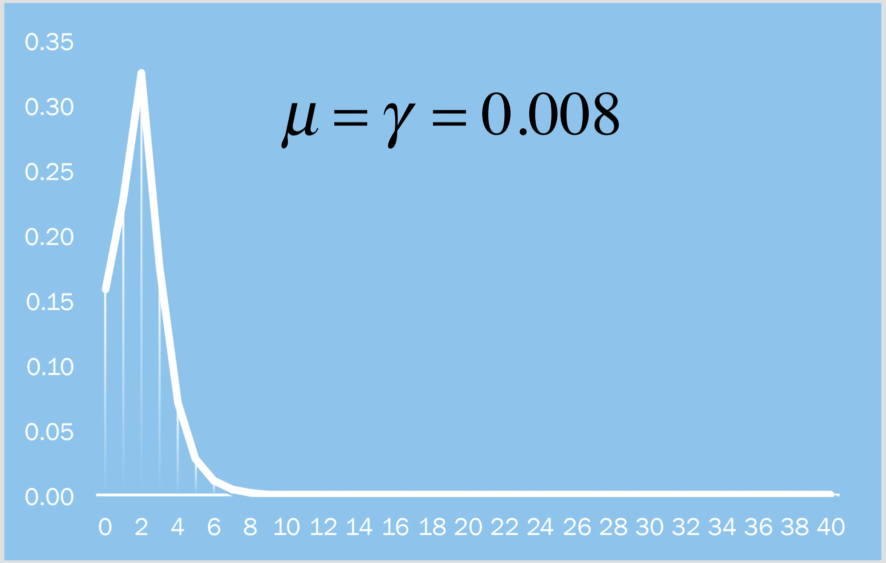


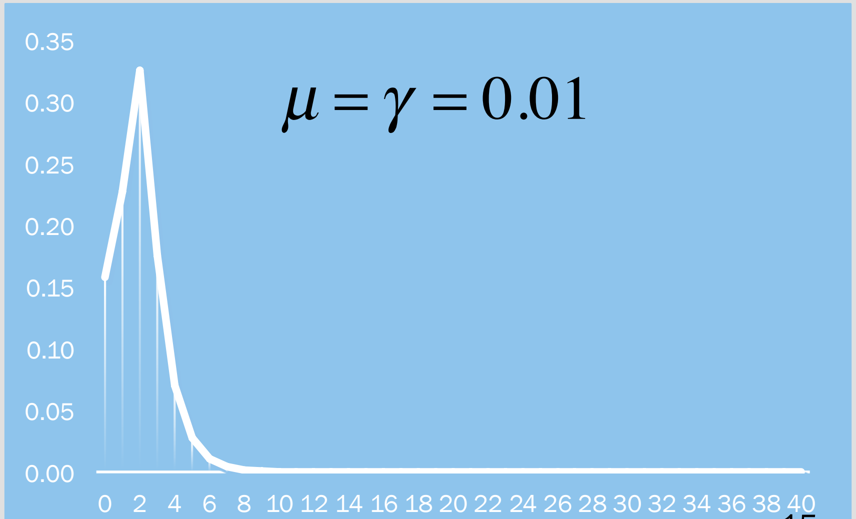
$\mu = \gamma = 0.01$

Marginal Probability of Inventory Level

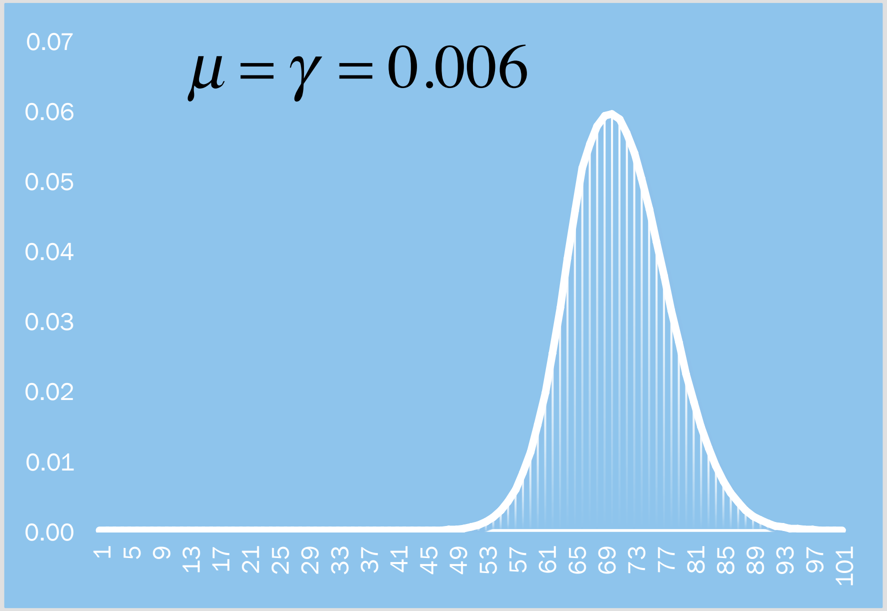


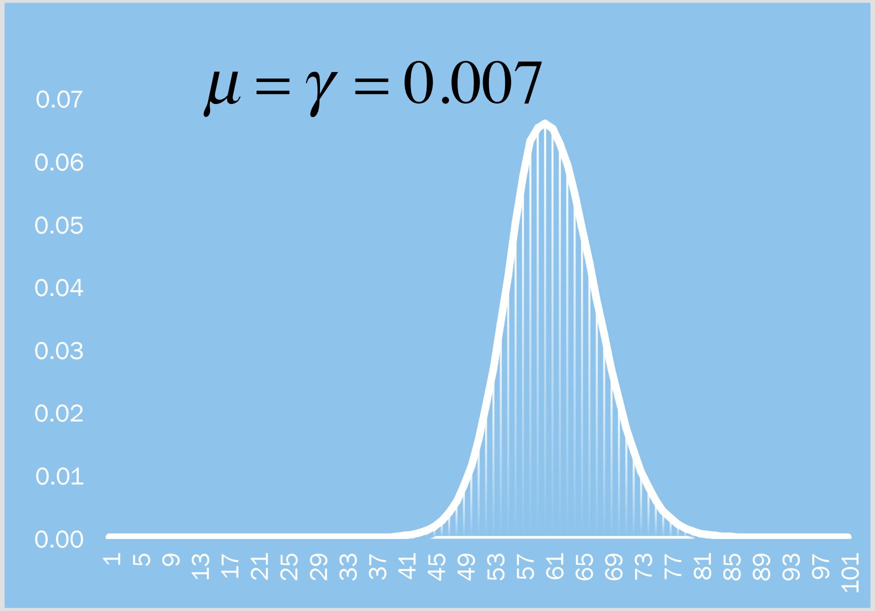


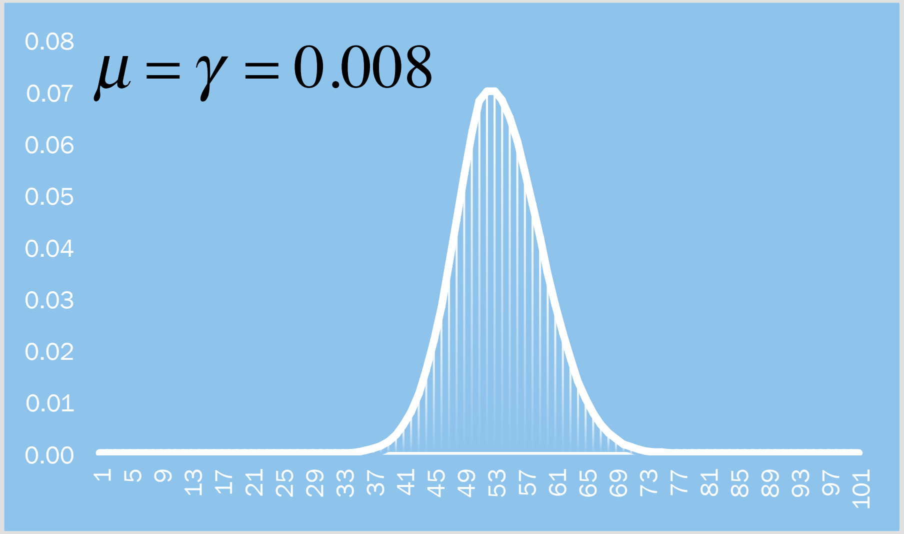


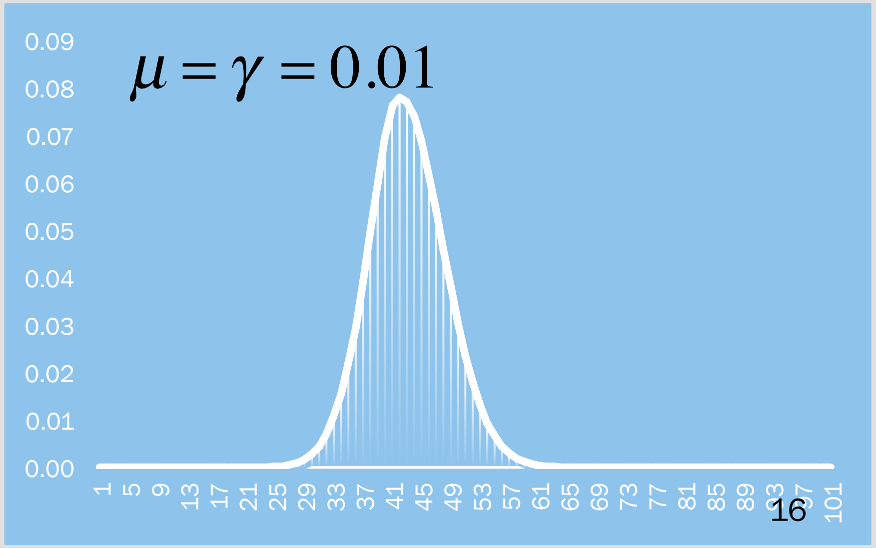


Marginal Probability of Orbit Size





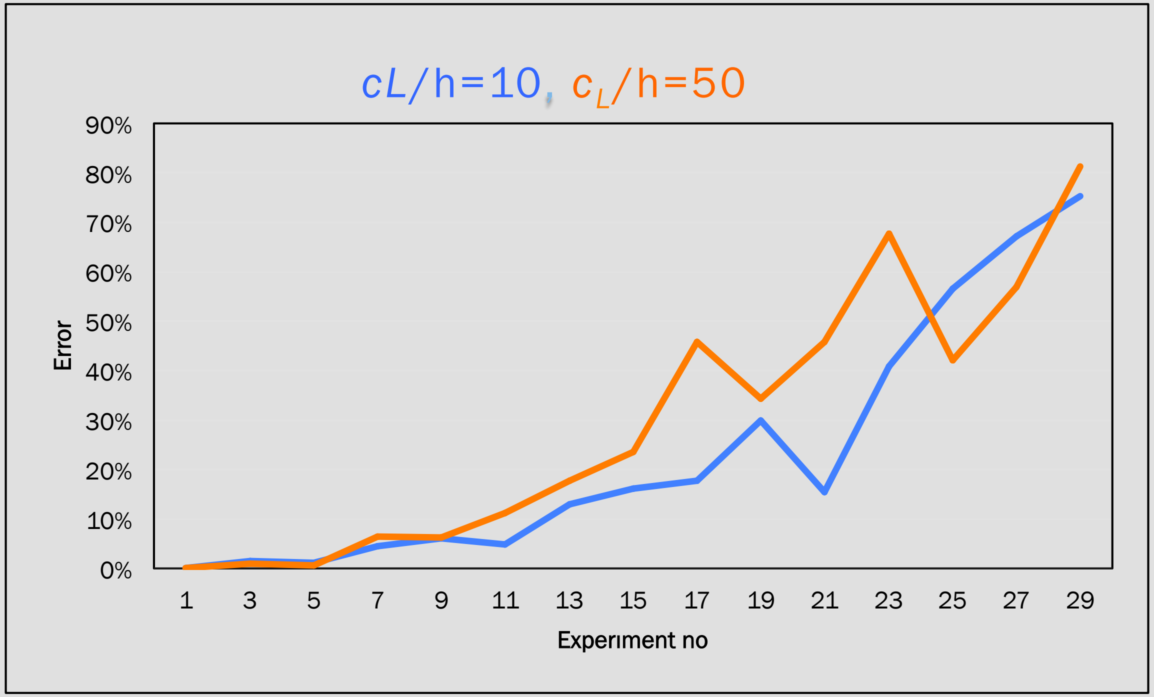




Pure vs. Hybrid Production

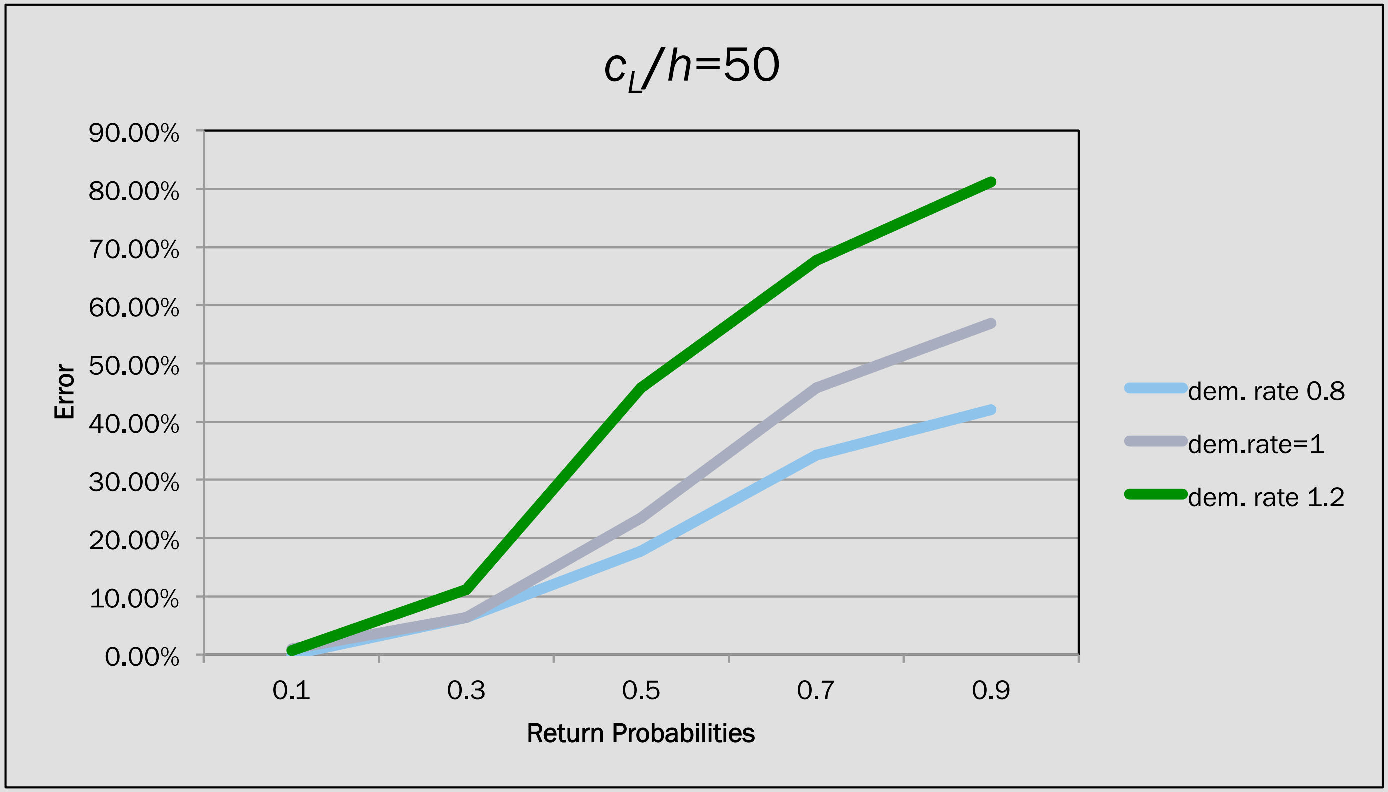
For $\alpha = 0$ we can observe the added value of using returns.
If no returns are accepted S_{max} is the optimal control policy.

$$\theta = 1, \lambda = \{0.8, 1, 1.2\}, \; p = \mu/(\mu + \gamma) = \{0.1, 0.3, 0.5, 0.7, 0.9\},$$
$$\mu = \{0.002, \cdots, 0.018\}, c_L/h = \{10, 50\}, h = 1$$



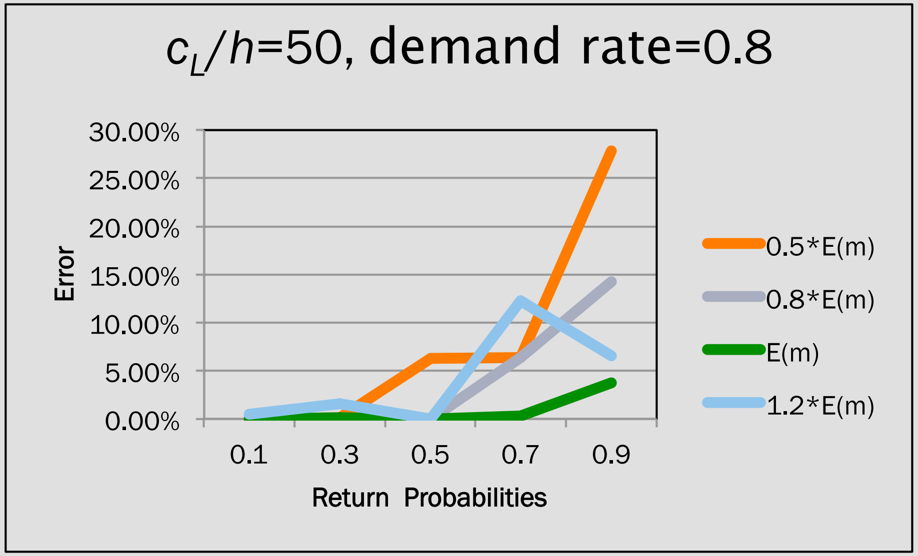
Pure vs. Hybrid Production

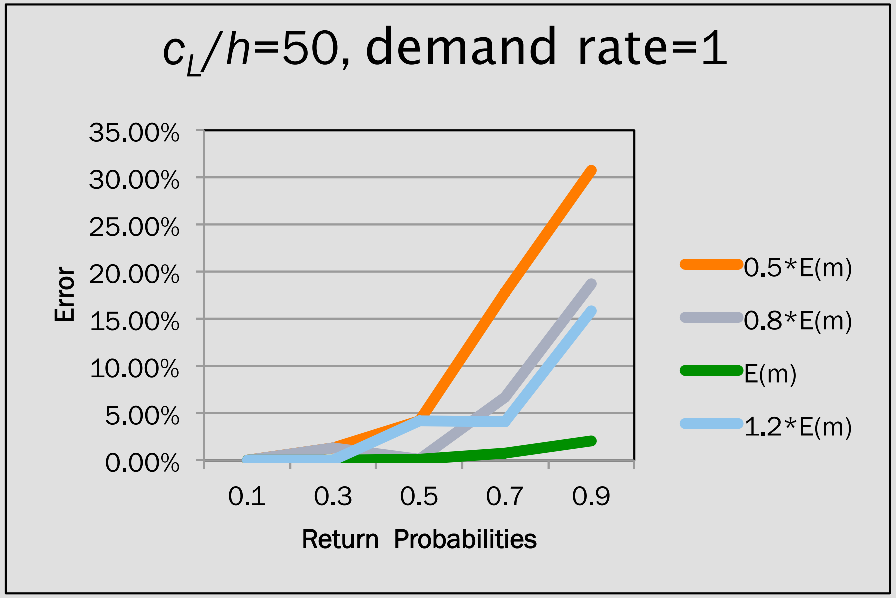
- The impact of p and λ/θ
- As λ/θ increases the impact of returns increase since the orbit expands. For p=0.9 and $\lambda = 1.2, \bar{\mu} = 1.047$

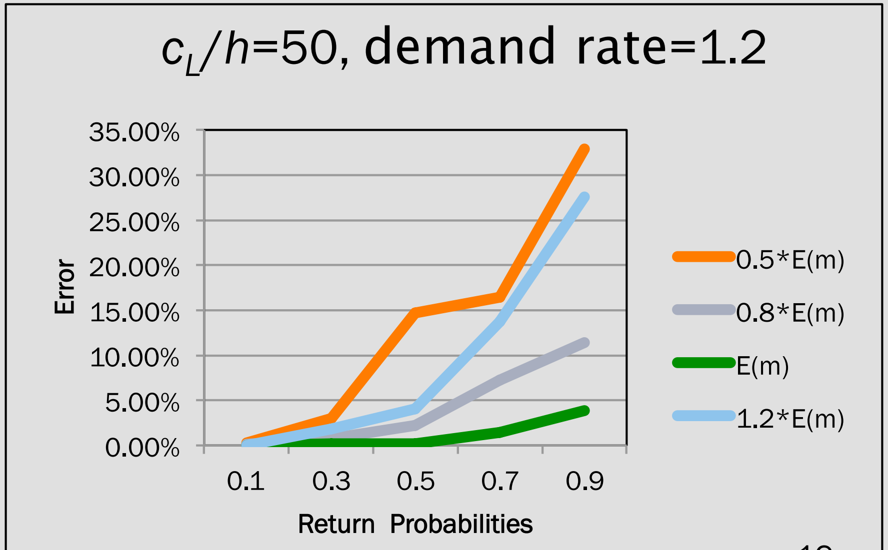


Value of Orbit State

$$\hat{\mu} = \bar{\mu} \cdot \{0.5, 0.8, 1, 1.2,\}$$







Remanufacturing vs Manufacturing Cost

Control levels for $\bar{\mu}$ model under the impact of production costs.

$$\theta = 1, c_L / h = 50, c_M = 5, c_R = 2$$

	$\lambda = 0.8$		$\lambda = 0.8$		$\lambda = 0.8$	
p	S with $C_{R--}C_{M-}$	S	S with $C_{R--}C_{M-}$	S	S with $C_{R--}C_{M-}$	S
0.1	4	5	6	7	10	10
0.3	3	4	5	5	7	7
0.5	3	3	4	4	5	5
0.7	2	2	3	3	3	4
0.9	1	1	1	1	2	2

Conclusions

- A testbed for measuring the impact of the number of products in use (orbit) on the production control of hybrid production systems is introduced.
- The added value of using returns for supplying the demand is clearly displayed.
- Results indicate that the base stock policy is heavily dependent on the orbit size and return probabilities.
- The difference between complete information and no information present the value of the partial observation effort.

Future Work

Model structure extensions:

- Return admission control
- Remanufacturing lead time
- Demand differentiation
- The feedback relation between the orbit and the demand rate.

Testbed extensions:

- Automated LP model
- LP model for upto S levels
- Measure the effectiveness of proposed methods so far

Thank You

References:

Toktay et. *al* (2000), Inventory Management Of Remanufacturable Products, Mgt. Sci., 46.

Flapper et. *al* (2012), Control Of A Production-inventory System With Returns Under Imperfect Advance Return Information, EJOR, 218.

Zerhouni et. *al* (2013), Influence Of Dependency Between Demands And Returns In A Reverse Logistics System, IJPE 143.

Büyükdağlı and Fadıloğlu (2016) A Mathematical Programming Framework for Control of Tandem Production Lines, Research Report.

Distributions of the given example.

$$\theta = 1, \lambda = 1.2, \gamma = 0.002, \mu = 0.018, (p = 0.9), c_L / h = 50$$

$$E\left[x_I\right] = 6{,}195 \qquad \bar{\mu} = 1{,}047,\ E\left[x_0\right] = 58$$