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Capacity considerations in the bullwhip effect in supply chains: The effect on lead times

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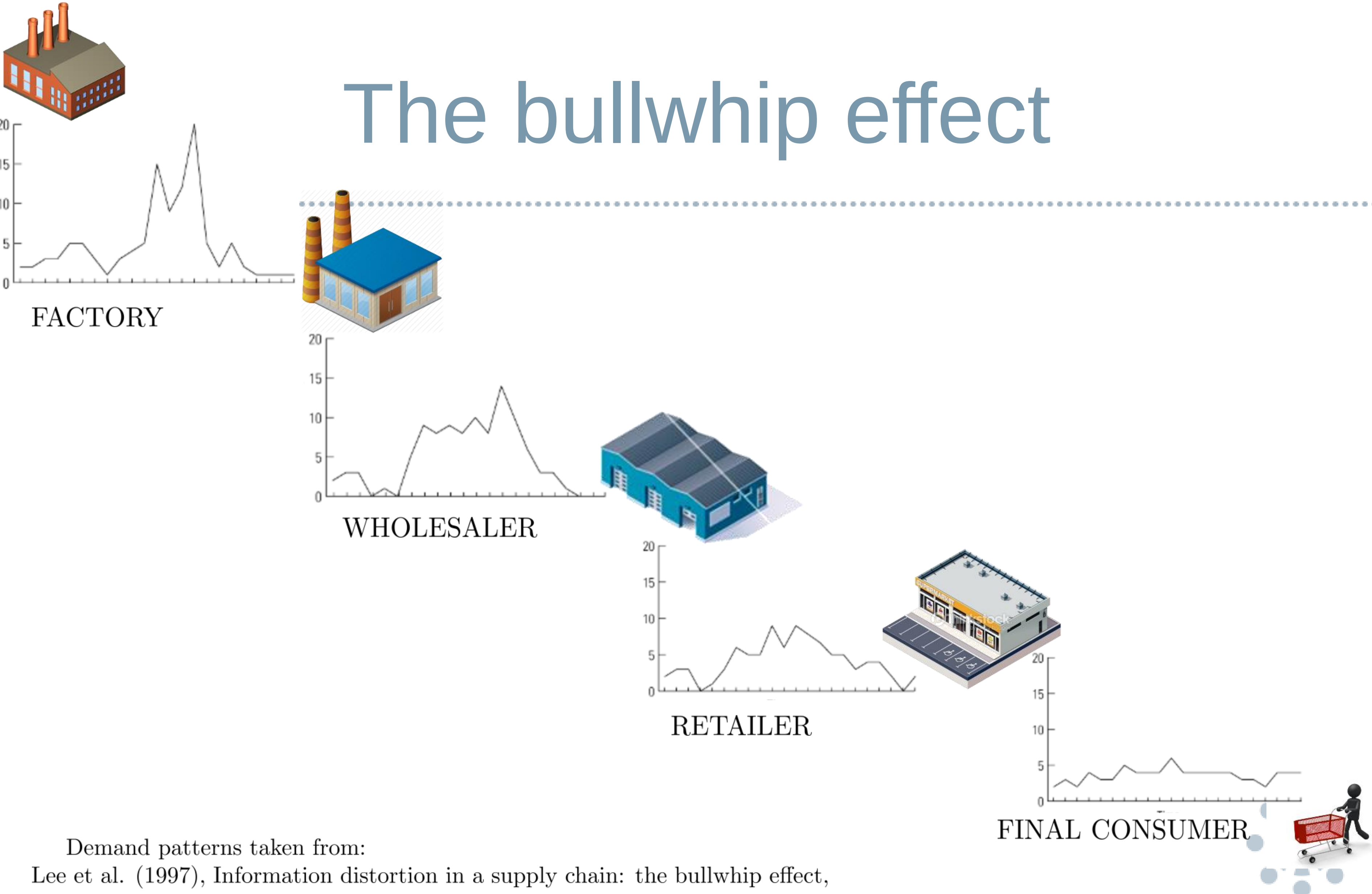
The bullwhip effect



Demand patterns taken from:
Lee et al. (1997), Information distortion in a supply chain: the bullwhip effect, *Management Science*, 40, 546558.

Impact of BWE

- Extension:
  - USA (Bray and Mendelson, 2012):

    Bray, R.L., Mendelson, H. (2012), Information transmission and the bullwhip effect: An empirical investigation, *Management Science*, 58 (5), 860-875.

    - 4,689 companies
    - BWE present in around 2/3
  - China (Shan et al. 2014)

    Shan, J et al. (2014), An empirical study of the bullwhip effect in China, *Production and Operations Management*, 23 (4), 537-551

    - 1,200 companies
    - BWE in more than 67 % of them
- Costs (Turrisi 2013):

  Turrisi M. et al. (2013) Impact of reverse logistics on supply chain performance, *International Journal of Physical Distribution & Logistics Management*, 43 (7), 564 - 585.

  - Operating costs increase: 25 %
  - Loss of profit: 15-30 %
  - Inventory excess: 33 %

Causes of BWE

- Behavioural (Sterman, 1989):

  Sterman, J (1989), Modeling managerial behavior: misperceptions of feedback in a dynamic decision making experiment. *Management Science*, 35 (3), 321-339.

  - *Irrational* decisions
  - *Beer game*

- *Rational* causes (Lee et al. 1997):

  Lee et al. (1997), Information distortion in a supply chain: the bullwhip effect, *Management Science*, 40, 546558.

  - Lead time
  - Demand forecasting
  - Price fluctuations
  - Rationing game
  - ...

Main assumptions in literature

- Most studies quantifying bullwhip effect assume fixed lead times no matter the order size.
  - Undelying assumption: the node has no limits in the capacity (*Exogenous* supply chain)
- However:
  - Chen & Lee (2012) model a node in a supply chain where, for each period, the orders cannot exceed a given threshold.
    - This assumption makes the BWE to be smaller than in uncapacitated supply chains.
  - Boute et al. (2008,2009) model explicitly the capacity using queuing theory
    - The distribution of the lead times is obtained under rather restrictive assumptions

Motivation

- Spare parts warehouse for aircraft
    - Spare parts are manufactured by the company in a different facility
- Amplification of demand variance higher than predicted by models
    - Of course, many other causes could be possible ...
    - ... still, figures where too high, particularly since it was not the un-capacitated case
        - There were frequent variations in the lead times, apparently due to capacity problems in the manufacturing facility
    - ... and it was not possible to link the lead times with the (current/past) order size
        - The manufacturing facility also processes parts for the assembly line

Capacity in the supply chain

Capacity of a node in a supply can be used to represent (at least) the following situations:

- Case I: The order size at any time cannot exceed a fixed capacity C.
  - The node receiving the order has the ability to reject orders exceeding its regular capacity.
  - In practice, such constraint means rejecting some customers.
- Case II: Capacity as a factor influencing the lead times, but that cannot be directly linked to current/past orders and/or demand.
  - The node cannot determine the precise lead time of the orders, but it should be estimated. Lead time can be consider a random variable.
- Case III: Capacity that can directly linked to current/past orders and/or demand.
  - The node can guess the precise lead time of the order and there would be no need to forecast the lead times.

General model: Demand modelling

- Single-product supply chain consisting of nodes serially linked
- D_t the demand that a node sees in period t follows a stationary first-order autoregressive process, AR(1), process:

$$D_t = d + \rho D_{t-1} + \epsilon_t$$

where $|\rho| < 1$ and ϵ_t are iid random variables with $E[\epsilon_t] = 0$ and $V[\epsilon_t] = \sigma^2$

It follows that:

$$E[D_t] = \frac{d}{1-\rho} = \mu$$

$$E[V_t] = \frac{\sigma^2}{1-\rho^2}$$

$$cov(D_t, D_{t-k}) = \rho^k \cdot V[D_t]$$

General model: Sequence of events

1. Product arrives from the upstream node, and demand is received from the downstream node.

2. Work in progress and net stocks are updated

3. An order is issued to the upstream node to replenish the inventory.

   a) We assume that the node adopts a stock-base policy (or Order-Up-To, OUT), resulting in:

$$O_t = D_t + \hat{D}_t^L - \hat{D}_{t-1}^L \tag{1}$$

with

$$\hat{D}_t^L = \sum_{h=0}^{\hat{L}_t} \hat{D}_t(h)$$

$\hat{D}_t(h)$ is the estimation of the demand in period $t + h$ produced in period t, and $\hat{L}_t$ the estimation of the lead time produced in period t

General model: Lead time modelling

We assume some dependency between past lead times and current lead times:

$$L_t = L_0 + \rho_L L_{t-1} + \epsilon_t^L$$

with $\epsilon_t^L \sim N(0, \sigma_L^2)$

$$E[L_t] = \mu_L = \frac{L_0}{1 - \rho_L}$$

$$V[L_t] = \frac{\sigma_L^2}{1 - \rho_L^2}$$

$$cov(L_t, L_{t-k}) = \rho_L^k \cdot V[L_t]$$

General model: Demand & lead time forecasts

Among the different methods that can be employed to forecast both lead times and demand, we consider:

- Minimum Mean Squared Error (MMSE) estimation
  - From a practical viewpoint, a benchmark as the error in the forecast is minimal
- Moving Average (MA).
  - In this case, node uses information regarding the past m_L periods to forecast lead time and/or the past m periods to forecast demand:

$$\hat{L}_t = \frac{1}{m_L} \sum_{j=1}^{m_L} L_{t-j}$$

$$\hat{D}_t = \frac{1}{m} \sum_{i=0}^{m-1} D_{t-i}$$

General model: Indicators

Our interest is to quantify the bullwhip effect on the node, for which the most extended measure is the ratio between the variance of the orders and the variance of the demand, i.e.:

$$BWE = \frac{V[O_t]}{V[D]}$$

Some alternative measures can be employed (Zhang 2005), but these are essentially equivalent to the one that we use here.

Case IIa

MMSE forecast of demand and lead times

$$BWE = 1 + \frac{2\rho(1-\rho^{\mu_L+1}) \cdot (1-\rho^{\mu_L})}{1+\rho}$$

- There is demand amplification if $\rho > 0$
  - The intensity of the amplification is related to μ_L
- The case $\rho = 0$ (iid demands) reduces to the no amplification case where the demand is 'chased'
- For the case $\rho < 0$, the bullwhip effect is dampened
  - The intensity of this dampening effect is oscillating with μ_L

Case IIb

MMSE forecast of demand, MA forecast of lead time (demand is iid)

$$BWE = 1 + 2\left(\frac{\mu\sigma_L}{\sigma m_L}\right)^2 \frac{1-\rho_L^2}{1-\rho_L^{m_L}}$$

- There is always amplification of the variance regardless the sign of ρ_L
- Larger values for m_L tend to reduce the amplification
- BWE does not depend on the average value of the lead time, but on its variance

Case IIc

MA forecast of demand, MMSE forecast of lead times

$$BWE = 1 + 2(1 - \rho^m)\frac{\mu_L}{m}\left(1 + \frac{\mu_L}{m}\right)$$

- There is always an amplification of the variance
- The value of BWE depends not only on the estimated lead time and on the length m of the MA estimation of demand, but also on the demand autocorrelation

Case IId

MA forecast of demand, MA forecast of lead time (demand is iid, lead time is iid)

$$BWE = 1 + \underbrace{2\frac{\mu_L(m_L-1)}{m \cdot m_L}\left(1 + \frac{\mu_L(m_L-1)}{m \cdot m_L}\right)}_{\text{from case II.c}} + \underbrace{\frac{2\sigma_L^2\mu^2}{\sigma^2 \cdot m_L^2}}_{\text{from case II.b}} + \underbrace{(m + m_L - 2)\frac{2\sigma_L^2}{m^2 \cdot m_L^2}}_{\text{crossed relation}}$$

- All components are positive and contribute to the bullwhip effect
- The first term reflects the influence of the MA forecast of the demand
- The second term is due to the MA forecast of lead times
- The third factor reflects the crossed influence of the forecast
  - Zero if a single period is used to forecast demand and lead times
  - However, this comes at the expense of increasing the other terms of the bullwhip effect

Summary

	Lead time - MMSE	Lead time - MA
Demand - MA	$1+\frac{2\mu_L}{m}\left(1+\frac{\mu_L}{m}\right)$	$1+\frac{2\mu_L(m_L-1)}{m\cdot m_L}\left(1+\frac{\mu_L(m_L-1)}{m\cdot m_L}\right)+$ $+(m+m_L-2)\frac{2\sigma_L^2}{m^2\cdot m_L^2}+2\left(\frac{\sigma_L\mu}{\sigma\cdot m_L}\right)^2$
Demand - MMSE	1	$1+2\left(\frac{\mu\sigma_L}{\sigma m_L}\right)^2$

$$(\rho = 0 \text{ and } \rho_L = 0)$$

Conclusions & Future Research

- The consideration of capacity in the supply chain may refer to (at least) three different aspects
  - Limits in the order size
    - In this case, the BWE is smaller than in the uncapacitated case
  - (Indirect) influence in lead times
    - Estimation of lead times / demand becomes critical to assess the BWE
    - In any case, reduction of BWE only appears under MMSE estimation of lead time and demand, and negatively correlated demand
  - Direct influence in lead times
    - (Maybe) subject of future research

THANK YOU!

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