Ingolstadt School of Management

A Stochastic Dynamic Programming Approach for Assigning Inventories in Multi-Channel Retailing

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Agenda

1. **Motivation and omni-channel retailing**
2. Omni-channel inventory allocation problem
3. Model development
4. Results
5. Summary and future area of research

Omni-channel retailers serve customer with on- and offline channels

Example

Bricks-and-mortar store

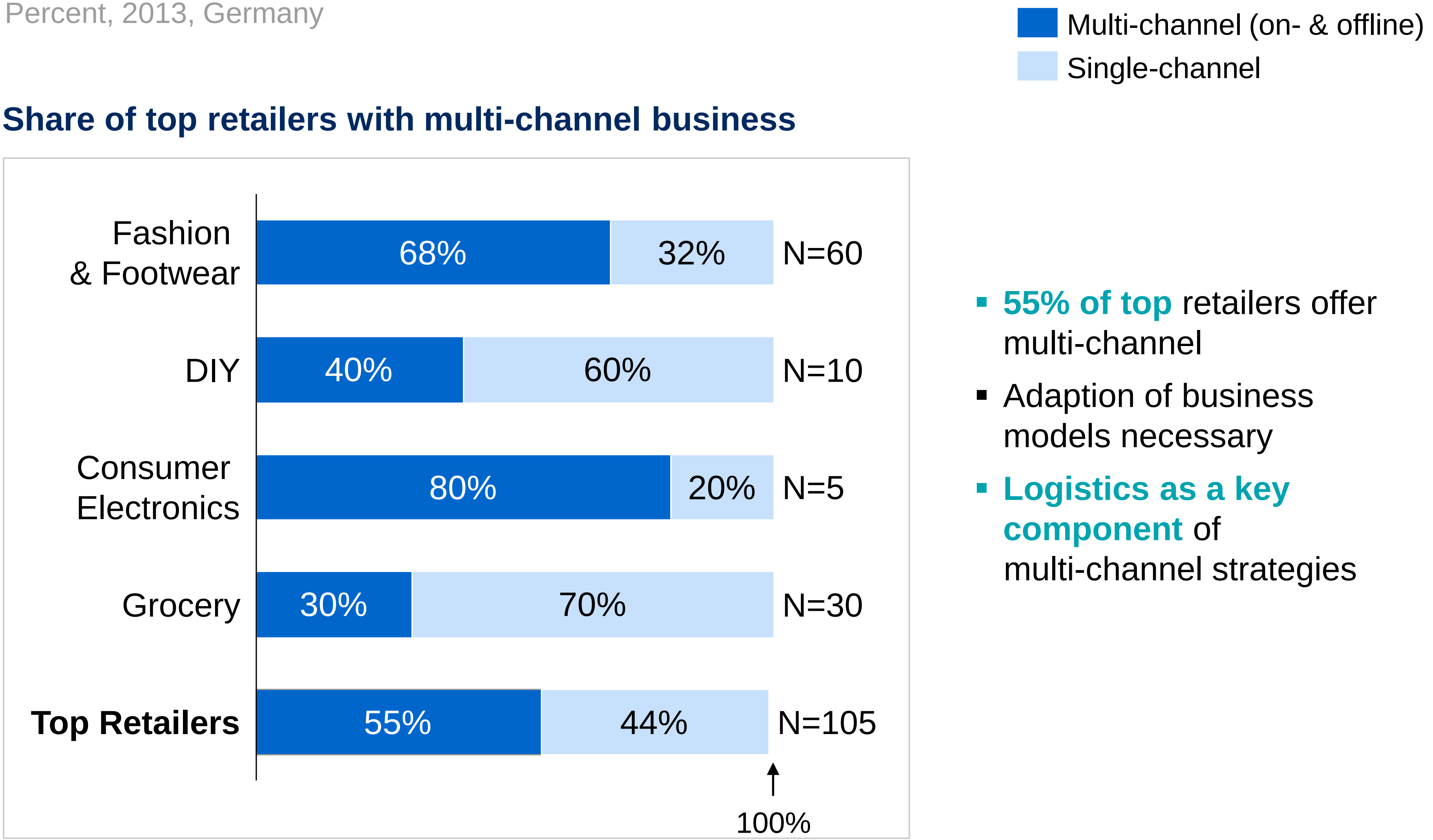
Online store

Buy online pick up instore and buy instore and get home delivery

Already 55% of top retailers operate in multi-channel business

Percent, 2013, Germany

Share of top retailers with multi-channel business

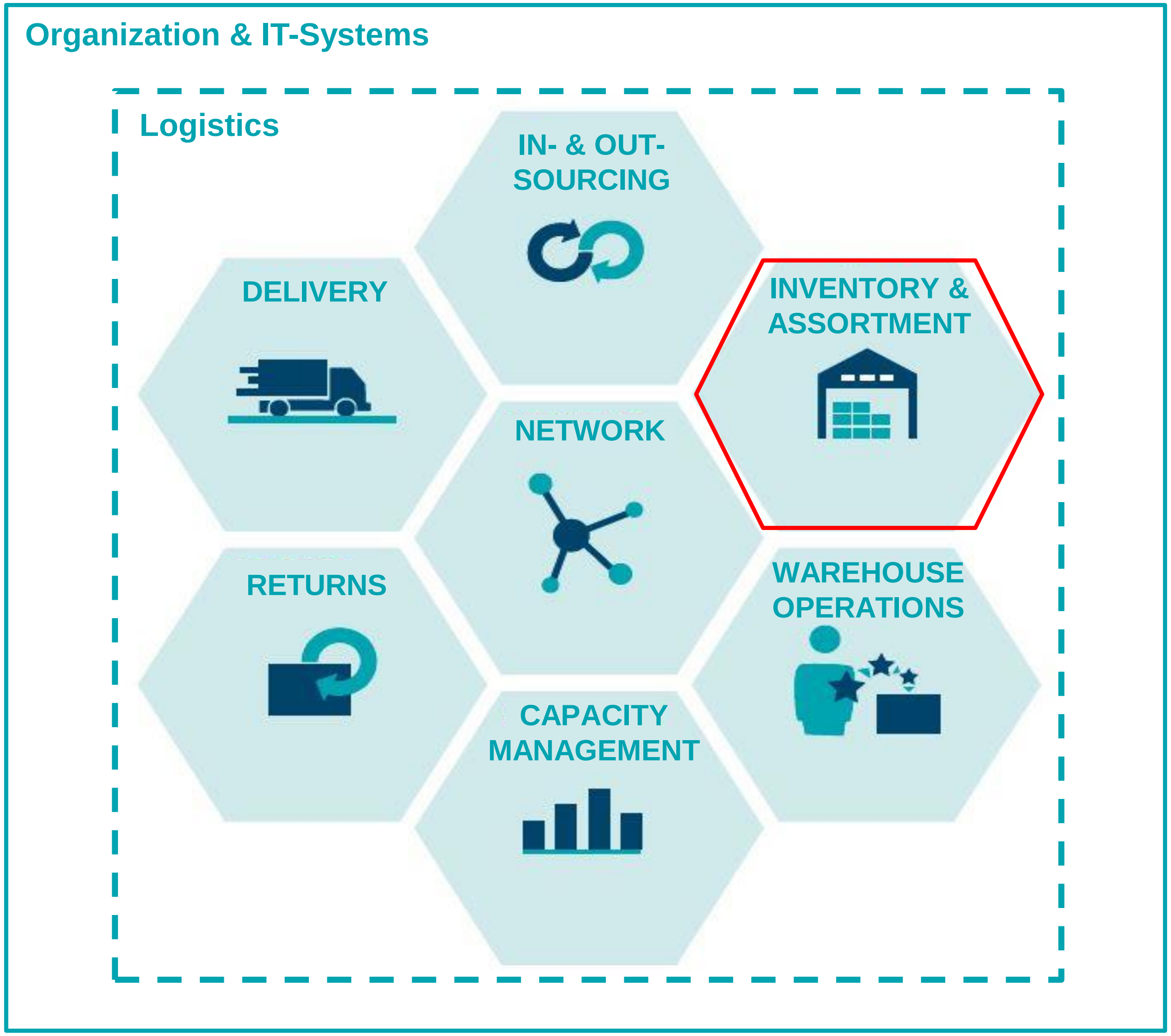


- **55% of top** retailers offer multi-channel
- Adaption of business models necessary
- **Logistics as a key component** of multi-channel strategies

Source: Kuhn/Hübner/Holzapfel (2013)

Seven logistics planning areas have been identified by means of qualitative interviews with >30 retailers

Planning areas in multi-channel retailing



Planning areas identified through ***face-to-face interviews with retailer managers***

Source: Hübner/Holzapfel/Kuhn (2014)

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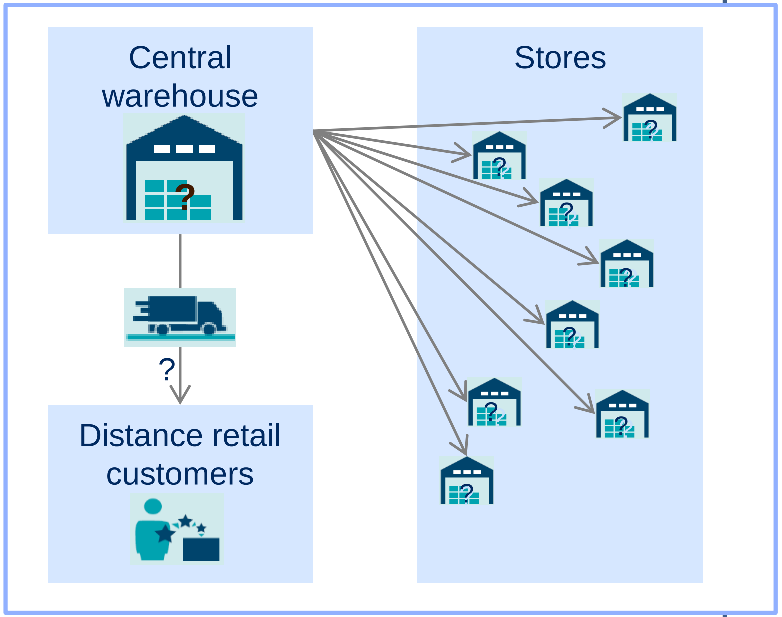
Allocation of inventories to different distribution channels is a central challenge in omni-channel retailing

Problem structure



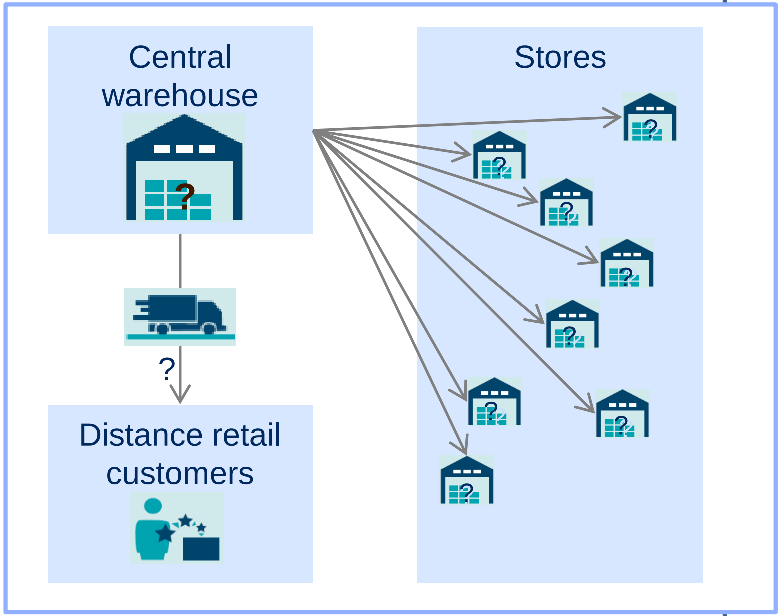
Motivation

- Adequate allocation of inventories is important to **prevent shortages** in one channel while there is a **surplus** in the other channel
- Inventories in omni-channel retailing:
  Each store is an **individual warehouse**, the **online shop** is an additional **"large store"** with an aligned warehouse

Sources: Hübner/Holzapfel/Kuhn (OMR, 2015)

We apply our model to omni-channel retailers that sell seasonal products and operate a central warehouse and multiple stores

Network structure

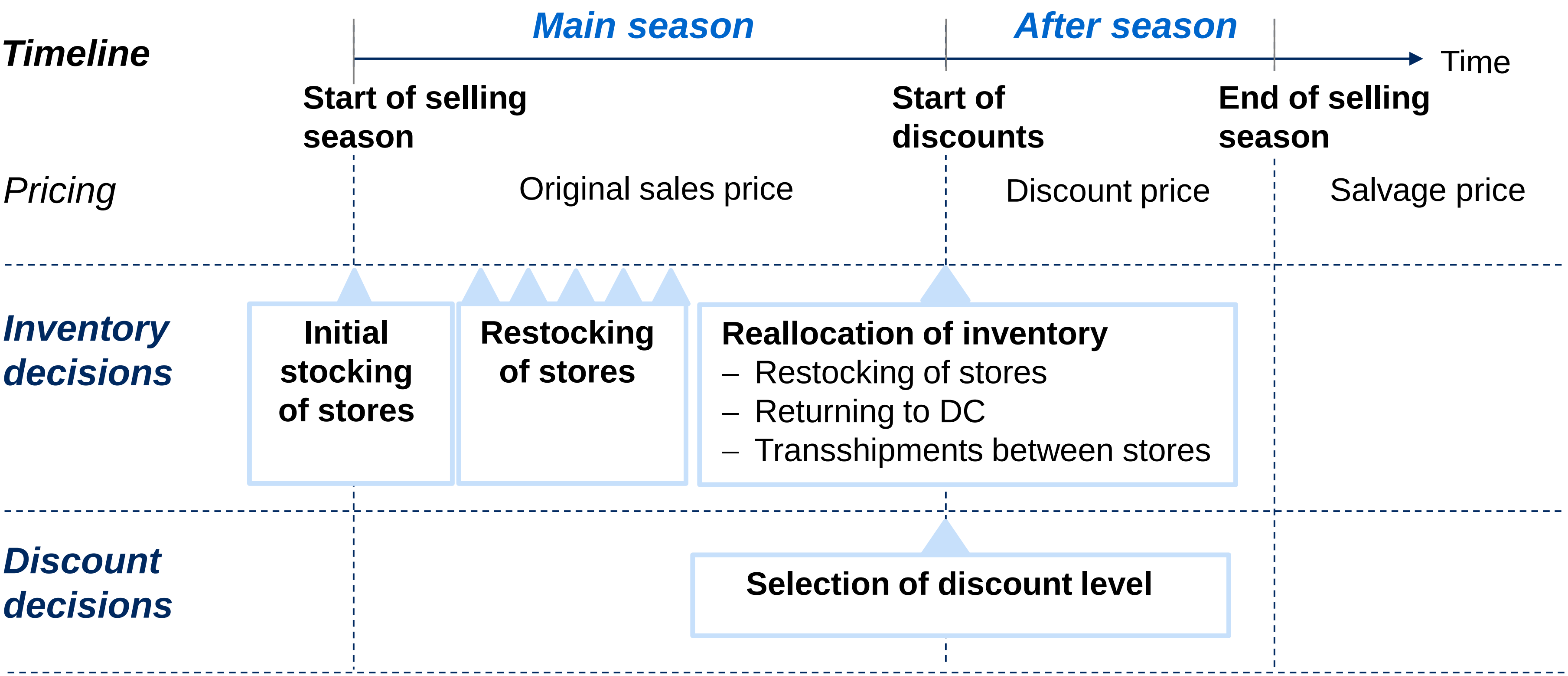


Application & case study

- **Product characteristics and examples**
  - **Seasonal products** with a **main season** and **discounted sales** afterwards
  - **Fashion products**, promotional items, etc.
- **Network structure of omni-channel retailers**
  - **One DC** for bricks-and-mortar and distance channel
  - Own branch network
- **Typical sourcing and purchasing policy**
  - Order placement **6 to 12 month in advance** of the selling season (e.g. in Far East)
  - All distributable **products arrive** at the central warehouse at the beginning of the selling season
  - **No reorders possible** during the selling season

Research project with a fashion retailer of the Otto Group, Germany

The phases of the selling season determine the structure of the decision problem and the decision alternatives

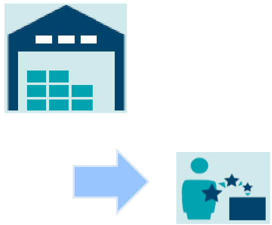


Literature on inventory allocation is not tailored to omni-channel problems, based on actual process costs and decision problems in this context

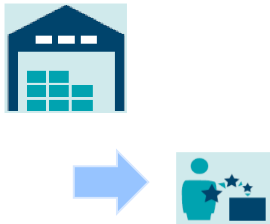
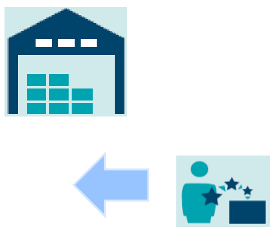
Literature	Contribution
• **Common literature** about inventory allocation Relevant **recent paper**: • **Alptekinoglu, Tang (2005):** A model for analyzing multi-channel distribution systems • **Agrawal, Smith (2013)**: Two-stage allocation of inventories to stores with different demand patterns	• **Omni-channel** retailing as area of application • Practice-oriented **analysis** of **processes** and **costs** which influence the allocation decision • **Integrative treatment** and **modeling** of different **allocation** alternatives and **pricing**

Sources: own research, Agrawal/Smith (2013)

The inventory allocation and discounting decisions cause different channel-specific (process) costs

Cost factor	Description	Influencing factors *(selection)*
Forward logistics 	Costs for initial **stocking** and **restocking of stores** and **shipment of customer orders**	• Picking and packaging system • Mode of shipment • …

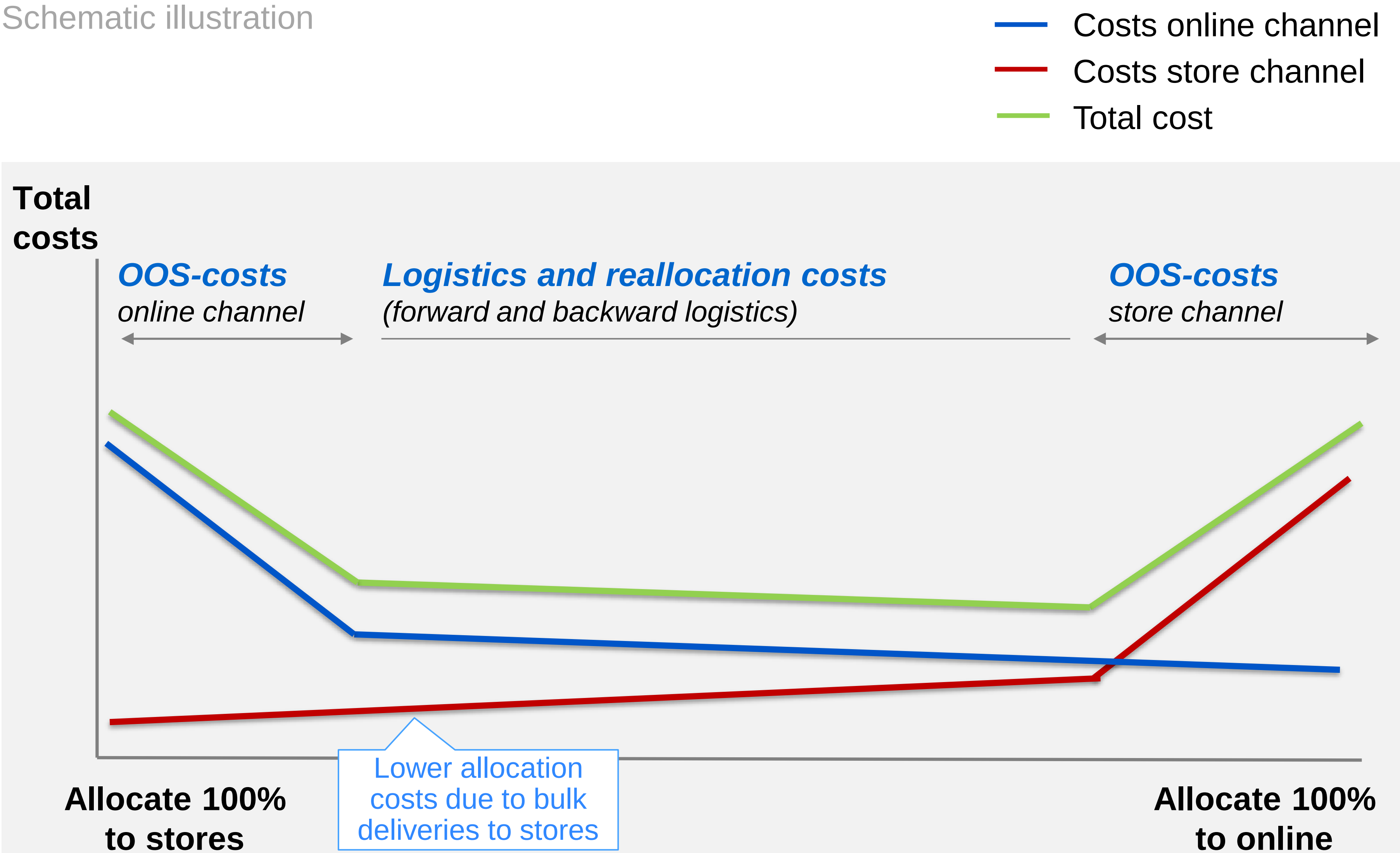
The inventory allocation and discounting decisions cause different channel-specific (process) costs

Cost factor		Description	Influencing factors *(selection)*
Forward logistics		Costs for initial **stocking** and **restocking of stores** and **shipment of customer orders**	• Picking and packaging system • Mode of shipment • …
Backward logistics		Costs for **customer return** handling and shipment	• Return quota • Rework effort • …
Out-of-stock and -prevention		Costs for **unsatisfied demand** and **prevention strategies** (like reallocation and transshipments)	• Shipment • Handling effort • …
Discounts and remnants		Reduction of **sales margin** and effort clearance	• Level of discount • Handling effort • …

Sources: Data case company, Hübner /Holzapfel/Kuhn (OMR, 2015)

Inventory allocation deals with the cost trade-off between savings in bulk shipments to stores and risk of stock-out costs

Schematic illustration



Source: Data case company

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A stochastic DP minimizes the process, out-of-stock and discount costs considering the various decision stages and both sales channels

Notation

Indices

l **locations**
r **discount** levels
t **periods**

Parameter

c allocation **costs** (DC to store, store to DC, transsphipment between stores)

q initial **inventory**

π **unit profit** for each sales phase and price level

Random variables

D **Demand** at location for each sales phase and price level

Auxilliary variables

A, B Start/end **inventory at location**

Z Realized **sales**

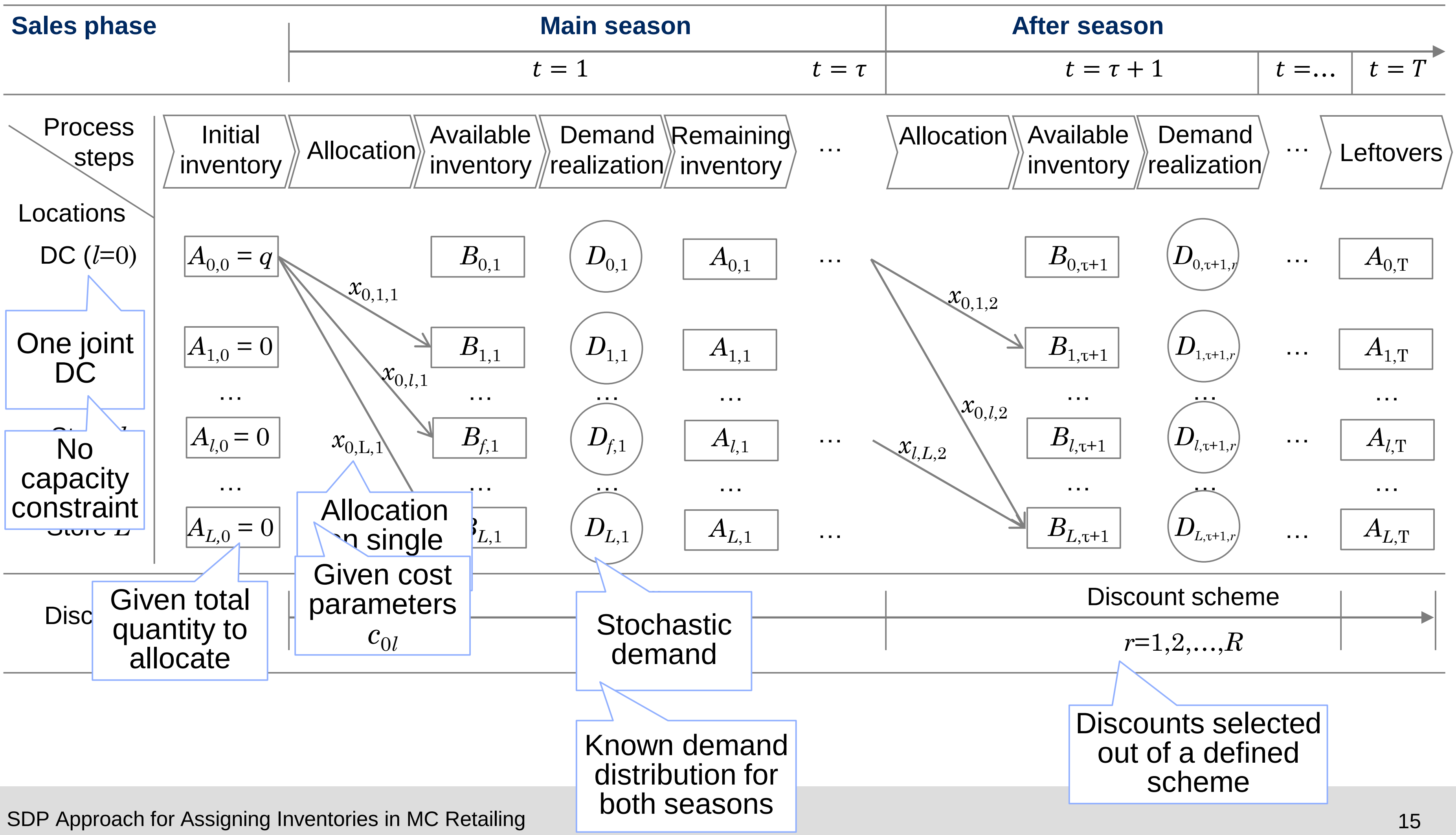
Decision variables

x **Reallocation volume**

y **Discount level**

The decisions during the planning horizon can be represented as stochastic dynamic program

Inventory related decisions and auxiliary variables of the SDP



A stochastic DP maximizes the total marginal profit considering the various decision stages and both sales channels

Objective function

$$\max!\ \mathrm{TP} =$$

$$\sum_{t=1}^{\tau}\sum_{l=0}^{L}\pi_{lt}^{sale}\cdot E[Z_{lt}]$$

Realized profit *from sales during* ***main season***

$$+\sum_{t=\tau+1}^{T}\sum_{l=0}^{L}\sum_{r=1}^{R}\pi_{ltr}^{disc}\cdot E[Z_{lt}]\cdot y_r$$

Realized profit *from sales during* ***after-season*** *and for selected discount*

$$+\sum_{l=0}^{L}\pi_{l}^{remn}\cdot E[A_{lT}]$$

Realized profit *from* ***remnant sales*** *at the end of after-season*

$$-\sum_{l=0}^{L}\sum_{l=0,l\neq k}^{L}\sum_{t=1}^{T}c_{lk}\cdot x_{lkt}$$

Total costs *for* ***reallocation*** *of items between locations at the beginning of different periods*

A stochastic DP minimizes the process, out-of-stock and discount costs considering the various decision stages and both sales channels

Constraints

Available inventory *at the* ***beginning*** *of period t*

$$B_{lt} = A_{l,t-1} - \sum_{k=0,k\neq l}^{L} x_{lkt} + \sum_{l=0,l\neq k}^{L} x_{klt} \qquad l = 0,1,\dots,L;\ t = 1,2,\dots,T\ (2)$$

Sales volume *in* ***main season***

$$Z_{lt} = min[D_{lt}; B_{lt}] \qquad l = 0,1,\dots,L;\ t = 1,\dots,\tau (3)$$

Sales volume *in* ***after season***

$$Z_{lt} = min[D_{ltr}; B_{lt}] \cdot y_r \qquad l = 0,1,\dots,L;\ t = \tau + 1,\dots,T (4)$$

Inventory level *at the* ***end*** *of period t*

$$A_{lt} = B_{lt} - Z_{lt} \qquad l = 0,1,\dots,L;\ t = 1,2,\dots,T\ (5)$$

Discount scheme *and variables definition*

$$\sum_{r=1}^{R} y_r = 1 \qquad (6)$$

$$A_{lt} \epsilon \mathbb{Z}_0^+; B_{lt} \epsilon \mathbb{Z}_0^+; Z_{lt} \epsilon \mathbb{Z}_0^+ \qquad l = 0,1,\dots,L;\ t = 1,2,\dots,T\ (7),(8),(9)$$

$$x_{lkt}\ \epsilon \mathbb{Z}_0^+ \qquad l,k = 0,1,\dots,L;\ t = 1,2,\dots,T\ (10)$$

$$y_r \epsilon \{0,1\} \qquad r = 1,2,\dots,R\ (10)$$

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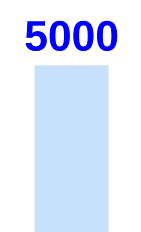
The case study covers a data setting with one DC, 60 stores, different inventory levels and a broad set of cost constellations …

Network

Initial inventory



Financials

Values tested [in currency units]

Item price	**14 (low)**	**28 (medium)**	**56 (high)**
Logistics costs			
• Shipment costs to customers	2.8		
• Initial bulk stocking of stores	0.01		
• Restocking of stores	0.3		
• Return from store to DC	1.0		
• Transshipment between stores	1.2		
Lost sales costs			
• in distance channel	margin – shipment costs to customers		
• in store channel	margin – restocking costs of stores		
Discounts in discount phase	{10%, 20%, 30%}		
Remnant costs after end of selling			
• Remnant value for remnant items	50%		
• Remnant cost distance channel	margin*rem.value+customer shipment costs		
• Remnant cost store channel	margin*rem.value+restocking costs of stores		

… the numerical study covers a data setting with multiple demand constellations

Demand

Mean demand ratios	Values tested [ratios]		
Total demand *as % of initial total stock at DC*	**50% (low)**	**100% (medium)**	**150% (high)**
Demand share of main season *as % of total demand*	50%		
Demand share of online channel *as % of total demand*	10%		
Demand elasticity			
Additional demand on discounts, *as a factor of the discount*	1.0		

Demand is assumed to be uniformly distributed with a spread of 40 in distance channel and 20 for each store and sales phase.

Different combinations of financials and demand data results in

9 different data sets

simulated 100 times each

We apply different solution approaches to the inventory allocation problem

Focus on following slides (shaded columns)

Allocation volume determined by

Frequency of decisions	Integrated allocation problem (exact) *Application of OCIAP model* ***Solutions only for two-store cases possible***	Two-phase allocation problem (AP-heuristics) *OCIAP applied to each phase with known demand distribution*[1]	Demand ratio (DR-heuristics) *Proportional allocation based on expected mean demand*	Lot-for-lot *Replenishment after sales based on first-come-first serve logic*
2xbulk (2B) — *main*, *after*	**2B-exact**	**2B-AP**	**2B-DR**	---
1xbulk (1B)	**1B-exact**	**1B-AP**	**1B-DR**	---
continuous	---	---	---	**Lot-for-Lot**

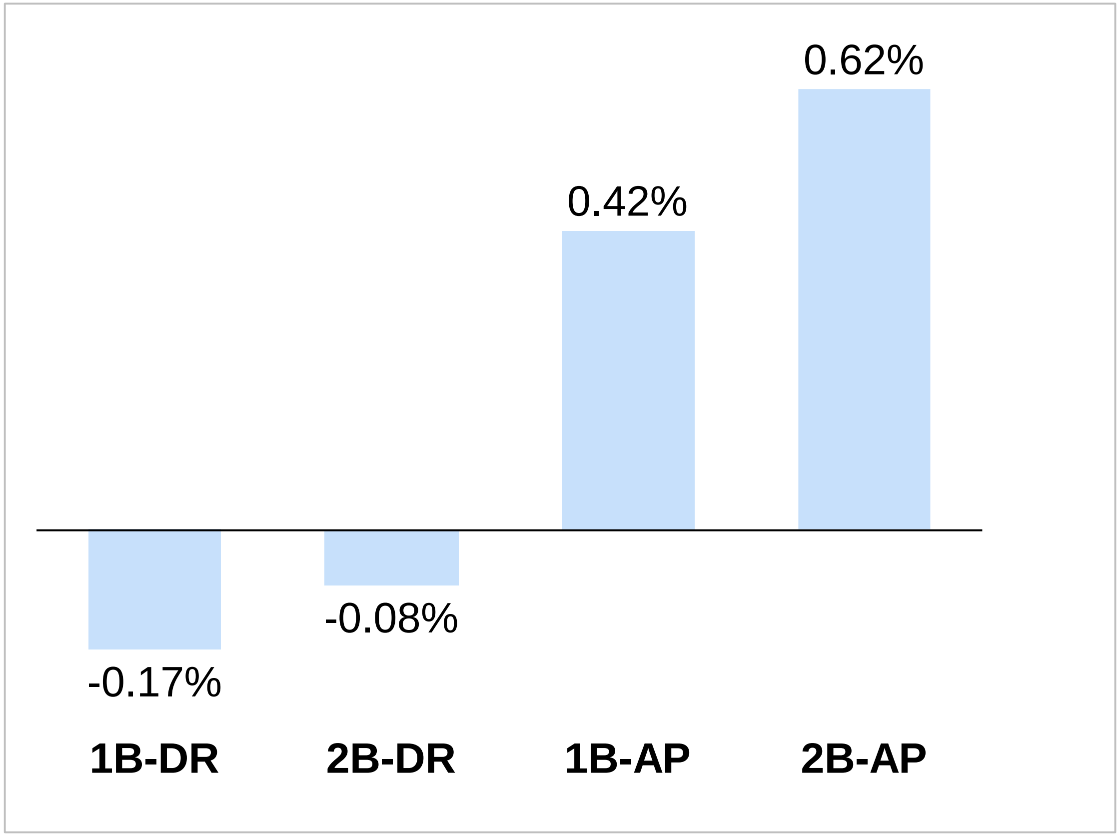
1: first allocation includes demand for all phases, second allocation is only reallocation based on realized demand

Total profit can be increased with efficient allocation methods

Overview of case study results

Average profit change of 900 examples with varying demand and price ratios, Case company

Profit change vs. lot-for-lot policy



1. AP - allocation heuristics improves profit on average **by 0.7 ppt. in comparison to demand ratio allocation** (AP vs. DR)
2. A **second bulk allocation improves** profit on average **by 0.2ppt** (2B vs. 1B)

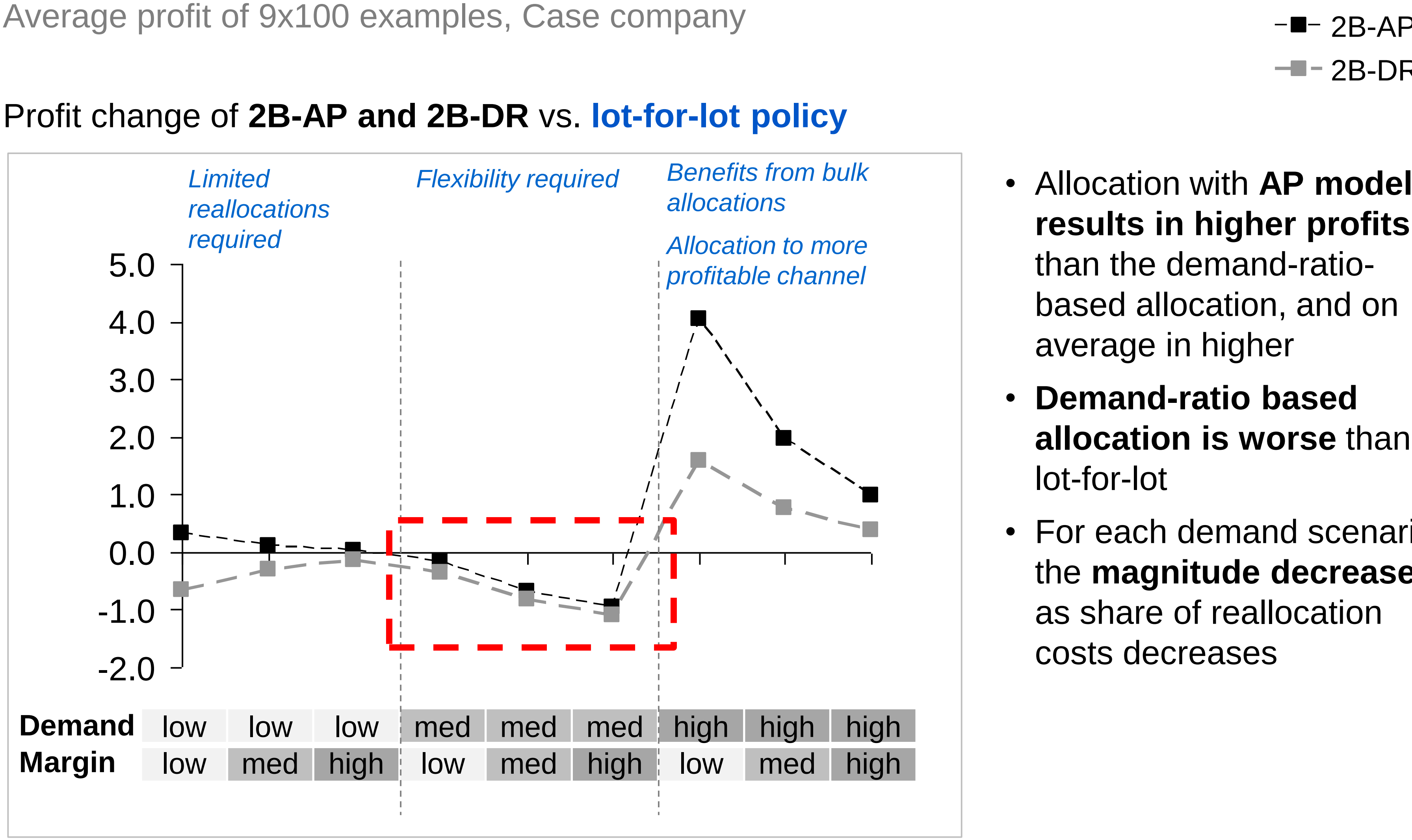
But, this does not hold true in general – see next slides

1 Optimal policy highly depends on the demand ratio and margins (1/2)

Allocation with AP model vs. allocation by demand ratio

Average profit of 9x100 examples, Case company

Profit change of **2B-AP and 2B-DR** vs. **lot-for-lot policy**



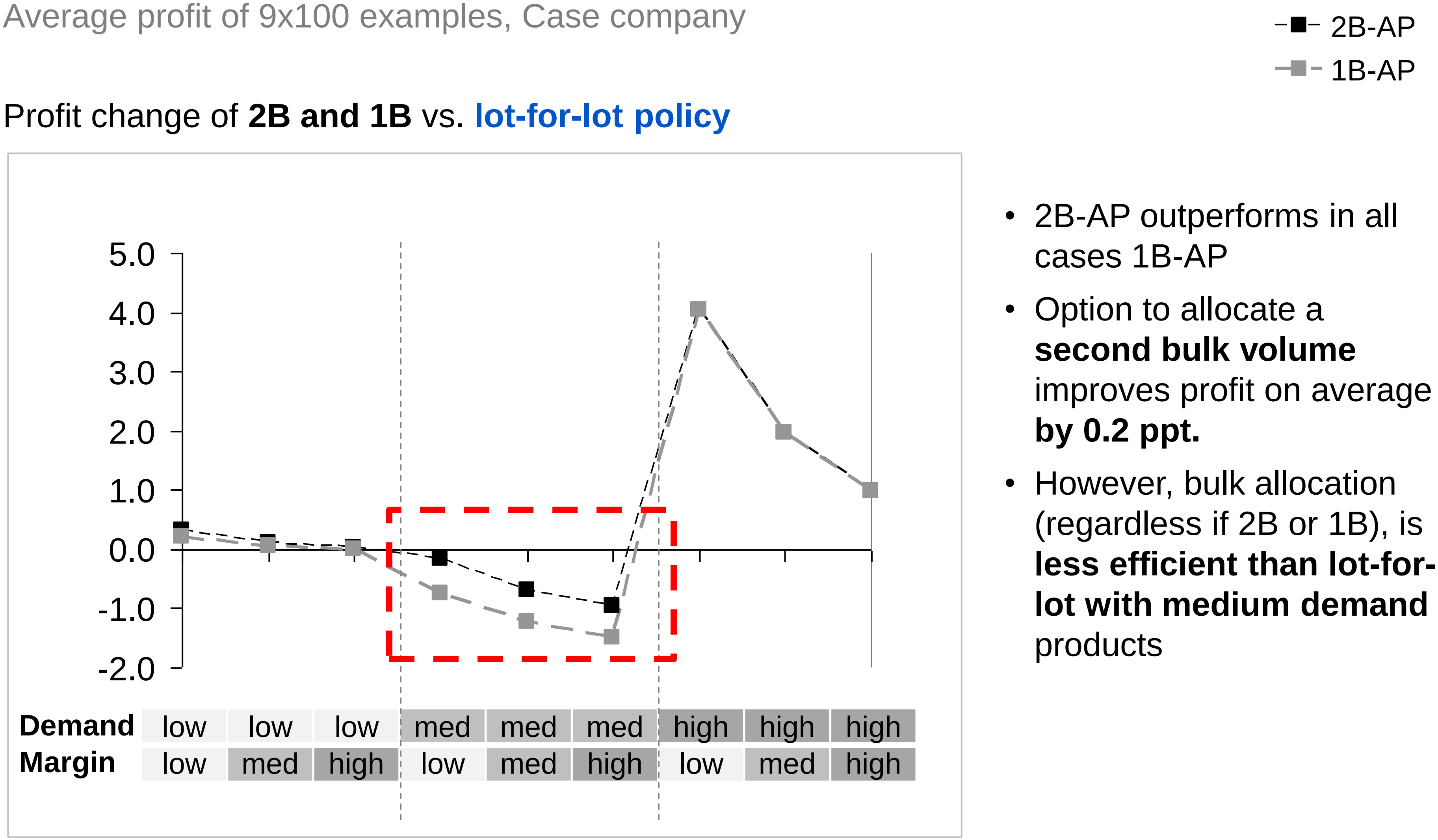
- Allocation with **AP model results in higher profits** than the demand-ratio-based allocation, and on average in higher
- **Demand-ratio based allocation is worse** than lot-for-lot
- For each demand scenario the **magnitude decreases** as share of reallocation costs decreases

2 Optimal policy highly depends on the demand ratio and margins (2/2)

Two vs. one bulk allocation

Average profit of 9x100 examples, Case company

Profit change of **2B and 1B** vs. **lot-for-lot policy**



- 2B-AP outperforms in all cases 1B-AP
- Option to allocate a **second bulk volume** improves profit on average **by 0.2 ppt.**
- However, bulk allocation (regardless if 2B or 1B), is **less efficient than lot-for-lot with medium demand** products

We extend the bulk allocation approach with a flexible buffer

Allocation volume determined by

Additional approach (shaded)

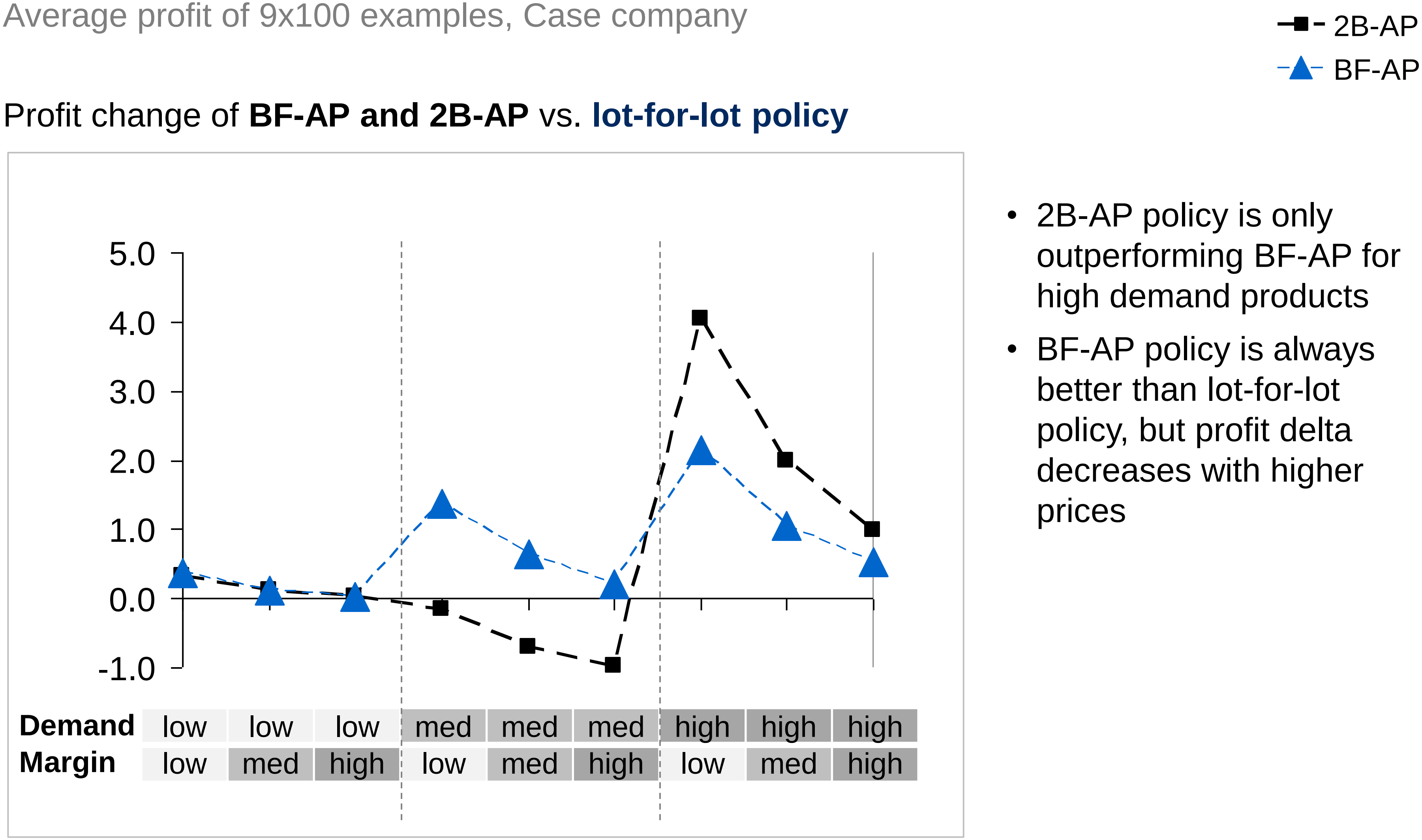
Frequency of decisions	Integrated allocation problem (exact)	Two-phase allocation problem (AP-heuristics)	Demand ratio (DR-heuristics)	Lot-for-lot
	Application of OCIAP model	*OCIAP applied to each phase with known demand distribution*	*Proportional allocation based on expected mean demand*	*Replenishment after sales based on first-come-first serve logic*
2xbulk (2B) — *main*, *after*	**2B-exact**	**2B-AP**	**2B-DR**	---
1xbulk (1B)	**1B-exact**	**1B-AP**	**1B-DR**	---
1xbulk plus flexible buffer	**BF-exact**	**BF-AP**	**BF-AP**	---
continuous	---	---	---	**Lot-for-Lot**

Optimal policy highly depends on the demand ratio and prices

Bulk allocation with vs. without puffer

Average profit of 9x100 examples, Case company

Profit change of **BF-AP and 2B-AP** vs. **lot-for-lot policy**



- 2B-AP policy is only outperforming BF-AP for high demand products
- BF-AP policy is always better than lot-for-lot policy, but profit delta decreases with higher prices

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Key learnings and managerial insights

Preliminary results based on case study

1. Introduction of **flexible puffers matters**!
2. **Efficient allocation approach** outperforms proportional allocation!
3. **Bulk allocation improves logistics costs** (two bulk allocations are better than one bulk and better than lot-for-lot replenishment)

However, **improvement potential depends** mainly on **demand levels**, **gross margin** and **logistics costs**

References

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More information at: ***www.multichannellogistik.net***

Many thanks for your attention!

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