Dynamic capacitated Lot Sizing with Random Demand and Random Yield

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Outline

1. The Problem
2. Model Formulation
3. Solution
4. Numerical Results
5. Conclusion

Problem definition

Dynamic Lotsizing, random demand, random yield

- A single resource with setups and limited capacity
- Multiple products with dynamic and random demand (normally distributed)

Period t	6/2017	7/2017	...	12/2017
μ_t	...	...	...	...
σ_t	...	...	...	...

- Random yield (Binomial yield with success probability p per unit produced)
- 'Static-Dynamic Uncertainty' policy (usually applied in a rolling planning environment)
- Service level: Fill rate per cycle (β_c)

Industrial practice

Planning approach (MRP, APS, OM textbooks)

Current industrial practice

- Forecast future demands and add a scrap allowance
- Use a lotsizing model (EOQ, SIULSP, CLSP, . . .).

Result: Uncontrollable safety stock and service level

Model SCLSP$^{Y}_{\beta^c}$

Non-linear model formulation, I

$$\min Z = \sum_{k=1}^{K}\sum_{t=1}^{T}(s_k \cdot \gamma_{kt} + h_k \cdot E\left\{I_{kt}^{p,\text{end}}\right\})$$

s.t.

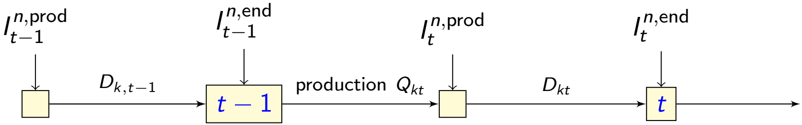
$$I_{k,t-1}^{n,\text{end}} + \underbrace{Q_{kt}(\overbrace{q_{kt}}^{\text{planned}}, p_k)}_{\text{observed}} - D_{kt} = I_{kt}^{n,\text{end}} \qquad \forall k, \forall t$$

$$q_{kt} - M \cdot \gamma_{kt} \leq 0 \qquad \forall k, \forall t$$

$$\sum_{k=1}^{K}(tb_k \cdot q_{kt} + tr_k \cdot \gamma_{kt}) \leq b_t \qquad \forall t$$

Net inventory I^n

Sequence of events



Model $\text{SCLSP}^{Y}_{\beta^c}$

Non-linear model formulation, II

Inventory on hand

$$I^{p,\text{end}}_{kt} = [I^{n,\text{end}}_{kt}]^{+} \qquad \forall k, \forall t$$

Backlog

$$I^{f,\text{prod}}_{kt} = -[I^{n,\text{end}}_{k,t-1} + Q_{kt}(q_{kt}, p_k)]^{-} \qquad \forall k, \forall t$$

$$I^{f,\text{end}}_{kt} = -[I^{n,\text{end}}_{kt}]^{-} \qquad \forall k, \forall t$$

Backorders

$$B_{kt} = I^{f,\text{end}}_{kt} - I^{f,\text{prod}}_{kt} \qquad \forall k, \forall t$$

Model SCLSP$^{Y}_{\beta^c}$

Non-linear model formulation, III

$$1 - \frac{E\left\{\sum_{\tau=t-\ell_{kt}}^{t} B_{k\tau}\right\}}{E\left\{\sum_{\tau=t-\ell_{kt}}^{t} D_{k\tau}\right\}} \geq \beta_k^c \qquad \forall k, \forall t \epsilon \{t | \gamma_{k,t+1} = 1\}$$

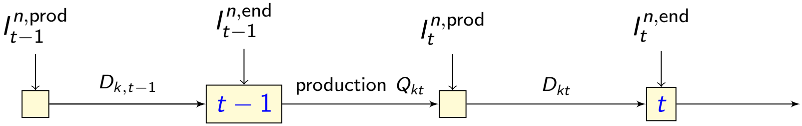
$$\ell_{kt} = (\ell_{k,t-1} + 1) \cdot (1 - \gamma_{kt}) \qquad \forall k, \forall t$$

$$\ell_{k0} = -1 \qquad \forall k$$

$$\ell_{kt} \geq 0 \qquad \forall k, \forall t$$

$$q_{kt} \geq 0 \qquad \forall k, \forall t$$

Net inventory I^n



$$I_{kt}^{n,\text{prod}} = I_{k,t-1}^{n,\text{prod}} + Q_{kt}(q_{kt}, p_k) - D_{k,t-1}$$

$$I_{kt}^{n,\text{end}} = I_{k,t-1}^{n,\text{end}} + Q_{kt}(q_{kt}, p_k) - D_{kt}$$

...

Net inventory I_{kt}^n

Parameters (**normal** distribution)

Expected values

$$\mu_{I_{kt}^{n,\text{prod}}} = \mu_{I_{k,t-1}^{n,\text{prod}}} + \mu_{Q_{kt}} - \mu_{D_{k,t-1}}$$

$$\mu_{I_{kt}^{n,\text{end}}} = \mu_{I_{k,t-1}^{n,\text{end}}} + \mu_{Q_{kt}} - \mu_{D_{kt}}$$

Variances

$$\sigma^2_{I_{kt}^{n,\text{prod}}} = \sigma^2_{I_{k,t-1}^{n,\text{prod}}} + \sigma^2_{Q_{kt}} + \sigma^2_{D_{k,t-1}}$$

$$\sigma^2_{I_{kt}^{n,\text{end}}} = \sigma^2_{I_{k,t-1}^{n,\text{end}}} + \sigma^2_{Q_{kt}} + \sigma^2_{D_{kt}}$$

Performance criteria

Inventory on hand and backorders

Inventory on hand at the end of period t

$$E\{I_{kt}^{p,\text{end}}\} = E\{\left[I_{kt}^{n,\text{end}}\right]^{+}\}$$

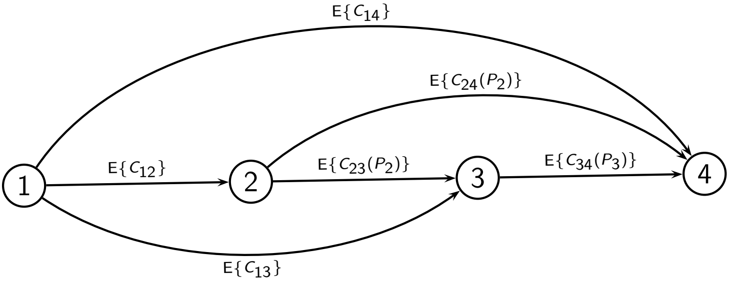
Backorders in period t

$$E\{B_t\} = E\{I_{kt}^{f,\text{end}}\} - E\{I_{kt}^{f,\text{prod}}\}$$

Backlog at the end of period t

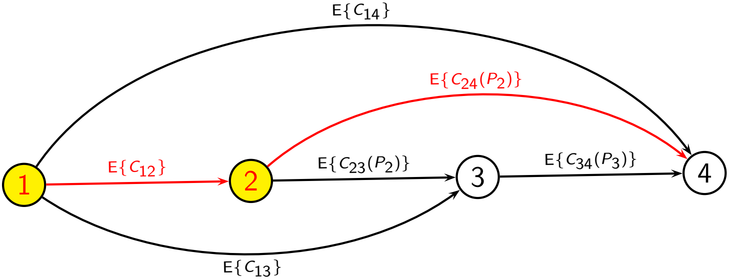
$$E\{I_{kt}^{f,\text{end}}\} = -\left(E\{I_{kt}^{n,\text{end}}\} - E\{I_{kt}^{p,\text{end}}\}\right)$$

Shortest-Path Representation



Shortest-Path Representation (3 periods)

Production in periods 1 and 2



Evaluation

of an edge from τ to t

$$E\{C_{\tau t}\} = s + h \cdot \sum_{\ell=\tau}^{t-1} E\left\{ \left[I_{\tau-1}^{\text{p,end}}(P_\tau) + \underbrace{Q[\overbrace{q^\star_{\tau t}(P_\tau)}^{\textit{lot size}}, p_k]}_{\textit{Effective outcome}} - \sum_{i=\tau}^{\ell} D_i \right]^+ \right\}$$

P_τ – Path from 1 to τ

Initial inventory at the beginning of period τ depends on path P_τ

$q^\star_{\tau t}$ – planned lot size required to meet target service level β_c

Multi-item problem

Solution approaches

- ABC^{Y}_{β} heuristic
- Column generation (set partitioning model)
- MIP with piecewise-linear approximation of inventory and backorder functions

Dynamic and random demand with finite capacities

ABC_{β}^{Y} Heuristic

Basic Principle

Transform the matrix of demands into a matrix of production quantities

d_{11}	d_{12}	d_{13}	$\cdots$
d_{21}	d_{22}	d_{23}	$\cdots$
d_{31}	d_{32}	d_{33}	$\cdots$
d_{41}	d_{42}	d_{43}	$\cdots$

look ahead

q_{11}	—	—	$\cdots$	
q_{21}	—	q_{23}	$\cdots$	T
q_{31}	q_{32}	q_{33}	$\cdots$	
q_{41}	q_{42}	—	$\cdots$	

Dynamic and Random Demand with Finite Capacities

ABC_{β}^{Y} Heuristic

procedure ABC_{β}^{Y}
  for $\tau = 1, 2, \ldots T$ **do**
    Call $\textsc{CreateLots}(\tau)$
    if $\left(\sum_{k=1}^{K} (tp_k \cdot Q[q_{kt}^{\star}, p_k] + ts_k \cdot \gamma_{kt}) < b_\tau\right)$ **then**
      Call $\textsc{ExtendLots}(\tau)$
    else
      Call $\textsc{ShiftProduction}(\tau)$
    end if
  end for
end procedure

Column Generation Heuristic

Generation of Production Plans (β_c service level)

For each product k create a set of alternative production plans and select exactly one plan.

		n 1	2	3	4	5	6
	c_{kn}	3487.18	2214.59	2601.19	2225.15	2688.98	2807.74
t	1	28.40	132.46	641.64	326.39	132.46	28.40
	2	106.65	–	–	–	–	386.39
	3	203.52	290.51	–	–	206.06	–
	4	111.73	–	–	338.52	228.11	–
	5	132.29	253.26	–	–	–	167.85
	6	118.47	–	–	–	–	–

Column Generation Heuristic

Set Partitioning Model: $\text{SCLSP}_{\text{SPP}}$

$$\text{Minimize } Z = \sum_{k=1}^{K} \sum_{n=1}^{P_k} c_{kn} \cdot \delta_{kn}$$

subject to

$$\sum_{k=1}^{K} \sum_{n=1}^{P_k} \kappa_{knt} \cdot \delta_{kn} \leq b_t \qquad t = 1, 2, \ldots, T \quad (\pi_t)$$

$$\sum_{n=1}^{P_k} \delta_{kn} = 1 \qquad k = 1, 2, \ldots, K \quad (\sigma_k)$$

$$\delta_{kn} = \{0, 1\} \qquad k = 1, 2, \ldots, K; n = 1, 2, \ldots, P_k$$

Column Generation Heuristic

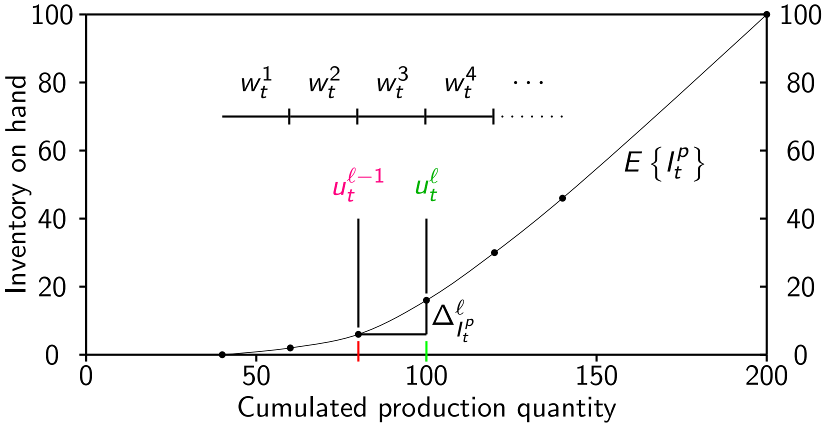
Solution Procedure

Basic Structure of the CG Heuristic

1. Generate production plans (e. g. with the ABC_{β}^{Y} heuristic)
2. Apply standard column generation procedure
3. For the remaining items: apply ABC_{β}^{Y} heuristic

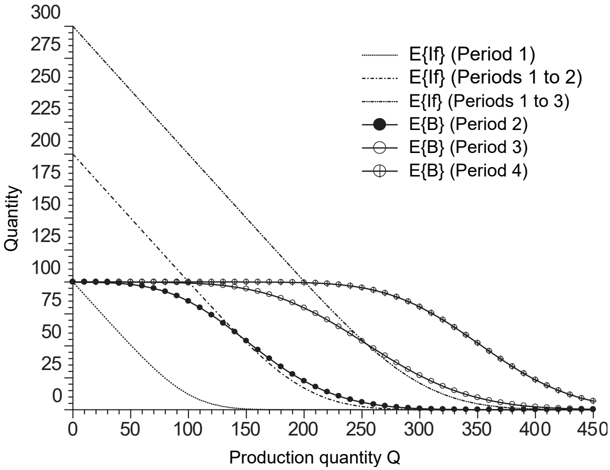
MIP based Heuristic

Piecewise Linear Approximation of Inventory on hand



MIP based Heuristic

Piecewise Linear Approximation of Backorders



MIP based Heuristic

Piecewise Linear Approximation of Inventory on hand

Slope of the *inventory on hand* function for line segment ℓ:

$$\Delta^{\ell}_{I^{p,\text{end}}_{kt}} = \frac{E\left\{I^{p,\text{end}}_{kt}(u^{\ell}_{kt})\right\} - E\left\{I^{p,\text{end}}_{kt}(u^{\ell-1}_{kt})\right\}}{u^{\ell}_{kt} - u^{\ell-1}_{kt}}$$

Slope of the function of the *backorders* for line segment ℓ is

$$\Delta^{\ell}_{B_{kt}} = \frac{E\left\{B_{kt}(u^{\ell}_{kt})\right\} - E\left\{B_{kt}(u^{\ell-1}_{kt})\right\}}{u^{\ell}_{kt} - u^{\ell-1}_{kt}}$$

MIP based Heuristic

Piecewise Linear Approximation; Multi-item

$$\text{Minimize } E\{C\} = \sum_{k=1}^{K} \sum_{t=1}^{T} \left(s_k \cdot \gamma_{kt} + h \cdot \left[\Delta^0_{I^p_{kt}} + \sum_{\ell=1}^{L} \Delta^\ell_{I^p_{kt}} \cdot w^\ell_{kt} \right] \right)$$

subject to

$$\sum_{k=1}^{K} (tb_k \cdot q_{kt} + tr_k \cdot \gamma_{kt}) \leq b_t \qquad t = 1, 2, \ldots, T$$

$$w^\ell_{kt} \leq u^\ell_{kt} - u^{\ell-1}_{kt} \qquad \begin{array}{l} t = 1, 2, \ldots, T; \\ \ell = 1, 2, \ldots, L; \\ k = 1, 2, \ldots, K \end{array}$$

MIP based Heuristic

Piecewise Linear Approximation

$$\sum_{\ell=1}^{L} w_{kt}^{\ell} - \sum_{\ell=1}^{L} w_{k,t-1}^{\ell} = q_{kt} \qquad \begin{array}{l} t = 1, 2, \ldots, T \\ k = 1, 2, \ldots, K \end{array}$$

$$q_{kt} \leq u_{kt}^{L} \cdot \gamma_{kt} \qquad \begin{array}{l} t = 1, 2, \ldots, T; \\ k = 1, 2, \ldots, K \end{array}$$

MIP based Heuristic

Piecewise Linear Approximation

$$w_{kt}^{\ell} \leq M \cdot \lambda_{kt}^{\ell} \qquad \begin{array}{l} t = 1, 2, \ldots, T; \\ \ell = 1, 2, \ldots, L; \\ k = 1, 2, \ldots, K \end{array}$$

$$w_{kt}^{\ell} \geq (u_{kt}^{\ell} - u_{kt}^{\ell-1}) \cdot \lambda_{kt}^{\ell+1} \qquad \begin{array}{l} t = 1, 2, \ldots, T; \\ \ell = 1, 2, \ldots, L-1; \\ k = 1, 2, \ldots, K \end{array}$$

MIP based Heuristic

Piecewise Linear Approximation: Service level

$$\underbrace{\frac{\sum_{i=\tau}^{t}(\Delta^0_{B_{ki}} + \sum_{\ell=1}^{L} \Delta^{\ell}_{B_{ki}} \cdot w^{\ell}_{ki})}{\sum_{i=\tau}^{t} E\{D_{ki}\}}}_{\text{proportion of backorders } (<1)} \leq 1 - \beta_{ck} + (1 - \gamma_{k\tau}))$$

$$k = 1, 2, \ldots, K; \tau = 1, 2, \ldots, T; t = \tau, \tau + 1, \ldots, T; \sum_{i=\tau}^{t} E\{D_{ki}\} > 0$$

$$\sum_{i=1}^{t} \gamma_{ki} \geq 1 \qquad k = 1, 2, \ldots, K; t = 1, 2, \ldots, T; \; \sum_{i=1}^{t} E\{D_{ki}\} > 0$$

MIP based Heuristic

$$\gamma_{kt} \in \{0,1\} \qquad t = 1,2,\ldots,T; k = 1,2,\ldots,K$$

$$w_{kt}^{\ell} \geq 0 \qquad t = 1,2,\ldots,T; \ell = 1,2,\ldots,L; k = 1,2,\ldots,K$$

$$\lambda_{kt}^{\ell} \in \{0,1\} \qquad t = 1,2,\ldots,T; \ell = 1,2,\ldots,L; k = 1,2,\ldots,K$$

Numerical Experiment

Data

Parameter	**Value**
Number of products	10, 40
Number of periods	10, 20
Average utilization	40%, 60%, 80%
Demand (mean)	$E\{D_{kt}\} \sim DU(20, 40, 60, 80, 100)$
Demand (coefficient of variation)	$CV\{D_{kt}\} \sim DU(0.1, 0.3)$
Random yield	BI yield with $p_k \sim DU(0.7, 0.8, 0.9)$
Time between orders	$TBO_k \sim DU(2, 3, 4)$
Setup costs	$s_k = 1000$
Holding costs	$h_k = 2s_k / (TBO_k^2 \cdot \sum_{t=1}^{T} E\{D_{kt}\} / T)$
Setup time	$ts_k \sim DU(5, 10, 20)$
Processing time	$tb_k = 1$
Target fill rate	$\beta_k^c = 0.9$

Numerical Experiment

Solution Quality

[K; T]	Total	ABC_{β}^{Y}	CG	CG2	F&O
[10; 10]	254	0.0%	-12.5%	-13.0%	-11.3%
[10; 20]	230	0.0%	-15.2%	-15.4%	-14.7%
[40; 10]	282	0.0%	-13.2%	-13.3%	-11.4%
[40; 20]	270	0.0%	-16.1%	-16.2%	-14.6%
Total	1036	0.0%	-15.0%	-15.1%	-13.6%

Objective value compared to ABC_{β}^{Y} solution
CG2 is best, but F&O is more flexible ($CLSP_L$)

Numerical Experiment

Solution Time (sec)

[K; T]	Total	ABC_β^Y	CG	CG2	F&O
[10; 10]	300	0.01	0.47	0.38	20.70
[10; 20]	300	0.02	5.21	3.62	136.91
[40; 10]	300	0.04	3.22	2.33	226.32
[40; 20]	300	0.08	20.47	26.45	1007.80
Total	1200	0.04	7.34	8.2	347.93

Numerical Experiment

Number of Feasible Solutions

[K; T]	Total	ABC_{β}^{Y}	CG	CG2	F&O	CPLEX
[10; 10]	300	298	254	298	299	265
[10; 20]	300	300	230	300	300	278
[40; 10]	300	300	282	300	300	106
[40; 20]	300	300	270	300	300	0
Total	1200	1198	1036	1198	1197	649

Numerical Experiment

CPLEX versus F&O

[K; T]	Total	CPLEX
[10; 10]	265	-1.3%
[10; 20]	278	1.3%
[40; 10]	106	4.5%
[40; 20]	0	–

Endlich . . .

- In MRP and OM textbooks random yield is almost neglected
- Column generation approach performs best, but
- Linearization approach can be extended (e. g. include setup carry-overs: CLSP-L)
- By-product of our research: a new variant of ABC heuristic (setup times)
- Multi-level extensions