Performance Evaluation of Milkrun-Supplied Flow Lines

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11^{th} Conference on Stochastic Models of Manufacturing and Service Operations

A milkrun-supplied flow line

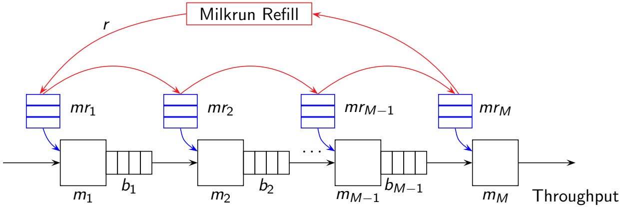

Figure 1: Milkrun-supplied flow line with $i = 1, \ldots, M$ machines

- Workpieces are matched with milkrun material for processing at m_i
- Machine-specific order up-to level S_i of milkrun material mr_i
- Uncapacitated milkrun supply happens every r time periods
- Starving and blocking of m_i refers to workpieces and milkrun material
- Milkrun shortages interrupt workpiece flow and milkrun material demand

Research questions

Evaluation

1. What is the fill rate of milkrun material supply for a given flow line configuration?
2. What is the impact of milkrun supply shortage on the throughput of the production system?

Optimization

1. What are the minimal milkrun storage areas subject to target milkrun supply fill rate?
2. What is the cost-minimal allocation of milkrun storage areas and buffers subject to target throughput?

Related literature and research gap

Related literature

- Transport consolidation: Özden (2011), Schwarz et al. (2015), . . .
- Supplied flow lines: Bukchin and Meller (2005), Weiss et al. (2017), . . .
- Handling: Kovacs (2011), Faccio et al. (2013), Alnahhal and Noche (2014), . . .
- Vehicle Routing: Kilic et al. (2012), Satoglu and Sahin (2013), Meyer (2015), . . .
- Flow line evaluation: Lagershausen (2013), Li and Meerkov (2009), . . .

Proposal of 2 new evaluation approaches

1. Flow line output as basis for calculation of milkrun supply fill rate
2. Production rate as function of milkrun supply and flow line configuration

1. Approach: fill rate of milkrun supply

1. Markov chain approach: times between processing starts $TBPS_i$

- Closed flow line with general processing times and finite buffers
- Calculate $TBPS_i$ with Markov chain approach by Lagershausen (2013)
- Problem size is limited due to Markov chain approach of $TBPS_i$

→ Hypothesis that $TBPS_i \sim$ gamma-distributed cannot be rejected for test data set using Kolmogoroff-Smirnov test and Chi-Square test

2. Counting process: demand for milkrun material during replenishment time

- Probability distribution of number of processing starts during the replenishment time r: $P\{N(r) = n\} = P\{Y_n \leq r\} - P\{Y_{n+1} \leq r\}$
- Whereby $Y_n = \sum_{j=0}^{n} TBPS_{ij} \; \forall \, i$ denotes the cumulative time until the occurrence of the n^{th} processing start

→ Basis for α-fill rate calculation (= prob. for no milkrun material shortages)

2a. Approach: production rate of two-machine line

Modeling approach and conventions according to Li and Meerkov (2009)

- Discrete time slots (= cycle time of machines)
- Machine m_i is up during time slot with probability p_i $\rightarrow$ time-dependent failures
- Buffer capacity $b_1 = 1, \ldots, N$ and milkrun storage area $mr_i = 1, \ldots, S_i$, $i \epsilon \{1, 2\}$
- Blocking before service
- First machine never starved, last machine never blocked
- Machine states are determined at the beginning and buffer states at the end of each time slot

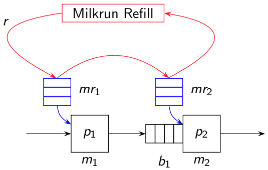

Figure 2: Milkrun-supplied two-machine line

2a. Approach: production rate of two-machine line

Generate transition probability matrix

- States of the system are modeled through:
  - State of the buffer between the machines $b_{1;0,\dots,N}$
  - States of milkrun storage areas $mr_{1;0,\dots,S_1}$ and $mr_{2;0,\dots,S_2}$
  - Time-dependence in relation to replenishment period r
- State space consists of $(N+1)*(S_1+1)*(S_2+1)*r$ states
- Transition probabilities expressed through p_1 and p_2
- Extension to milkrun-supplied three-machine line follows the same approach with $(N_1+1)*(N_2+1)*(S_1+1)*(S_2+1)*(S_3+1)*r$ states

Reduction of the number of states

- In replenishment period: milkrun material equals order up-to level S
- Minimum milkrun material: order up-to level - time counter $(S-t)$
- Maximum milkrun material: order up-to level - time counter * machine availability * safety factor $(S-t*p*sf)$

→ State reduction can amount to 50% of state number in dependence of milkrun storage area size

2a. Approach: production rate of two-machine line

Computation

- Transition matrix generation: VB.Net $\rightarrow$ most computational effort!
- Numerical solution: MATLAB $\rightarrow$ eigs-function to calculate the eigenvector for the largest eigenvalue of a sparse stochastic matrix

$\rightarrow$ Derive steady-state probabilities: $b_{1;0}$, $b_{1;N}$, $mr_{1;0}$, $mr_{2;0}$

Performance measures of two-machine line

$$PR = P[m_2 \text{ is up at the beginning of a time slot} \cap \text{buffer is not empty at the end of previous time slot} \cap 2^{nd} \text{ milkrun buffer is not empty at the end of previous time slot}]$$

$$= p_2 * (1 - Pb_{1;0} - Pmr_{2;0} + Pb_{1;0} * Pmr_{2;0}) = p_2 * (1 - P^S)$$

$$= P[m_1 \text{ is up at the beginning of a time slot} \cap \text{buffer is not full at the beginning of this time slot} \cap 1^{st} \text{ milkrun buffer is not empty at the end of the previous time slot}]$$

$$= p_1 * [1 - (1 - p_2) * Pb_{1;N} - p_2 * Pb_{1;N} * Pmr_{2;0}] * (1 - Pmr_{1;0}) = p_1 * (1 - P^B)$$

2b. Approach: production rate of $M > 2$-machine line

Modeling approach according to Li and Meerkov (2009)

Backward: aggregate each two last machines of the line into a single Bernoulli machine

- Use (2a) to calculate probability for full buffers and empty milkrun material P^B
- Calculate virtual production rate $p_i^{backward} = p_i * [1 - P^B(p_{i+1}^{backward}, p_i^{forward}, b_i)]$
- Exception for boundary condition $p_M = p_M^{backward}$

Forward: aggregate each first machine with aggregated version of the rest of the line into single Bernoulli machine

- Use (2a) to calculate probability for empty buffers or empty milkrun material P^S
- Calculate virtual production rate $p_i^{forward} = p_i * [1 - P^S(p_{i-1}^{forward}, p_i^{backward}, b_{i-1})]$
- Exception for boundary condition $p_1 = p_1^{forward}$

$\rightarrow$ **Repeat forward and backward aggregation until production rate converges**

2b. Approach: production rate of $M > 2$-machine line

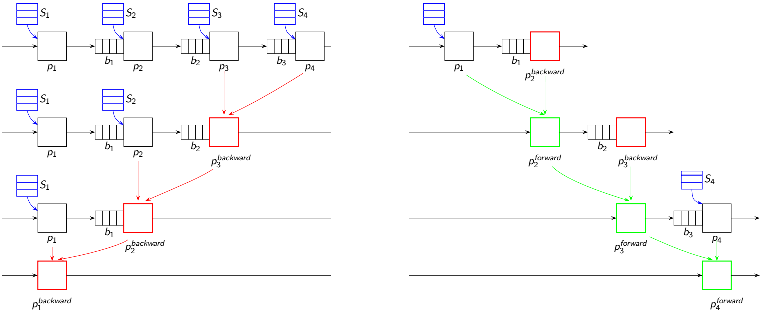

Figure 3: 1^{st} iteration: backward and foward aggregation of 4-machine line

- Recursive calculation of virtual production rates: boundary machines (in this example 1 and 4) are not starved respectively blocked by workpieces, but disrupted by milkrun.
- Approach can also be applied to special case: $M \geq 2$ machines where only the 1^{st} machine is supplied by (r, S)-policy, application as in Weiss et al. (2017).

2b. Approach: production rate of $M > 2$-machine line

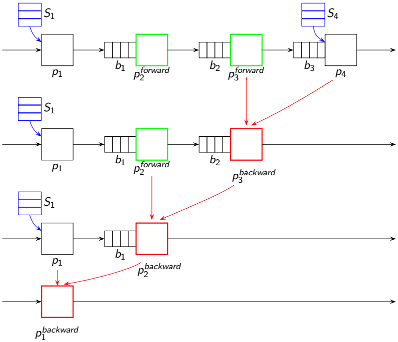

Figure 4: 2^{nd} iteration: backward aggregation of 4-machine line

To investigate: milkrun extension does not seem to cause problems with the algorithm's convergence proved by Li and Meerkov (2009)

Performance measures of milkrun-supplied 2-machine line vs. S_2

Throughput grows with the order up-to level S as long as there is milkrun supply shortage.

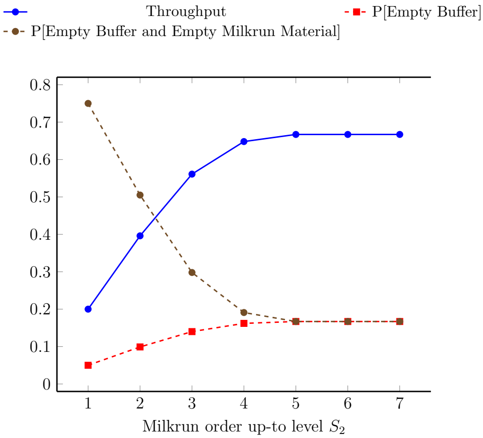

Figure 5: Performance measures of 2^{nd} machine versus milkrun order up-to level S_2, input parameters: $S_1 = 5$, $p_1 = p_2 = 0.8$, $b_1 = 1$, $r = 5$

Performance measures of milkrun-supplied 2-machine line vs. b_1

Throughput growth due to increasing buffer size is limited by milkrun parameters.

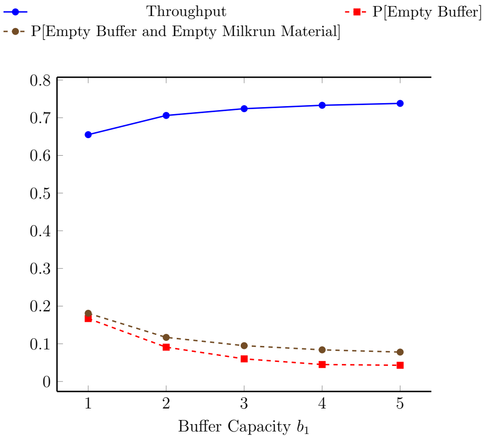

Figure 6: Performance measures of 2^{nd} machine versus buffer capacity b_1, input parameters: $S_1 = S_2 = 5$, $p_1 = p_2 = 0.8$, $r = 6$

Computation time for milkrun-supplied 2-machine line

Approach rather fits small-scale supply consolidation of flow lines.
Computation time due to transition matrix generation-solved within seconds!

$b_1, r, S = S_1 = S_2$	Computation time in mm:ss	No of States (after reduction)
$b_i = 1, r = 3, S = 2$	00:00	34
$b_i = 1, r = 4, S = 3$	00:01	96
$b_i = 1, r = 6, S = 5$	00:57	272
$b_i = 1, r = 7, S = 6$	00:54	270
$b_i = 1, r = 8, S = 7$	03:32	368
$b_i = 2, r = 3, S = 2$	00:00	51
$b_i = 2, r = 4, S = 3$	00:04	144
$b_i = 2, r = 5, S = 4$	00:07	162
$b_i = 2, r = 6, S = 5$	05:14	408
$b_i = 3, r = 3, S = 2$	00:00	68
$b_i = 3, r = 4, S = 3$	00:14	192
$b_i = 3, r = 5, S = 4$	03:50	378

Table 1: Computation time for 2-machine line with $p_1 = p_2 = 0.8$

Computations performed on an Intel(R) Core(TM) i7-4800 MQ CPU with 2.7 GHz and 8 GB RAM

Performance measures of milkrun-supplied 2-machine line

Insights

- Throughput rises with the order up-to level S (and decreasing replenishment interval r) as long as there is milkrun material shortage
- Throughput rises with increasing buffer size b but milkrun material parameters may limit the growth
- Opposite trend of state probabilities for empty buffer and empty milkrun material
- Throughput is limited to $PR^{max} = \frac{S_i^{\min}}{r}$ where $S_i^{\min}$ is the minimum order up-to level within the flow line
- Milkrun parameter ratio $\frac{S_i}{r}$ leads to different probabilities of empty milkrun material (e.g. $\frac{S_i}{r} = \frac{2}{3}$ vs. $\frac{S_i}{r} = \frac{4}{6}$)
- Evaluation approach is limited to small-scale supply consolidation

Quality of throughput approximation for $M > 2$ milkrun-supplied lines

Approximation quality is better for higher replenishment intervals r and larger buffers b_i.

Number of Machines $i, p_i, b_i, S_i, r \; \forall \, i$	PR_{Approx}	PR_{Sim}	$\frac{PR_{Sim} - PR_{Approx}}{PR_{Sim}}$
$i = 3, p_i = 0.8, b_i = 3, S_i = 4, r = 10$	0.40	0.40	0.00%
$i = 3, p_i = 0.8, b_i = 3, S_i = 3, r = 4$	0.67	0.66	-1.52%
$i = 3, p_i = 0.8, b_i = 3, S_i = 2, r = 3$	0.61	0.61	0.00%
$i = 3, p_i = 0.8, b_i = 3, S_i = 1, r = 2$	0.47	0.47	0.00%
$i = 3, p_i = 0.5, b_i = 1, S_i = 1, r = 2$	0.25	0.25	0.00%
$i = 3, p_i = 0.8, b_i = 1, S_i = 1, r = 2$	0.42	0.44	4.55%
$i = 3, p_i = 0.9, b_i = 1, S_i = 1, r = 2$	0.46	0.48	4.17%
$i = 4, p_i = 0.8, b_i = 1, S_i = 1, r = 2$	0.41	0.42	2.38%
$i = 5, p_i = 0.8, b_i = 1, S_i = 1, r = 2$	0.41	0.41	0.00%
$i = 7, p_i = 0.8, b_i = 1, S_i = 1, r = 2$	0.40	0.40	0.00%
$i = 15, p_i = 0.8, b_i = 1, S_i = 1, r = 2$	0.39	0.38	-2.63%
$i = 3, p_i \epsilon \{0.8, 0.6, 0.9\}, b_i = 1, S_i = 1, r = 2$	0.39	0.39	0.00%
$i = 4, p_i \epsilon \{0.8, 0.6, 0.9, 0.7\}, b_i = 1, S_i = 1, r = 2$	0.37	0.38	2.63%
$i = 5, p_i \epsilon \{0.7, 0.8, 0.9, 0.8, 0.7\}, b_i = 1, S_i = 1, r = 2$	0.40	0.40	0.00%
$i = 7, p_i \epsilon \{0.6, 0.65, 0.7, 0.75, 0.8, 0.9, 0.95\}, b_i = 1, S_i = 1, r = 2$	0.34	0.33	-3.03%

Table 2: Quality of aggregation procedure for $M > 2$ milkrun-supplied lines

Summary

- Milkrun material shortages change workpiece flow and the demand for milkrun material $\rightarrow$ α-fill rate as performance measure
- Production rate is determined by:
  - either flow line configuration $\rightarrow$ no milkrun shortages
  - or milkrun parameters $\rightarrow$ maximum production rate is limited to $\frac{S_i^{min}}{r}$

$\rightarrow$ Milkrun supply can only restrict the production rate of a flow line

Future research

- Fast evaluation approach for $M > 2$ milkrun-supplied line $\rightarrow$ approximation or simulation of milkrun-supplied 2-machine line?
- Solve optimization problems
  - Minimize milkrun storage areas subject to target milkrun supply fill rate
  - In case of milkrun material shortages: find cost-optimal milkrun material and buffer capacity subject to target throughput

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