Production/Inventory Control with Correlated Demand

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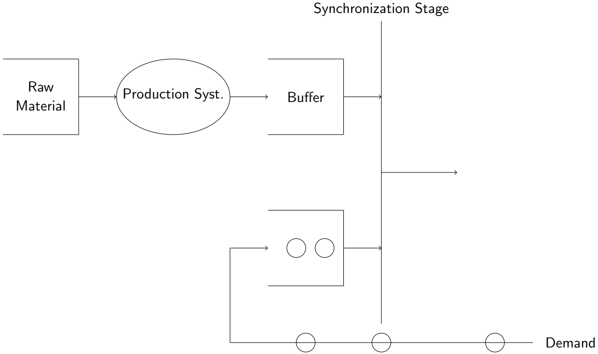
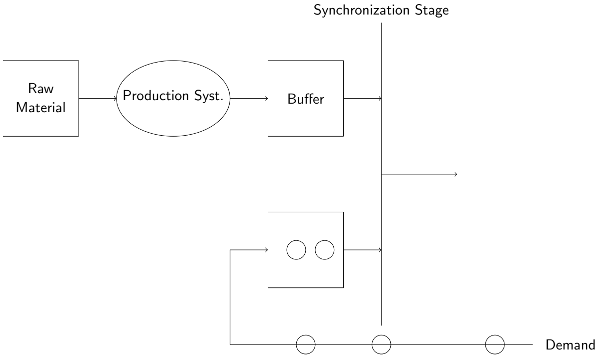
June 6, 2017

How to match supply and demand in an uncertain environment in manufacturing systems?

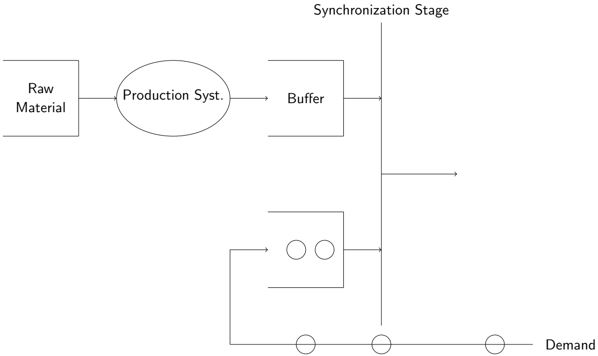


How to match supply and demand in an uncertain environment in manufacturing systems?



- Control-point policy (Gershwin, 2000),
- Base-Stock Policy,
- Production Authorization Card (Buzacott and Shanthikumar, 1993),
- Generalized Kanban Policy (Duri et al., 2000),
- Extended Kanban Control System (Liberopoulos and Dallery, 2000).

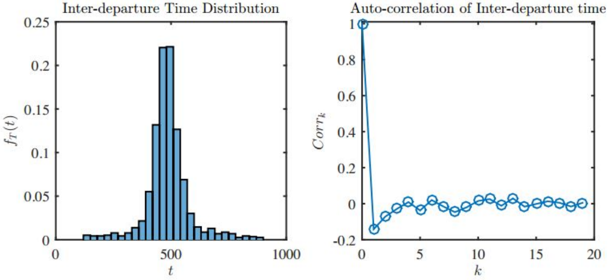
Demand inter-arrival and service times are usually modeled as an i.i.d. random variables



The output process from a production system is shown to be serially correlated

- Empirical studies: Schomig and Mittler (1995), Inman (1999).
- Analytical studies: Hendricks and McClain (1993), Tan and Lagershausen (2016)

Frequency Distribution of the Inter-departure Time and Sample Autocorrelation of Inter-departure Times for a Car Assembly Line



(Tan and Lagershausen, 2016)

How does autocorrelation impact the control of the system?

- ignoring auto-correlation in arrivals and service time leads to significant errors in setting the input buffers and base-stock levels in production/inventory systems.

(Dizbin, 2016)

Research questions

- What is the optimal control policy of a manufacturing system with correlated demand and service time?
- How does autocorrelation impact the optimal performance measures?

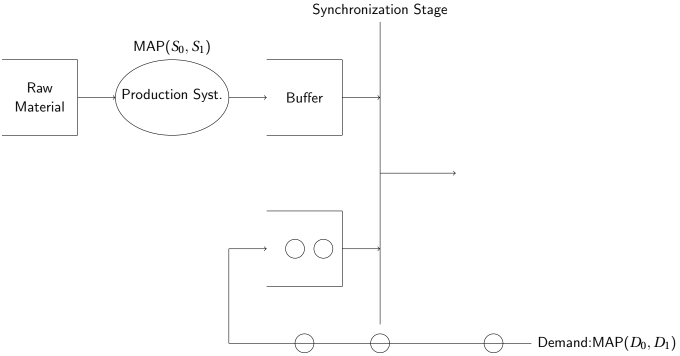
Literature of the inventory management problem with Markov-modulated demand

Paper	Demand Process	Lead Time	Production Capacity	Modeling Horizon	Cost Criterion
Song and Zipkin (1993)	MMPP	Stochastic	Infinite	Continuous (Finite & Infinite)	Discounted
Song and Zipkin (1996a)	MMPP	Stochastic	Infinite	Continuous (Infinite)	-
Song and Zipkin (1996b)	MMPP	Stochastic	Infinite	Continuous (Infinite)	-
Nasr and Maddah (2015)	MM	Deterministic	Infinite	Continuous (Infinite)	Average
Bayraktar and Ludkovski (2010)	MM	Stochastic	Infinite	Continuous (Infinite)	-
Muharremoglu and Tsitsiklis (2008)	MM	Markov Modulated	Infinite	Discrete (Finite & Infinite)	Average and Discounted
Janakiraman and Muckstadt (2009)	MM	Stochastic	Finite	Discrete (Finite)	Discounted
Sethi and Cheng (1997)	MM	Deterministic	Infinite	Discrete(Finite& Infinite)	Discounted
Beyer and Sethi (1997)	MMPP	Deterministic	Infinite	Discrete(Finite)	Discounted
Özekici and Parlar (1999)	MMPP	Deterministic	Finite	Discrete(Finite)	Discounted
Cheng and Sethi (1999)	MMPP	Deterministic	Finite	Discrete(Finite)	Discounted
Chen and Song (2001)	MM	Deterministic	Infinite	Discrete(Finite & Infinite)	Average
Hu et al. (2016)	MM	Deterministic	Infinite	Discrete(infinite)	Discounted
This Work	MM	MM	Finite	Continuous(finite & infinite)	Average

MM: Markov Modulated Demand

MMPP: Markov Modulated Poisson Process

Model



- Demand and Production Processes are modeled as Markov Arrival Processes (MAP)
- The system is continuously reviewed, and the state of the system is fully observed at any time t

Problem formulation

Find a control policy π that authorizes production based on the state of the system to minimize expected inventory and backlog costs

$$\inf_{\pi} V_T^{\pi}(x, s) = \lim_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T C(x(t))dt | x(0) = x, s_0 = s\right], \quad (1)$$

$$C(x(t)) = \begin{cases} -bx(t), & \text{if } x(t) < 0, \\ hx(t), & \text{otherwise} \end{cases} \quad (2)$$

The cost structure consists of the holding (h) and backlog costs (b).

Main result

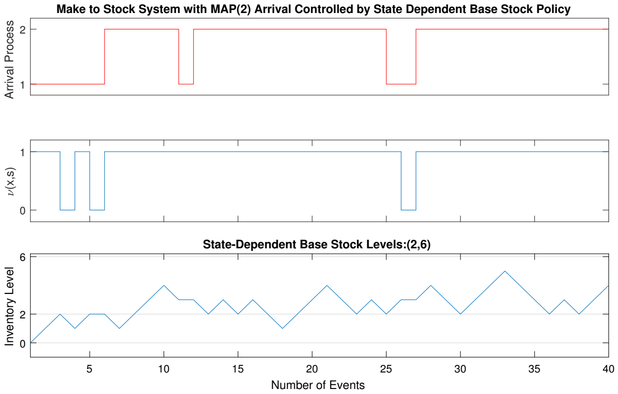
Theorem 1

The set of state-dependent base-stock policies is optimal for the production model with MAP arrival and production process.

$$\nu(x(t) = x, s_t = s) = \begin{cases} 1, & \text{if } x < Z_s, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

where ν is the control variable,
x is the inventory level,
s is the state of the Markovian process,
and Z_s is the base stock level associated with state s.

State-dependent base-stock policy



Dynamic programming formulation of a production system with MAP(D_0, D_1) arrival and exponential (μ) service processes

Let $\lambda_m = -\min(D_0(i,i))$, and $\alpha = \lambda_m + \mu$

$$V(x,i) + \frac{g}{\alpha} = \frac{C(x)}{\alpha} + \sum_{j \in S - \{i\}} P_0(i,j) V(x,j) + \sum_{j \in S} P_1(i,j) V(x-1,j) + \frac{\lambda_m + D_0(i,i)}{\alpha} V(x,i) + (\frac{\mu}{\alpha}) min\{V(x,i), V_n(x+1,i)\} \quad (4)$$

where $P_0 = \frac{1}{\alpha} D_0 + I$ and $P_1 = \frac{1}{\alpha} D_1$ and g is the average total cost.

Performance evaluation of a production control model with state-dependent base-stock policy

Infinitesimal Generator Matrix of a Production control model with State-Dependent Base-Stock Policy

$$Q = \begin{pmatrix} L^{(m)} & F^{(m)} & & & & & & & & & \\ B^{(m)} & L^{(m)} & F^{(m)} & & & & & & & & \\ & B^{(m)} & L^{(m)} & F^{(m)} & & & & & & & \\ & & \ddots & \ddots & \ddots & & & & & & \\ \hdashline & & & B^{(m-1)} & L^{(m-1)} & F^{(m-1)} & & & & & \\ & & & & \ddots & \ddots & \ddots & & & & \\ & & & & & B^{(2)} & L^{(2)} & F^{(2)} & & & \\ \hdashline & & & & & & B^{(1)} & L^{(1)} & F^{(1)} & & \\ & & & & & & & \ddots & \ddots & \ddots & \end{pmatrix}$$

$$\pi Q = 0$$

$$\pi 1 = 1$$

Approximated generator matrix of a process with MAP(D_0, D_1) demand and PH(β, S_0, s) service time

$$
Q = \begin{pmatrix}
D_0 & D_1 \otimes \beta & & & & & & & & & & \\
I_p \otimes s & D_0 \oplus S_0 & D_1 \otimes I & & & & & & & & & \\
 & I_p \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I & & & & & & & & \\
 & & \ddots & \ddots & \ddots & & & & & & & \\
 & & & I_p \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I & & & & & & \\
 & & & & I_p^{'} \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I & & & & & \\
 & & & & & \ddots & \ddots & \ddots & & & & \\
 & & & & & & I_p^{'} \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I & & & \\
 & & & & & & & I \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I & & \\
 & & & & & & & & \ddots & \ddots & \ddots & \\
 & & & & & & & & & I \otimes s\beta & D_0 \oplus S_0 & D_1 \otimes I \\
 & & & & & & & & & & I \otimes s\beta - (D_1 \otimes I)G & D_0 \oplus S + (D_1 \otimes I)G
\end{pmatrix}
$$

(Horvath et al., 2010)

Performance measures

$$E[I] = \sum_{z=0}^{Z^{(m)}} \pi_z (Z^{(m)} - z)$$

$$E[B] = \sum_{z=Z^{(m)}+1}^{\infty} \pi_z (z - Z^{(m)})$$

$$TC = h.E[I] + b.E[B]$$

$$P(I < 0) = \sum_{z=Z^{(m)}+1}^{\infty} \pi_z$$

Experiment Design

The behavior of the performance measures in processes with

- Negative Autocorrelation
- Positive Autocorrelation
- $cv > 1$
- $cv < 1$

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How Does Approximating Correlated Arrival by mean of its

- First Moment (Exponential Distribution (M))
- First Two Moments ($C_{2:b}$ Distribution)
- Marginal Distribution (PH Distribution)

approximates the optimal control policy.

$$h = 1 \quad b = 5$$

Impact of autocorrelation on the inter-event times

- Positive autocorrelations create clusters of long and short inter-event time.
- With negative autocorrelations, a short inter-event time is followed by a long inter-event time.

Impact of negative autocorrelation on performance measures of the system

$$D_0 = \begin{bmatrix} -1.5 & 1.5 & 0 \\ 0 & -3 & 1.5 \\ 0 & 0 & -1.5 \end{bmatrix} \qquad D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \end{bmatrix}$$

Table: Moment and autocorrelation structure of a negatively correlated process with $cv < 1$

	1	2	3	4	5
Moment	1.0	1.8	4.4	14.2	55.3
ith Lag AC	-0.1429	0	0	0	0

Impact of negative autocorrelation on optimal inventory levels

Table: Performance Measures of a Negatively Correlated System with $cv < 1$ and Its Un-Correlated Approximations with $\rho = 0.5$ Controlled By State-Dependent and State-Independent Base-Stock Policies

		$\bar{Z}^*$	TC	Error	$E[B]$	Error	$E[I]$	Error	$Pr(I < 0)$	Error
Opt	MAP/M/1	(1,2,2)	1.9200		0.1797		1.0214		0.1089	
App1	MAP/M/1	2	2.0006	4%	0.1446	-20%	1.2774	25%	0.0877	-20%
App2	PH/M/1	2	2.0006	4%	0.1446	-20%	1.2774	25%	0.0877	-20%
App3	$C_{2:b}/M/1$	2	2.0006	4%	0.1446	-20%	1.2774	25%	0.0877	-20%
App4	M/M/1	2	2.0006	4%	0.1446	-20%	1.2774	25%	0.0877	-20%

Impact of negative autocorrelation on optimal inventory levels

Table: Performance Measures of a Negatively Correlated System with $cv < 1$ and Its Un-Correlated Approximations with $\rho = 0.8$ Controlled By State-Dependent and State-Independent Base-Stock Policies

		$\bar{Z}^*$	TC	Error	$E[B]$	Error	$E[I]$	Error	$Pr(I < 0)$	Error
Opt	MAP/M/1	(5,6,6)	6.1660		0.6114		3.1088		0.1543	
App1	MAP/M/1	6	6.1775	0%	0.5692	-7%	3.3316	7%	0.1437	-7%
App2	PH/M/1	7	6.3155	2%	0.4255	-30%	4.1879	35%	0.1074	-30%
App3	$C_{2:b}/M/1$	7	6.3155	2%	0.4255	-30%	4.1879	35%	0.1074	-30%
App4	M/M/1	8	6.6711	8%	0.3181	-48%	5.0805	63%	0.0803	-48%

Impact of positive autocorrelation on optimal inventory levels

$$D_0 = \begin{bmatrix} -1.4964 & 0 & 0.0426 \\ 0.0033 & -1.4351 & 1.4209 \\ 0 & 0 & -1.5336 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1.4209 & 0.0329 & 0 \\ 0 & 0 & 0.1093 \\ 0.0533 & 1.4803 & 0 \end{bmatrix}$$

Table: Moment and autocorrelation structure of a positively correlated process with $cv < 1$

	1	2	3	4	5
Moment	1.1	1.95	5.0	16.4	65.0
ith Lag AC	0.1226	0.1099	0.1004	0.0918	0.0839

Impact of positive autocorrelation on optimal inventory levels

Table: Performance Measures of a Positively Correlated System with $cv < 1$ and Its Un-Correlated Approximations with $\rho = 0.5$ Controlled By State-Dependent and State-Independent Base-Stock Policies

		$\bar{Z}^*$	TC	Error	$E[B]$	Error	$E[I]$	Error	$Pr(I < 0)$	Error
Opt	MAP/M/1	(1,2,4)	2.7794		0.3077		1.2409		0.1172	
App1	MAP/M/1	2	3.2454	17%	0.3967	29%	1.2617	2%	0.1359	16%
App2	PH/M/1	2	3.2454	17%	0.3967	29%	1.2617	2%	0.1359	16%
App3	$C_{2:b}/M/1$	2	3.2454	17%	0.3967	29%	1.2617	2%	0.1359	16%
App4	M/M/1	2	3.2454	17%	0.3967	29%	1.2617	2%	0.1359	16%

Impact of positive autocorrelation on optimal inventory levels

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		$\bar{Z}^*$	TC	Error	$E[B]$	Error	$E[I]$	Error	$Pr(I < 0)$	Error
Opt	MAP/M/1	(11,12,19)	15.5317		1.6262		7.4006		0.1606	
App1	MAP/M/1	13	15.8697	2%	1.5373	-5%	8.1829	11%	0.1518	-5%
App2	PH/M/1	7	17.8717	15%	2.8710	77%	3.5166	-52%	0.2846	77%
App3	$C_{2:b}/M/1$	7	17.8717	15%	2.8710	77%	3.5166	-52%	0.2846	77%
App4	M/M/1	8	17.1643	11%	2.5864	59%	4.2320	-43%	0.2559	59%

Conclusions

- We show that the optimal control policy that minimizes the expected holding and backlog cost for a production/inventory system with arrival and service times modeled with Markov arrival processes is a state-dependent base-stock policy

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- We show that the optimal control policy that minimizes the expected holding and backlog cost for a production/inventory system with arrival and service times modeled with Markov arrival processes is a state-dependent base-stock policy
- We show how to evaluate performance measures of a production system by using matrix analytic methods

Conclusions

- The optimal performance measures of a system with correlated demand arrival are highly dependent on the traffic intensity of the system, coefficient of variation, and the autocorrelation structure of the arrival process

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- The optimal performance measures of a system with correlated demand arrival are highly dependent on the traffic intensity of the system, coefficient of variation, and the autocorrelation structure of the arrival process
- The state-independent base-stock policies give a good approximation of the performance measures in systems with negatively correlated demand. However, when the demand is positively correlated, using a state-dependent base-stock policy improves the performance

Future research

- What is the relation between Geometric Matrix (R,G) and autocorrelation structure of the arrival and service process?
- What is the optimal policy in systems with partially observable states?
- How to use shop floor arrival and service data to control manufacturing systems more efficiently?
- How include additional types of data such as inter-failure time, and inter-repair time in modeling?

Questions?

Bayraktar, E. and Ludkovski, M. (2010). Inventory management with partially observed nonstationary demand. *Annals of Operations Research*, 176(1):7–39.

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