Modeling and Analysis of a Collaborative Network

Behnaz Hosseini * & Barış Tan *+

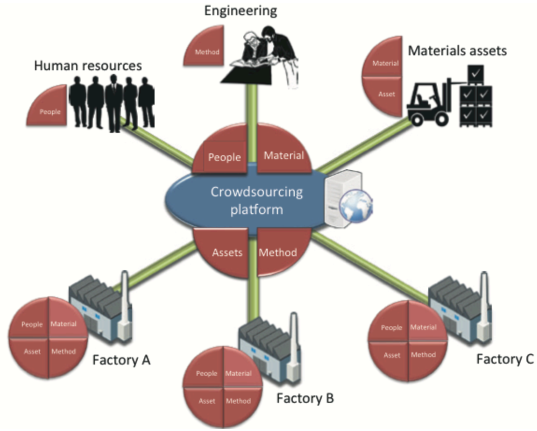
*College of Engineering
+ College of Administrative Sciences and Economics

KOÇ UNIVERSITY

Outline

- Motivation
- Introduction to the model of collaborative network
- Analysis of the model
- Conclusion
- Future work

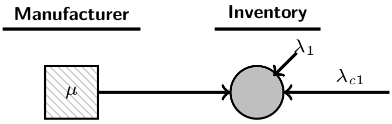
Motivation

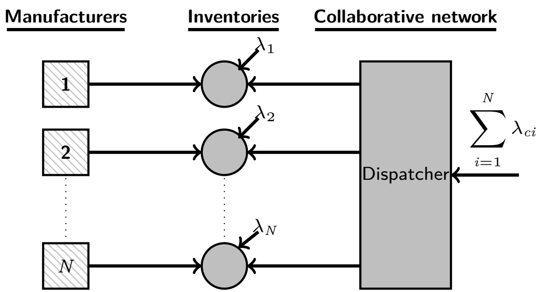


"Factory of the future" IEC (2014)

- *Crowd Manufacturing* term is used for a **collaborative production network** that is created by a number of manufacturers cooperating in a flexible arrangement.
- Creating collaborative network supports both fast-changing customer needs and high utilization rate of production systems.
- Analyzing business models for *crowd manufacturing* is important to realize the financial and service level benefits of cooperation.

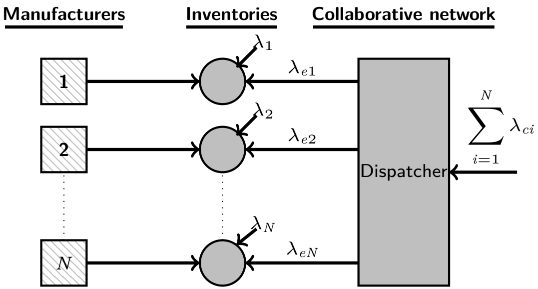
The Collaborative Network





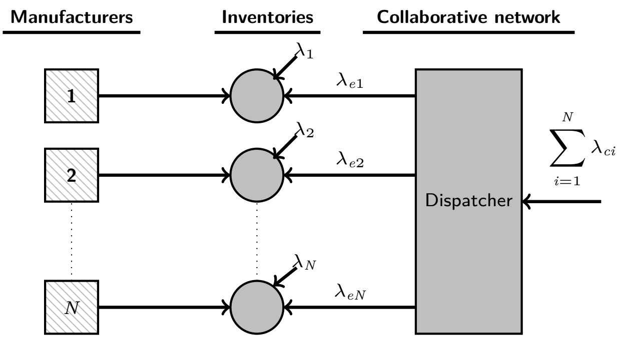
- We consider manufacturers producing for their own customers and also for external customers that require a lower price at a desired service level (e.g. private label producers).
- Manufacturers enter a **collaborative network** that consolidates external customers for its members and dispatch an arriving external customer to an available manufacturer.
- We analyze the financial and service-level benefits of collaborative network provided to the participant manufacturers.

Research Questions



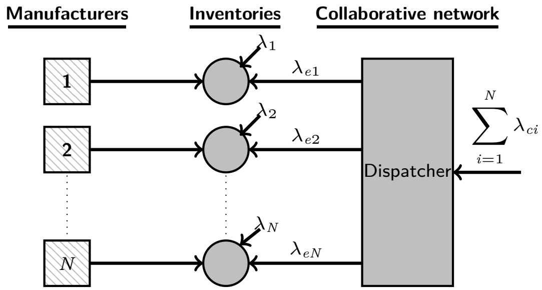
- What is the **optimal dispatching policy** of the network to maximize the profit?
- What are the **optimal rationing and production policies** for the manufacturers to maximize the profit?
- Is the collaborative network **beneficial** or not?

Analytical Model for the Collaborative Network



- N producers with their own and external customers.
- Exponential production and interarrival times.
- External customers place an order if the network offers their required service level γ^*.
- Network sends an arriving external customer to one of the producers according to the dispatching policy.
- Network should decide on the routing policy.
- Manufacturers should decide on whether to produce or not and whether to accept or reject the arriving customers.

The Model is Different from the Others in the Literature



- External customers arrival does not follow a Poisson stream.
- λ_{ei} depends on rationing and production policies of all participants.

This Work is Related to Two Streams of Literature: Stock Allocation and Cooperation

Stock allocation problem:

- Ha (1997)
- De Vericourt et al. (2001)
- De Vericourt et al. (2002)
- Frank et al. (2003)
- Deshpande et al. (2003)
- Huang and Iravani (2007)
- Gayon et al. (2009)
- Pang et al. (2014)
- Yu et al. (2015)
- Liu et al. (2015)

Cooperation:

- Bucklin and Sengupta (1993)
- Park and Russo (1996)
- Benjaafar et al. (2005)
- Yu et al. (2006)
- Tan (2006)
- Akcay and Tan (2008)
- Swaminathan and Moorman (2009)
- Tan(2014)

- Analysis of the model
  - Analysis of the independent operation (no cooperation case) $\Leftarrow$
  - Exact analysis of optimal production, rationing and routing policies of the collaborative network with heterogenous participants using mathematical programming
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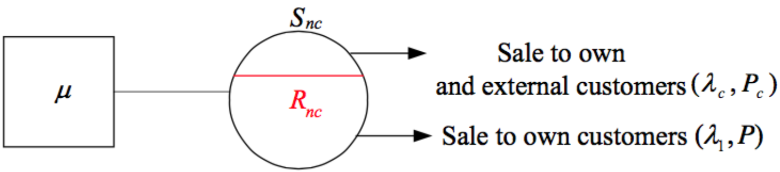
Analysis of the Independent Operation



- Maximizing profit rate function:

$$\underset{S_{nc},R_{nc}}{\text{Maximize}} \;\; \pi_{nc} = \lambda_1(Pr(I_{nc} > 0))P + \lambda_c(Pr(I_{nc} > R_{nc}))P_c - h\overline{I}_{nc}$$

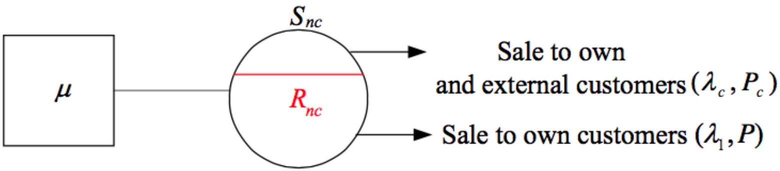
Subject to:

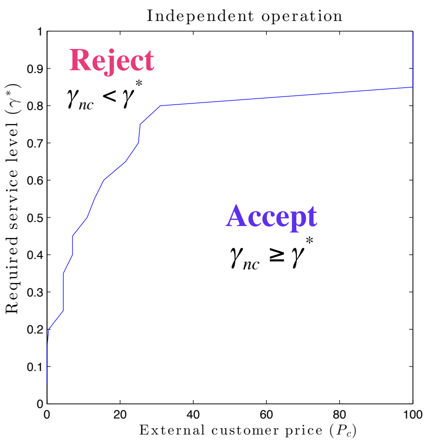
$$\gamma_{nc} \geq \gamma^*.$$

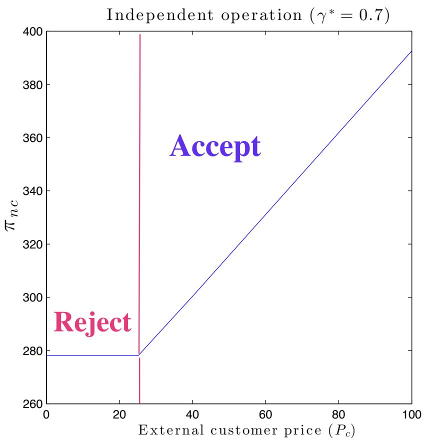
- Service level: $\gamma_{nc} = Pr(I_{nc} > R_{nc})$.

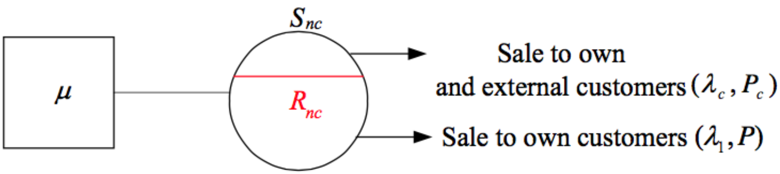
(nc: No Cooperation Case)

Manufacturers can't Offer Low Price and High Service Level to External Customers Independently









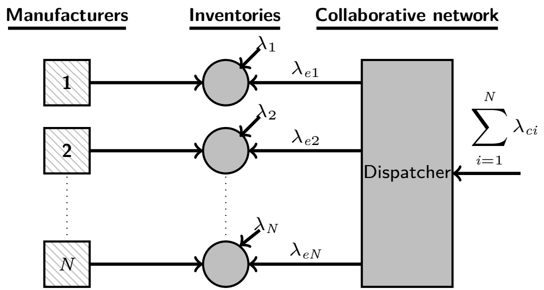
$$Pr(I > R) = 1 - \frac{(1-\xi)\xi^{S-R}(1-\rho^{R+1})}{(1-\rho)(1-\xi^{S-R}) + (1-\xi)\xi^{S-R}(1-\rho^{R+1})},$$

$$Pr(I > 0) = 1 - \frac{(1-\xi)(1-\rho)\xi^{S-R}\rho^{R}}{(1-\rho)(1-\xi^{S-R}) + (1-\xi)\xi^{S-R}(1-\rho^{R+1})},$$

$$\overline{I} = E[I(t)] = S - \frac{(\xi(1-\rho)/(1-\xi))(1-\xi^{S-R} - (1-\xi)(S-R)\xi^{S-R-1})}{(1-\rho)(1-\xi^{S-R}) + (1-\xi)\xi^{S-R}(1-\rho^{R+1})}$$
$$- \frac{(\rho(1-\xi)/(1-\rho))(\xi/\rho)^{S-R}(\rho^{S-R} - \rho^{S+1} + (1-\rho)[(S-R)\rho^{S-R-1} - (S+1)\rho^{S}])}{(1-\rho)(1-\xi^{S-R}) + (1-\xi).\xi^{S-R}(1-\rho^{R+1})}$$

(Ha,1997)

- Analysis of the model
  - Analysis of the independent operation (no cooperation case)
  - Exact analysis of optimal production, rationing and routing policies of the collaborative network with heterogeneous participants using mathematical programming $\Leftarrow$
  - Numerical results for the LP formulation of collaborative network with two heterogenous manufacturers
  - Approximate analysis of the collaborative network with identical participants
  - Numerical results for the collaborative network with a mass number of identical manufacturers



$$\text{Maximize} \;\; \pi_c$$

Subject to:

$$\gamma_c \geq \gamma^*.$$

- The average reward per unit time over an infinite planning horizon:

$$\lim_{T\to\infty} \sup_{v\in\mathbb{V}(x^0)} \left\{ \frac{1}{T} E_{x^0}^{v} \left[\sum_{i=1}^{N} \int_0^T P_i n_i^o(t)\,dt + \int_0^T P_c n_e(t)\,dt - \sum_{i=1}^{N} \int_0^T h_i I_i(t)\,dt \right] \right\}$$

Parameters:

- State of the system (Inventories): $x = [I_1 I_2 \dots I_N], I_i \in \mathbb{I} = \{0, 1, \dots, M_i\}$.
- Production decision: $u = [u_1 u_2 \dots u_N], u_i \in \mathbb{U} = \{0, 1\}$.
- Accept or reject decision made by each producer for own customers : $f = [f_1 f_2 \dots f_N], f_i \in \mathbb{F} = \{0, 1\}$.
- Dispatching decision made by network for external customers: $d \in \mathbb{D} = \{0, 1, \dots, N\}$.
- Uniformization rate: $UR = N\lambda_c + \sum_{i=1}^{N} \lambda_i + \sum_{i=1}^{N} \mu_i$.

Decision variables:

- Steady state probability of being in state x: P_{xufd}.

- Profit rate function (π_c):

$$\pi_c = \sum_x \sum_u \sum_f \sum_d PR_{xufd} P_{xufd}.$$

$$PR_{xfd} = \sum_{i=1}^{N} f_i \lambda_i P_i + \sum_{i=1}^{N-1} g(d,i) N \lambda_c P_{ci} + w(d) N \lambda_c P_{cN} - \sum_{i=1}^{N} I_i h_i.$$

$$g(d,i) = \frac{\prod_{\substack{j=0 \\ j\neq i}}^{N} (d-j)}{\prod_{\substack{j=0 \\ j\neq i}}^{N} (i-j)}$$

$$w(d) = \frac{\prod_{j=0}^{N-1} (d-j)}{N!}$$

$$\max \quad \pi_c$$

Subject to

$$\sum_u \sum_f \sum_d \Bigg[P_{xufd} - \frac{1}{UR} \Bigg\{ \sum_{i=1}^{N} u_i \mu_i P_{(x-e_i)ufd} + \sum_{i=1}^{N} f_i \lambda_i P_{(x+e_i)ufd}$$

$$+ \sum_{i=1}^{N-1} g(d,i) N \lambda_c P_{(x+e_i)ufd} + w(d) N \lambda_c P_{(x+e_N)ufd}$$

$$+ P_{xufd} \Bigg(UR - \sum_{i=1}^{N} u_i \mu_i - \sum_{i=1}^{N} f_i \lambda_i - \sum_{i=1}^{N-1} g(d,i) N \lambda_c - w(d) N \lambda_c \Bigg) \Bigg\} \Bigg] = 0 \qquad \forall x,$$

$$\sum_x \sum_u \sum_f \sum_d P_{xufd} = 1$$

$$P_{xufd} = 0 \qquad \forall i, I_i \in \{M_i\}, u_i \in \{1\}, \forall f, \forall d,$$

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$$P_{xufd} = 0 \qquad \forall i, I_i \in \{0\}, d \in \{1, \dots, N\}, \forall u, \forall f,$$

$$P_{xufd} = 0 \qquad \forall i, I_i \in \{-1, M_i + 1\}, \forall u, \forall f, \forall d,$$

$$\sum_x \sum_u \sum_f \sum_{d=1}^{N} P_{xufd} \geq \gamma^*.$$

Optimal Production, Rationing and Routing Policies for Two Identical Manufacturers in the Network

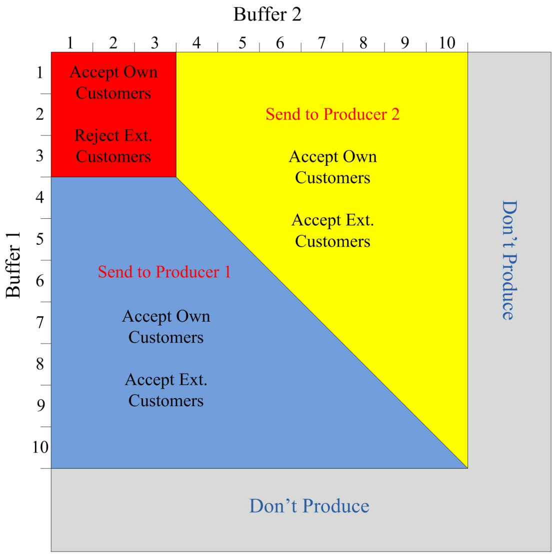

Figure 1: Optimal routing policy for network customers with identical manufacturers participating in the network
($\lambda_1 = 2, \lambda_2 = 2, \mu = 4, \lambda_c = 5, P = 100, P_c = 40, h = 1, N = 2, \gamma^* = 0.6$)

Optimal Production, Rationing and Routing Policies for Two Identical Manufacturers in the Network

- The optimal production policy is the base stock policy.
- The optimal rationing policy is to accept the second group of customers whenever the level of inventory is greater than the rationing level R.
- The optimal routing policy for identical participants is to send the network customer to the manufacturer whose inventory level is the highest one over the rationing level $R \implies$ Join the Shortest Queue Routing Policy

What are the Optimal Production, Rationing and Routing Policies for Two Heterogeneous Manufacturers?

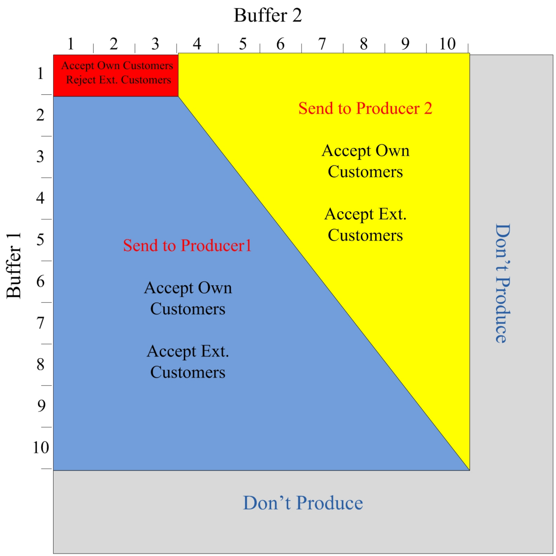

Figure 2: Optimal policies for network customers with heterogeneous manufacturers participating in the network
$(\lambda_1 = 2, \lambda_2 = 4, \mu = 4, \lambda_c = 5, P = 100, P_c = 40, h = 1, N = 2, \gamma^* = 0.6)$

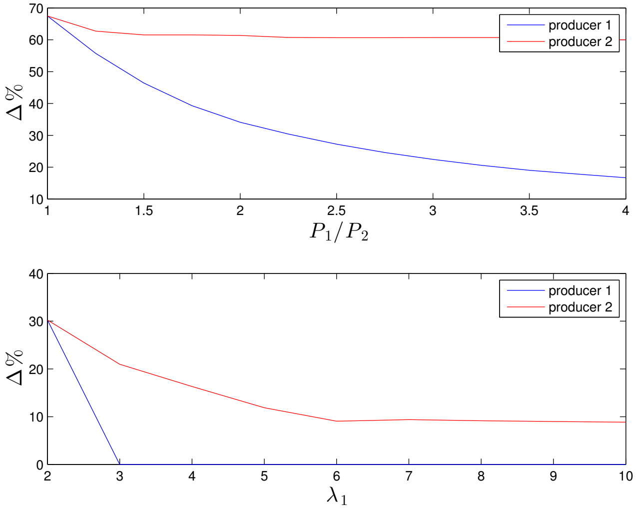

Figure 3: Profit increase for different values of price and arriving rate in heterogeneous network setting with optimal routing policy ($\lambda_2 = 2, \mu = 4, P_c = 70, h = 1, N = 2, \gamma^* = 0.8$)

Offered Service Level in the Network Setting with Identical Manufacturers

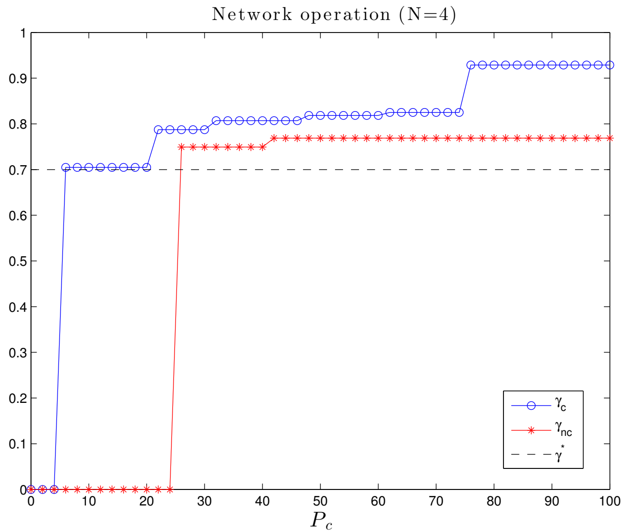

Figure 4: Service level for different values of external customers price in network setting with JSQ routing policy ($\lambda = 4, \mu = 5, P = 100, h = 30, N = 4, \gamma^* = 0.7$)

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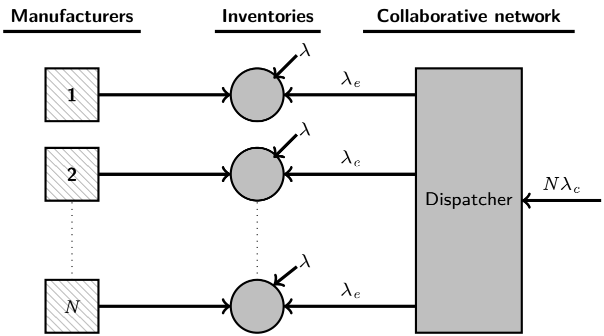
Approximate Analysis of the Collaborative Network with Identical Manufacturers



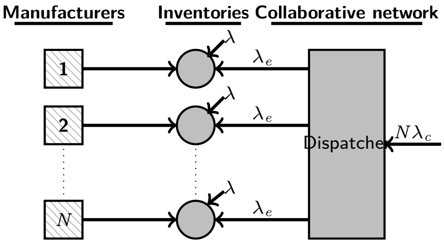
- N identical make-to-stock producers with own and external customers.
- Each producer uses a rationing policy to accept or reject arriving second-class customers and uses a base-stock policy for production.
- Network dispatches an arriving customer to one of the available producers with equal probability (the level of inventory for available producers is greater than the rationing level).
- If there are no available producers, the order is lost.

Approximate Analysis of the Collaborative Network with Identical Manufacturers

Proposition 1: Single Queue Approximation Method

The collaborative network with N identical participant manufacturers can be modeled as a $M/M/N/R$ queue where R is the rationing level routing policy of the network. Therefore, the single queue approximation with the exact state-dependent arrival rates leads to the same steady state queue length distribution as in the original $M/M/N/R$ model for the collaborative network model.

Approximate Analysis of the Collaborative Network with Identical Manufacturers



$$\underset{S_c, R_c}{\text{Maximize}} \;\; \pi_c$$

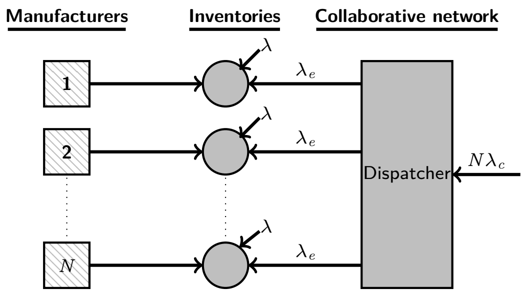
Subject to:

$$\gamma_c \geq \gamma^*.$$

$$\pi_c = \lambda(Pr(I_c > 0))P + \lambda_e(Pr(I_c > R_c))P_c - h\overline{I}_c.$$

$$\gamma_c = 1 - Pr(I_c^1 \leq R_c^1, I_c^2 \leq R_c^2, \cdots, I_c^N \leq R_c^N).$$

$$\gamma_c \approxeq 1 - [1 - Pr(I_c > R_c)]^N.$$



Assuming that there are $m+1$ available manufacturers among N participants:

$$\lambda_e = N\lambda_c\left(\sum_{m=0}^{N-1}\frac{1}{m+1}Pr(M=m)\right)$$

$$Pr(M=m) = \binom{N-1}{m}(Pr(I_c > R_c))^m(1-Pr(I_c > R_c))^{N-m-1}.$$

Closed form approximation for external customers conditional arrival rate:

$$\lambda_e = \lambda_c\frac{(1-(1-Pr(I_c > R_c))^N)}{Pr(I_c > R_c)}.$$

An Approximation for the Arrival Rate of Network Customers

$$\lambda_e = \lambda_c \frac{(1-(1-Pr(I_c > R_c))^N)}{Pr(I_c > R_c)}.$$

$$\xi_c = (\lambda_1 + \lambda_e)/\mu.$$

$$Pr(I_c > R_c) = 1 - \frac{(1-\xi_c)\xi_c^{S_c - R_c}(1-\rho^{R_c+1})}{(1-\rho)(1-\xi_c^{S_c-R_c}) + (1-\xi_c)\xi_c^{S_c-R_c}(1-\rho^{R_c+1})}.$$

$\implies$ Solving nonlinear Equation to find conditional arrival rate.

- Single Queue Approximation that uses the approximate arrival rate allows us to evaluate the performance of the network accurately.

Percentage Errors					
	$Pr(I_c > 0)$	$Pr(I_c > R_c)$	$\overline{I_c}$	λ_e	γ_c
Maximum	1.5172	9.7679	10.3453	0.045	0.0636
Average	0.4576	2.6615	3.8675	0.0215	0.0294
Minimum	0.0405	0.0208	0.3370	0.0052	0.0071

(Average results for 20 cases of networks up to 10 members)

Operation of the Collaborative Production Network with Rationing Level Routing Policy

Proposition 2

The profit obtained by each producer in the collaborative network with N identical participants and rationing level dispatching policy is always greater than or at least equal to the profit obtained in the independent setting where the producer receives the second-class requests directly.

Proposition 3

The total profit obtained when the collaborative network optimizes the base-stock levels and the rationing levels of all the members centrally is the same as the total profit when each member optimizes its own base-stock and rationing levels.

Network Offer Low Price and High Service Level to External Customers

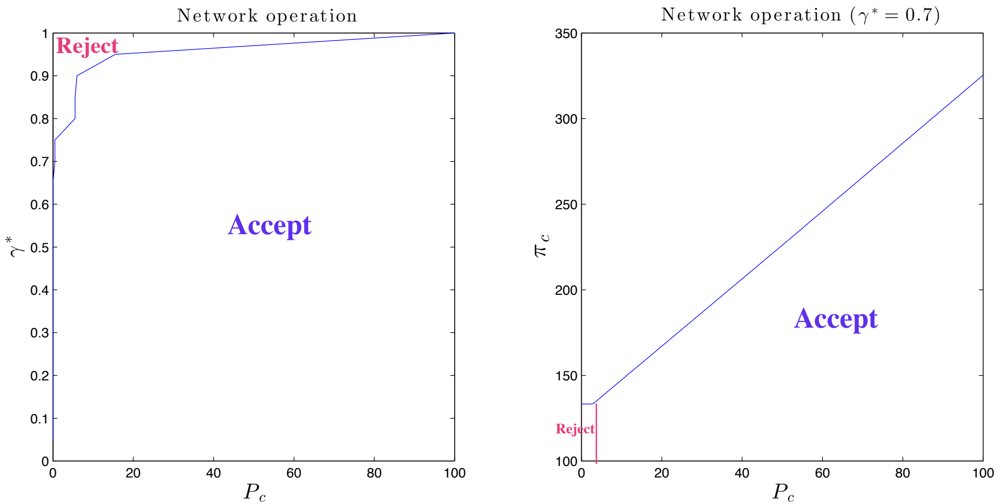


Figure 5: External customers acceptance area and profit for network based operations ($\lambda = 4, \mu = 5, N = 15, P = 100, h = 5$)

Accept or Reject?

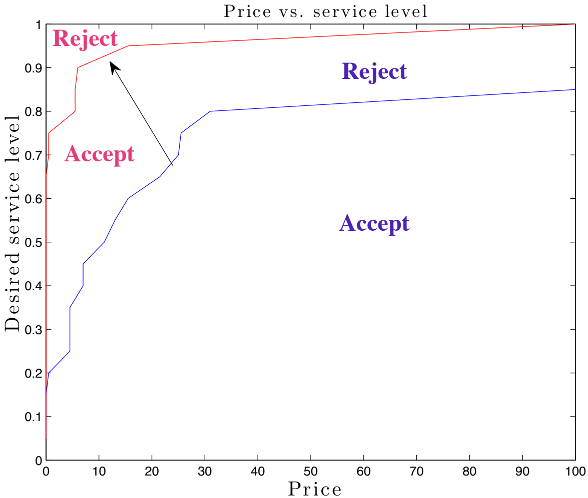


Figure 6: Acceptance and rejection areas for external customers ($\lambda = 4, \mu = 5, P = 100, h = 5$)

For the network with rationing level routing policy:

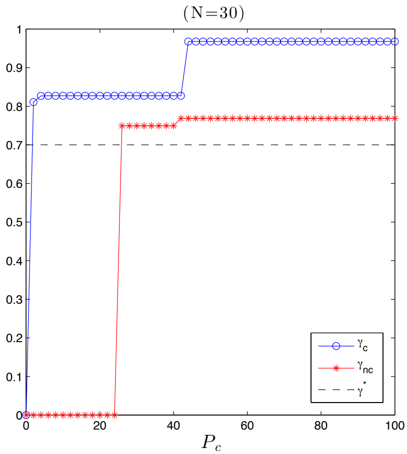


Figure 7: Network based service level and producer based service level for different values of price ($\lambda = 4, \mu = 5, \lambda_c = 2, P = 100, h = 30, \gamma^* = 0.7$)

Effect of Price and Desired Service Level on the Profit Increase

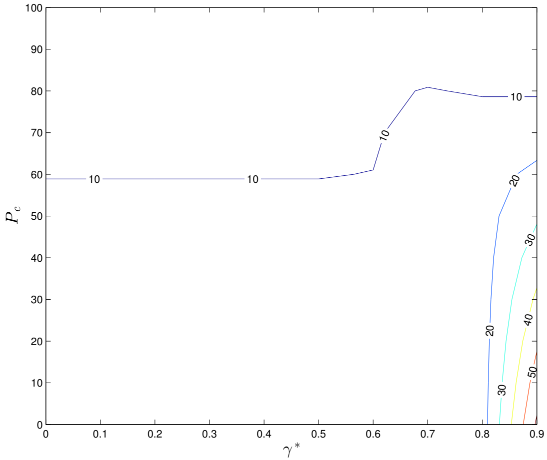


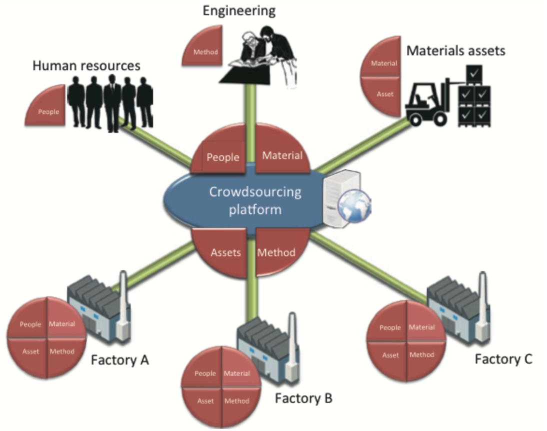
Figure 8: Graph for price (P_c), requested service level (γ^*) and profit increase (Δ) ($\lambda = 4, \mu = 5, \lambda_c = 2, P = 100, h = 30, N = 10$)

Conclusions

- We present an analytical model to analyze the business model of collaborative network.
- We analyzed the optimal production, rationing and routing policies of the network with heterogeneous participants.
- An approximate analysis is developed for the network with high number of identical manufacturers.
- Our results show that a collaborative production network increases the profit for producers and allows the network to offer higher service level and lower price at the same time.

Future Work

- Analyzing collaborative network for unsatisfied own customers.
- Analyzing collaborative production network.



"Factory of the future" IEC (2014)