

---

# Dynamic Lead Time Based Control Point Policy for Multi Stage Manufacturing Systems

**Marcello Colledani<sup>(1)</sup>, Stanley B. Gershwin<sup>(2)</sup>**

---

11<sup>th</sup> Conference on Stochastic Models of Manufacturing and  
Service Operations, SMMSO2017, Acaya, Lecce, June 5<sup>th</sup>.

(1) Department of Mechanical Engineering  
Politecnico di Milano

(2) Laboratory for Manufacturing and Productivity  
Massachusetts Institute of Technology

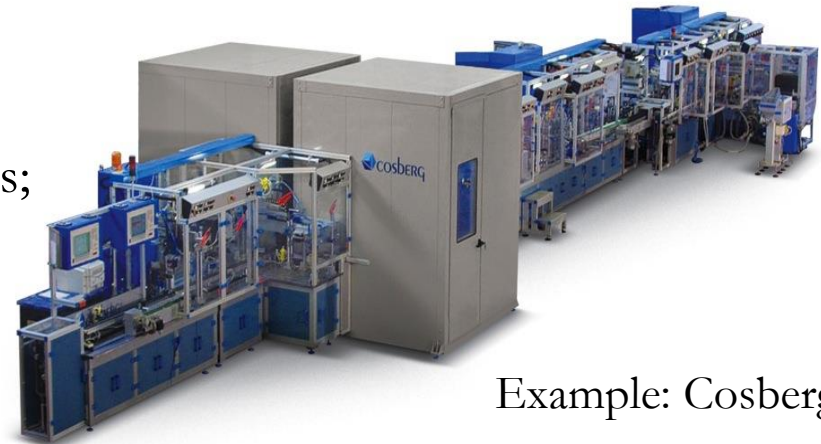
---

**SMMSO 2017**

# Industrial Motivation

## Modern manufacturing systems needs:

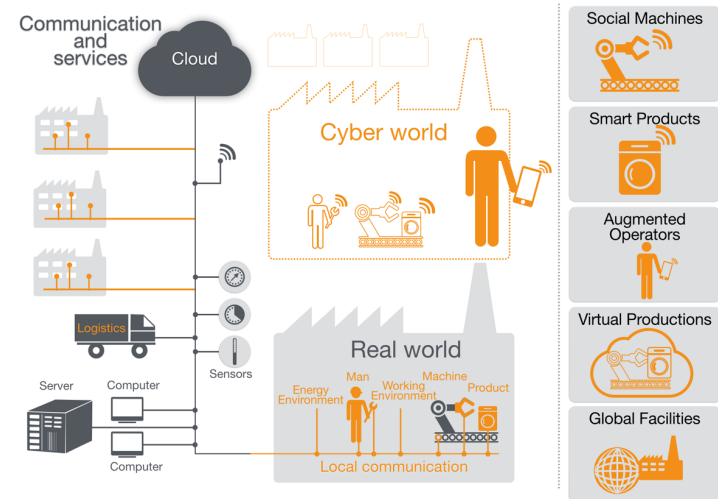
- Flexibility to produce many product variants;
- Small lot productions;
- Make-to-order.



Example: Cosberg.

## Industrie 4.0 technological enablers:

- Data availability;
- Real-time observability of the system state;
- High capabilities to implement real-time scheduling policies.



Can the observability of the system state be exploited for a more effective real-time scheduling of make-to-order, multiple part types systems?

# Production Control Policies

## Surplus-based production control policies (single machine):

- **Kimemia, J., S.B. Gershwin.** 1983. An algorithm for the computer control of a flexible manufacturing system. *AIIE Transactions* 15(4) 353–362.
- **Bielecki, T., Kumar, P.R.** 1988, Optimality of zero-inventory policies for unreliable manufacturing systems, *Operation Research*, 36,4, 532-541.

## Token-based production control policies:

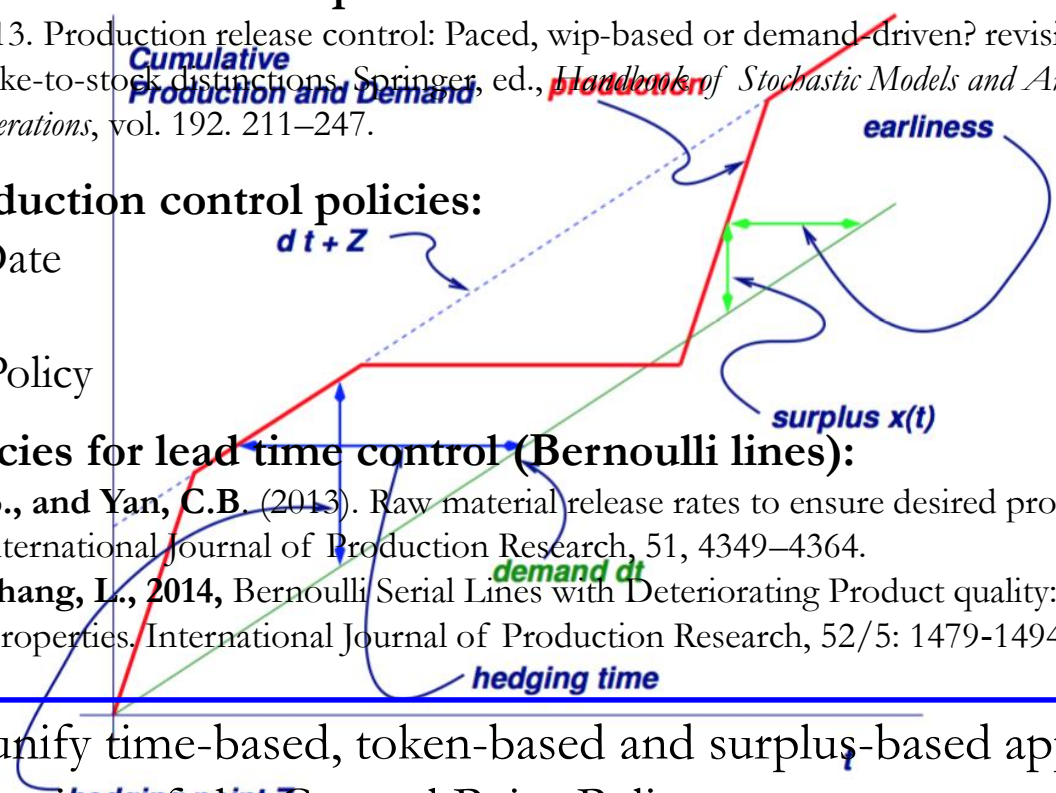
- **Liberopoulos, G.** 2013. Production release control: Paced, wip-based or demand-driven? revisiting the push/pull and make-to-order/ make-to-stock distinctions. Springer, ed., *Handbook of Stochastic Models and Analysis of Manufacturing System Operations*, vol. 192. 211–247.

## Time-based production control policies:

- Earliest Due Date
- Least Slack
- Critical Ratio Policy

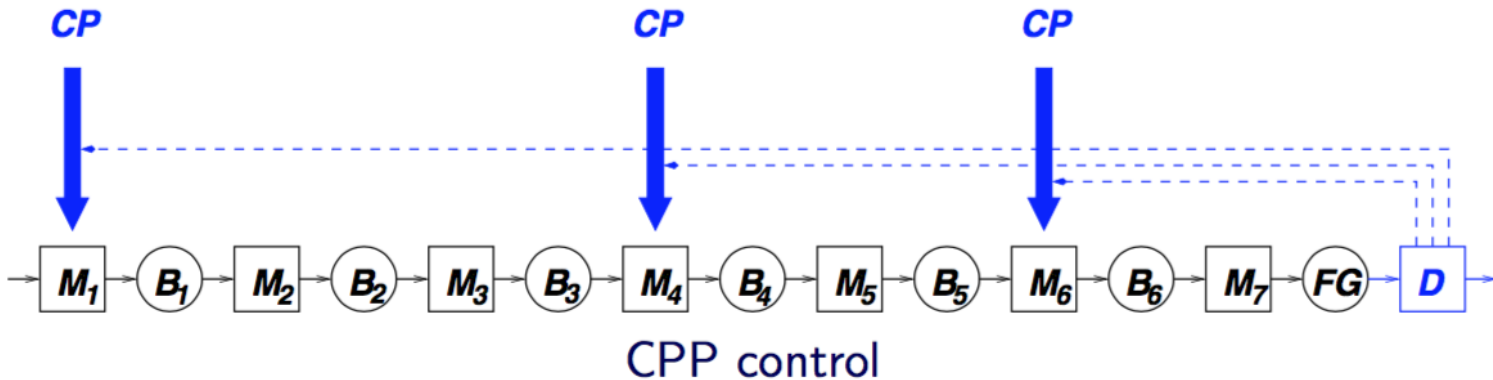
## Part-release policies for lead time control (Bernoulli lines):

- **Biller, S., Meerkov, S., and Yan, C.B.** (2013). Raw material release rates to ensure desired production lead time in bernoulli serial lines. *International Journal of Production Research*, 51, 4349–4364.
- **Naebulharam, R., Zhang, L., 2014**, Bernoulli Serial Lines with Deteriorating Product quality: Performance Evaluation and System-theoretic Properties. *International Journal of Production Research*, 52/5: 1479-1494.



An attempt to unify time-based, token-based and surplus-based approach was done with the introduction of the Control Point Policy [Gershwin, S.B. 2000. Design and operation of manufacturing systems: the control-point policy. *IIE Transactions* 32(10) 891–906].

# The Control Point Policy (CPP) – Time-based version



**Features of the CPP:** where the policy is applied.

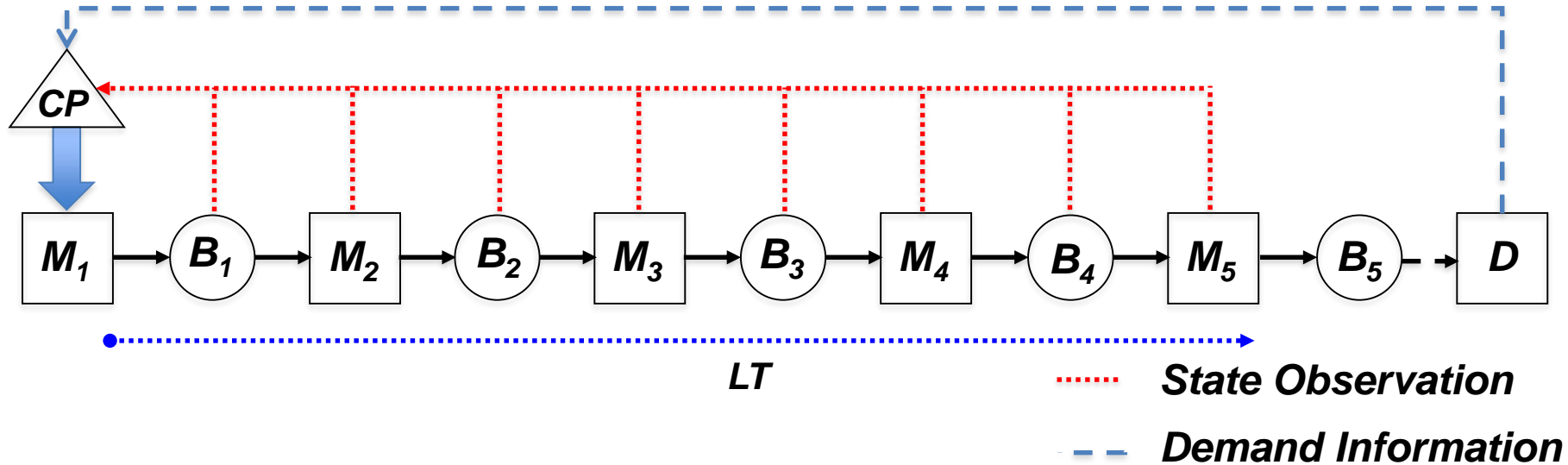
At each control point, whenever the machine becomes available, work on the highest ranking part that is ready. If no parts are ready, wait until a new part arrives or when an available part becomes ready.

- **Hedging time** is a conservative estimate of the lead time between the control point and the finished product buffer. In previous formulations, the hedging time was a constant parameter.
- Multiple part types (different importance).
- A part is **available** at a control point if it is present and not blocked.
- The hedging times need to be optimized by simulation or analytical methods [Vericourt, F., S.B. Gershwin, 2004. Performance evaluation of a make-to-stock production line with a two-parameter-per-machine policy: the control point policy. *IIE Transactions* 36(3) 221–236.]
- A part is **ready** if it is available and:

the current time + the hedging time  $\geq$  the due date

$$t + Z^3 D$$

# Objectives: new dynamic, state-based CPP

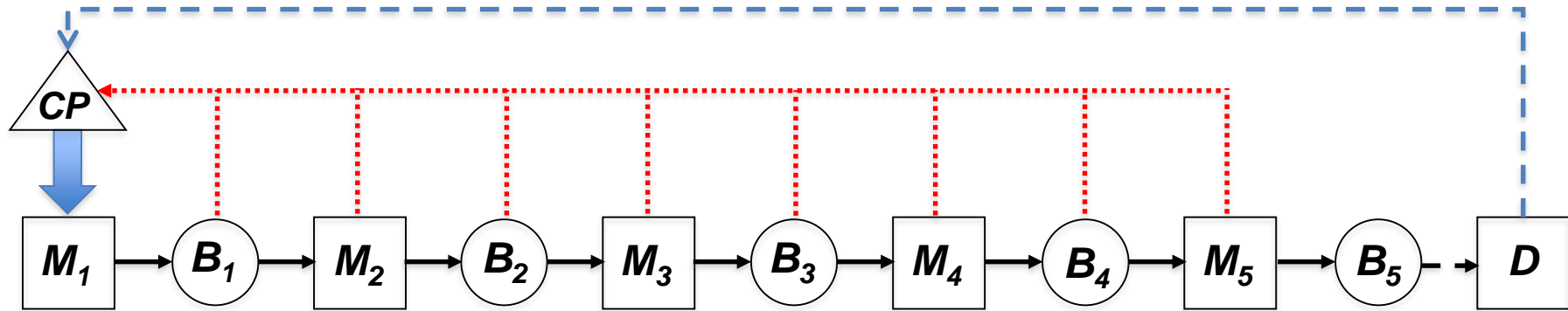


## Innovation sources:

- The hedging times are functions of the state of the system.
  - When the system is poorly populated: smaller hedging times.
  - When the system is highly populated: larger hedging times.
- The hedging times are calculated as target quantiles of the lead time distribution.

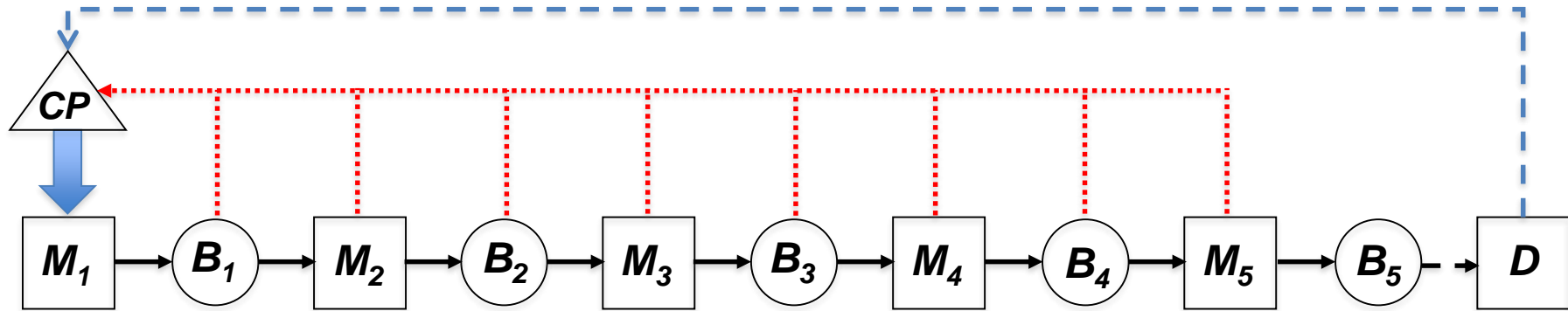
The goal is to meet target service levels while minimizing the inventory

# Modelling Assumptions – System Features



- Linear system with  $K$  unreliable machines and  $K-1$  buffers of finite capacity  $N_k$ .
- An infinite capacity buffer,  $B_K$ , store finished parts with early delivery.
- The flow of material in the system is discrete. Processing times are scaled to the time unit and are identical at different stages.
- The first machine,  $M_1$ , where loading decisions are made, is reliable and its operation time is 1.
- Machine  $M_k$ ,  $k=2, \dots, K$ , fail with probability  $p_k$ , and is repaired, once failed, with probability  $r_k$ .
- The system state can be described by the vector  $s=(n_1, n_2, \dots, n_{K-1}, \alpha_2, \dots, \alpha_K)$ . The set of states is denoted as  $S$ . The generic system state at time unit  $t$  is denoted as  $s_t$ .

# Modelling Assumptions – Part types



- In a finite time horizon, the system is assigned a batch of  $B$  parts to produce. The vector  $Q$  of size  $X$ , represents the number of parts of type  $x=1,..,X$  to be produced. For simplicity, the vector  $R$  of size  $B$  indicates the part type to which part  $b$  belongs.
- The system processes  $X$  part types,  $x=1,..,X$ . The part type 1 has the highest importance, part type 2 the second highest importance, and so on.
- For each part  $b$  in the batch, a due date  $D_b$  is assigned.
- At each time unit, a maximum lot size of 1 part can be loaded to the system.
- Set-up times while switching among parts are set to zero.
- A single control point is considered.

# Modelling Assumptions – Dynamic CPP

At the beginning of each time unit,  $t$ , the CPP defines one of the following actions:

- Load the **ready** part  $b$  of highest priority at  $M_1$  and start the processing.
- If no part is ready or  $n_1 = N_1$ , then do nothing.

The policy acts only on parts that are **ready**. A part  $b$  is said to be ready if:

$$t + Z_{R(b), s_t} \geq D_b \quad \rho(LT(s_t) > Z_{R(b), s_t}) \leq \gamma_{R(b)}$$

where  $LT(s_t)$  is the lead time when the system is in state  $s_t$ , conditioned to the loading of the part.  $\gamma_x$  is a fixed quantile of the lead time distribution.

The dynamic CPP has  $X$  parameters to be determined ( $\gamma_x$ )

The lead time distribution conditioned on the system state needs to be computed.



# Performance Measures

- The total tardiness **TT**, of the batch, calculated as the sum of the delay of parts with late delivery:

$$TT = \sum_{b=1}^B (E_b - D_b) I_b$$

Where  $E_b$  is the time unit at which part  $b$  is released in the finished parts buffer  $B_K$ , and  $I_b$  is an indicator function assuming value 1 if  $E_b > D_b$ .

- The total earliness, **TE**, of the batch, calculated as the sum of the time units parts with early delivery spend in the finished parts buffer,  $B_K$ .

$$TE = \sum_{b=1}^B (D_b - E_b)(1 - I_b)$$

# Performance Measures

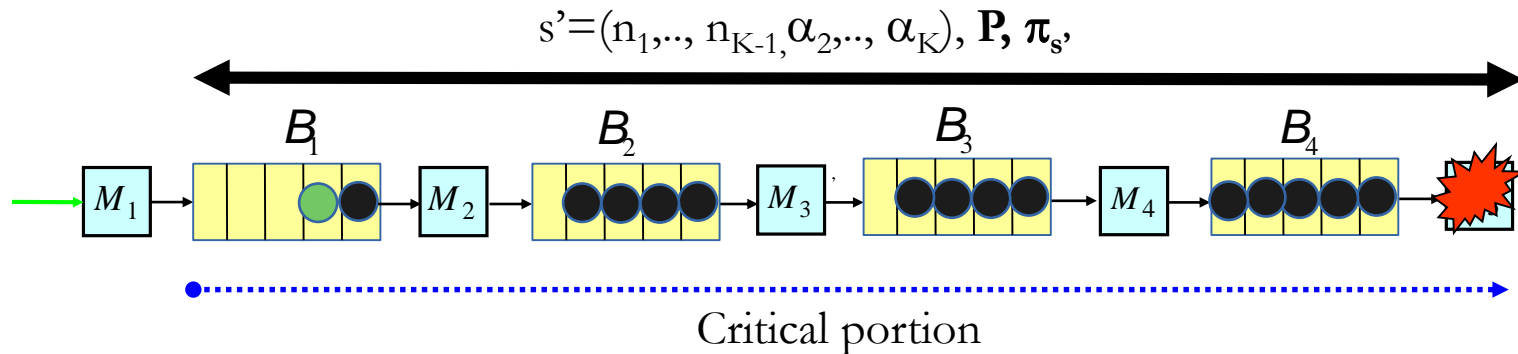
- The service level, **SL**, of the system, i.e. the fraction of parts delivered on time.
- The service level of part type  $x$ , **SL<sub>x</sub>**.
- The throughput, **TH**, calculated as:

$$TH = \frac{\max(E_b)}{B}$$

- The average Total Inventory, **WIP**, considering both the inter-stage inventory and the final parts inventory, used to store parts delivered before the due-date.

# Calculation of the lead time distribution

Colledani M., Gershwin S.-B., Angius A., Horvath A., "Lead Time Dependent Product Deterioration in Manufacturing Systems with Serial, Assembly and Closed-loop Layout", 10th Conference on Stochastic Models of Manufacturing And Service Operations, 1-6 June, 2015, Volos, Greece.



- Denote by  $\pi_{s',n}$  the transient probabilities after  $n$  steps, starting from state  $s'$ :

$$\rho_{s',n} = \rho_{s'} \mathbf{P}^n$$

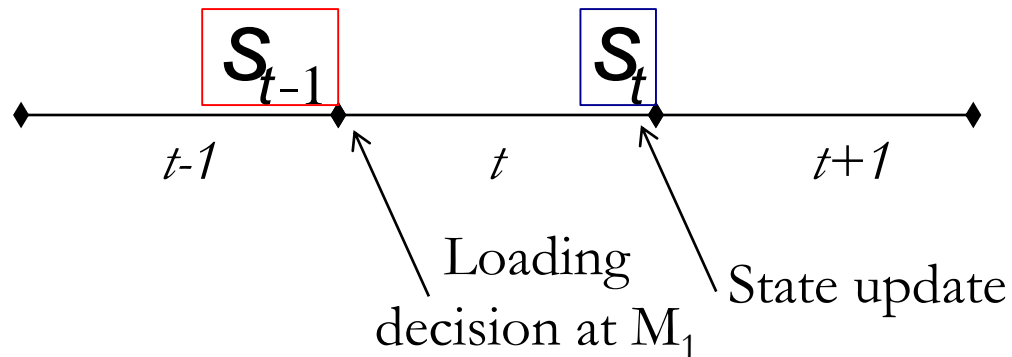
- The lead time distribution is:

$$P(LT(s') \leq n) = \rho_{s',n}(\mathbf{s}_{empty})$$

Where  $\mathbf{s}_{empty}$  is the state in which  $(n_1=0, n_2=0, n_{k-1}=0, \alpha_2=1, \dots, \alpha_K=1)$ .

## Calculation of the hedging times

- Lead time distribution, conditional to the loading of a part at time  $t$ .



$$P(LT(\mathbf{s}_t) \leq n) = \sum_{\mathbf{s}' \in S} \lambda(\mathbf{s}_{t-1}, \mathbf{s}') P(LT(\mathbf{s}') \leq n)$$

Where  $\lambda(\mathbf{s}_{t-1}, \mathbf{s}')$  is the probability of making a transition in one time unit from state  $\mathbf{s}_{t-1}$  to state  $\mathbf{s}'$  by loading a part at  $M_1$ .

- The hedging times  $Z_{x, \mathbf{s}_t}$  for each part type can be computed as:

$$P(LT(\mathbf{s}_t) > Z_{x, \mathbf{s}_t}) \leq g_x$$

# Dynamic CPP algorithm

Real time execution phase (at the generic time time unit  $t > 0$ ):  
Preparation phase:

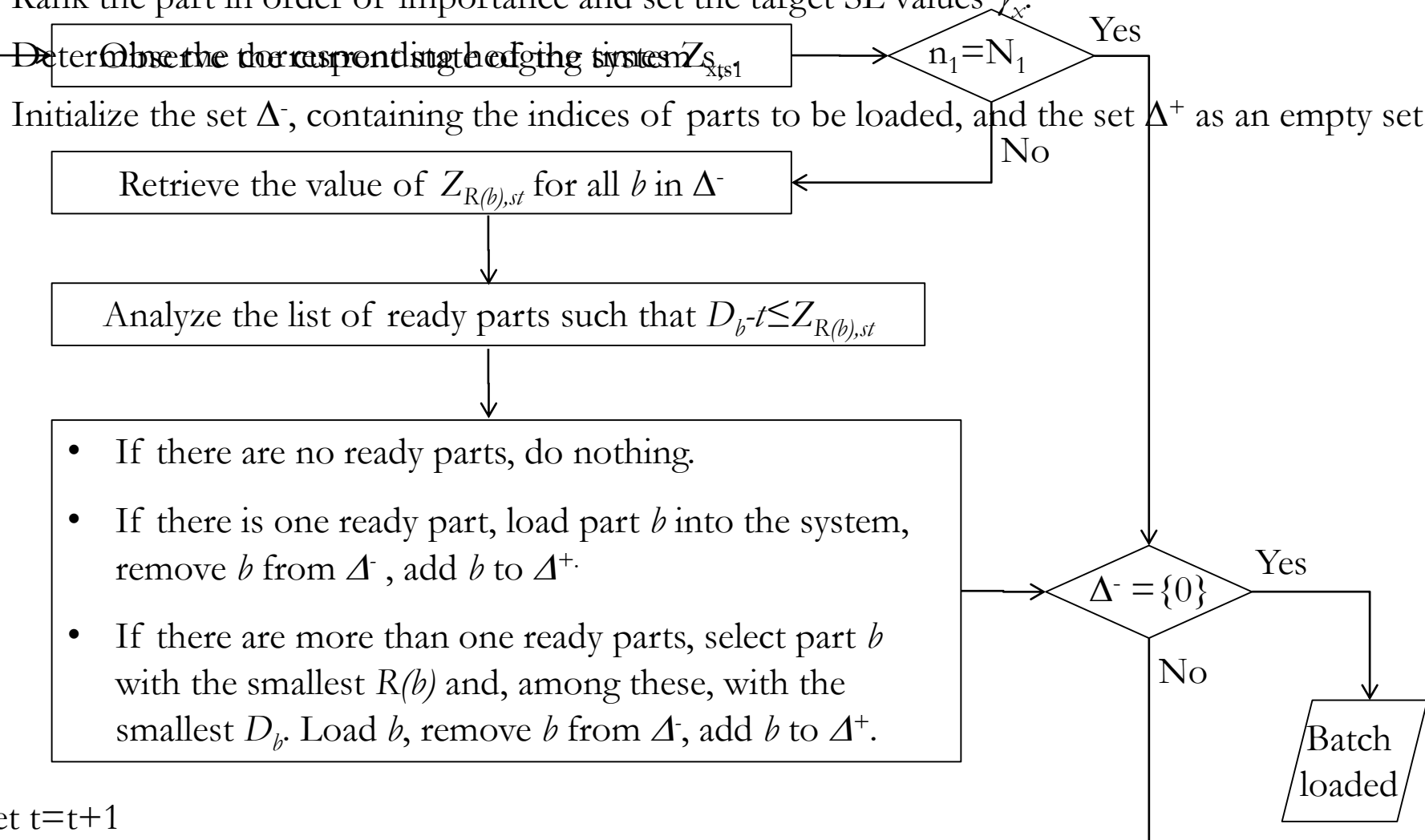
- Rank the part in order of importance and set the target SL values  $\gamma_x$ .
- Determine the corresponding state of the system  $Z_{s_{ts1}}$ .
- Initialize the set  $\Delta^-$ , containing the indices of parts to be loaded, and the set  $\Delta^+$  as an empty set.

Retrieve the value of  $Z_{R(b),st}$  for all  $b$  in  $\Delta^-$

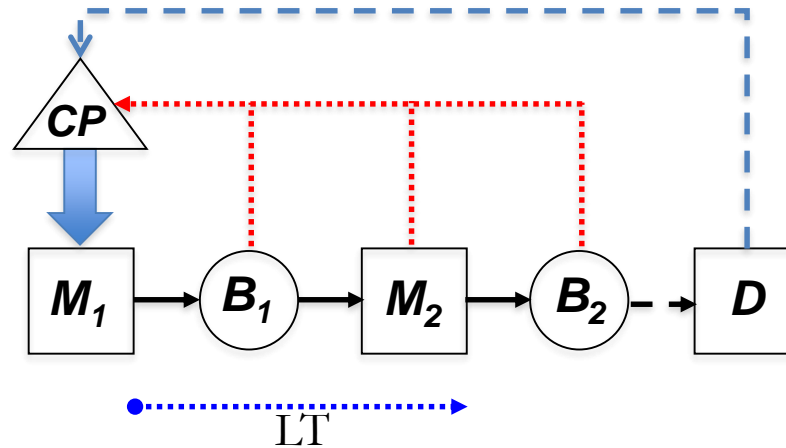
Analyze the list of ready parts such that  $D_b - t \leq Z_{R(b),st}$

- If there are no ready parts, do nothing.
- If there is one ready part, load part  $b$  into the system, remove  $b$  from  $\Delta^-$ , add  $b$  to  $\Delta^+$ .
- If there are more than one ready parts, select part  $b$  with the smallest  $R(b)$  and, among these, with the smallest  $D_b$ . Load  $b$ , remove  $b$  from  $\Delta^-$ , add  $b$  to  $\Delta^+$ .

Set  $t = t + 1$



# Numerical Results



The **D-CPP** policy is compared with the following policies:

- **EDDP** (Earliest Due Date Policy), neglecting the part type importance.
- **EDDP-IR** (with Importance Ranking): Applies EDDP to all parts of type 1, then 2, and so on.

## Questions:

- How does the D-CPP behave if compared to the other policies?
- How does the SL behaves with respect to the assigned target value  $\gamma_x$ ?
- How does the system saturation level affects the performance of the policy?

# Numerical Results – single part type

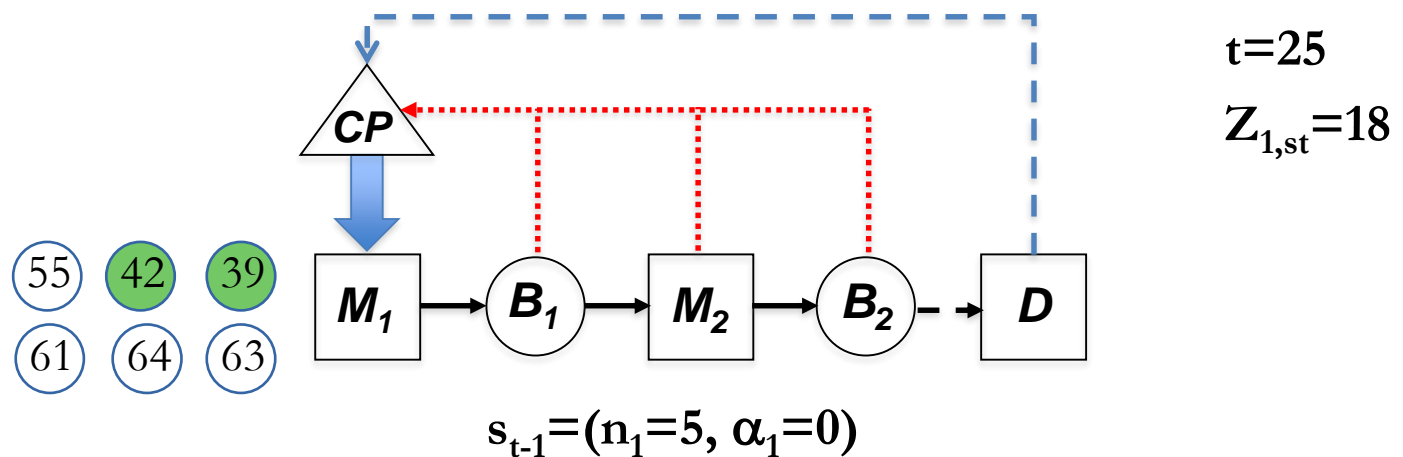
- $B=200$ ,  $\gamma=0.95$ ,  $l=d/E$ , where  $E$  is the throughput of the saturated system,  $l$  is the saturation level, and  $d$  is the demand rate (Poisson process).

ID	$p_2$	$r_2$	$e_2$	$l$	Policy	$SL$	$WIP$	$TE$	$TT$	$TH$
1	0.001	0.01	0.909	0.3	D-CPP	0.971 ± 0.005	0.64 ± 0.05	86 ± 1	676 ± 25	0.26
					EDDP	0.994 ± 0.002	72.59 ± 0.5	56880 ± 370	79 ± 5	0.9
2	0.001	0.01	0.909	0.8	D-CPP	0.93 ± 0.01	1.87 ± 0.11	178 ± 1	1407 ± 50	0.67
					EDDP	0.929 ± 0.011	25.66 ± 0.33	7180 ± 124	1290 ± 34	0.9
3	0.001	0.02	0.952	0.3	D-CPP	0.985 ± 0.003	0.561 ± 0.03	111 ± 1	157 ± 5	0.28
					EDDP	0.997 ± 0.002	69.29 ± 0.38	49724 ± 225	12 ± 1	0.95
4	0.001	0.02	0.952	0.8	D-CPP	0.93 ± 0.011	2.02 ± 0.09	167 ± 2	745 ± 20	0.76
					EDDP	0.931 ± 0.012	19.96 ± 0.21	4856 ± 88	689 ± 17	0.95
5	0.0001	0.002	0.952	0.3	D-CPP	0.989 ± 0.005	0.559 ± 0.06	112 ± 1	1147 ± 130	0.28
					EDDP	0.995 ± 0.003	67.13 ± 0.3	50953 ± 270	383 ± 24	0.95
6	0.0001	0.002	0.952	0.8	D-CPP	0.982 ± 0.005	1.6 ± 0.06	177 ± 1	1035 ± 69	0.76
					EDDP	0.983 ± 0.004	20.97 ± 0.1	5339 ± 37	2001 ± 52	0.95

- The D-CPP outperforms the EDDP in terms of WIP (savings in the range **-89%,-99%**).
- For low saturation levels ( $l=0.3$ ): the target service level is met.
- For high saturation levels ( $l=0.8$ ): the service level of the D-CPP drops below the target level but remains comparable to the EDDP. Why?

# Numerical Results – single part type

- For low saturation levels ( $l=0.3$ ): the target service level is met.
- For high saturation levels ( $l=0.8$ ): the service level of the D-CPP drops below the target level but remains comparable to the EDDP. Why?



For higher saturation level, the probability of having more than one ready parts increases, thus reducing the service level of the D-CPP policy.



# Numerical Results – single part type

- $B=200$ ,  $\gamma=0.95$ ,  $l=d/E$ , where  $E$  is the throughput of the saturated system,  $l$  is the saturation level, and  $d$  is the demand rate.

ID	$p_2$	$r_2$	$e_2$	$l$	Policy	$SL$	$WIP$	$TE$	$TT$	$TH$
1	0.001	0.01	0.909	0.3	D-CPP	$0.971 \pm 0.005$	$0.64 \pm 0.05$	$86 \pm 1$	$676 \pm 25$	0.26
					EDDP	$0.994 \pm 0.002$	$72.59 \pm 0.5$	$56880 \pm 370$	$79 \pm 5$	0.9
2	0.001	0.01	0.909	0.8	D-CPP	$0.93 \pm 0.01$	$1.87 \pm 0.11$	$178 \pm 1$	$1407 \pm 50$	0.67
					EDDP	$0.929 \pm 0.011$	$25.66 \pm 0.33$	$7180 \pm 124$	$1290 \pm 34$	0.9
3	0.001	0.02	0.952	0.3	D-CPP	$0.985 \pm 0.003$	$0.561 \pm 0.03$	$111 \pm 1$	$157 \pm 5$	0.28
					EDDP	$0.997 \pm 0.002$	$69.29 \pm 0.38$	$49724 \pm 225$	$12 \pm 1$	0.95
4	0.001	0.02	0.952	0.8	D-CPP	$0.93 \pm 0.011$	$2.02 \pm 0.09$	$167 \pm 2$	$745 \pm 20$	0.76
					EDDP	$0.931 \pm 0.012$	$19.96 \pm 0.21$	$4856 \pm 88$	$689 \pm 17$	0.95
5	0.0001	0.002	0.952	0.3	D-CPP	$0.989 \pm 0.005$	$0.559 \pm 0.06$	$112 \pm 1$	$1147 \pm 130$	0.28
					EDDP	$0.995 \pm 0.003$	$67.13 \pm 0.3$	$50953 \pm 270$	$383 \pm 24$	0.95
6	0.0001	0.002	0.952	0.8	D-CPP	$0.982 \pm 0.005$	$1.6 \pm 0.06$	$177 \pm 1$	$1035 \pm 69$	0.76
					EDDP	$0.983 \pm 0.004$	$20.97 \pm 0.1$	$5339 \pm 37$	$2001 \pm 52$	0.95

- The D-CPP outperforms the EDDP in terms of WIP (savings in the range **-89%,-99%**).
- For low saturation levels ( $l=0.3$ ): the target service level is met. For high saturation levels ( $l=0.8$ ): the service level of the D-CPP drops below the target level but remains comparable to the EDDP.
- The D-CPP behaves better for higher variability cases.

# Numerical Results – multiple part types

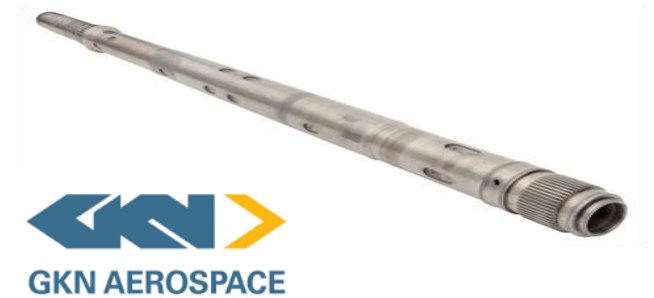
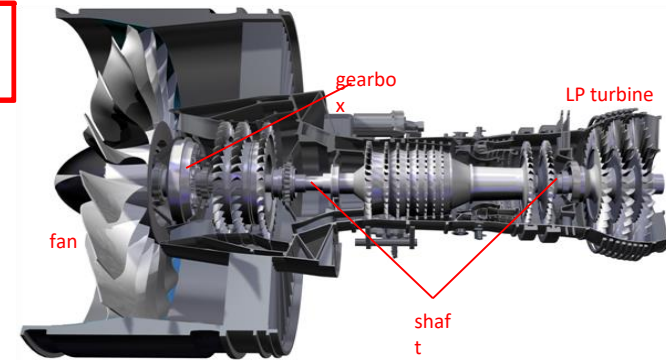
- $p=0.001, r=0.01, \gamma_x=0.95$  for each part type.

ID	$X$	$B$	$Q$	$l$	Policy	$SL_1$	$SL_2$	$SL_3$	WIP
1	3	1000	[100, 600, 300]	0.7	D-CPP	0.977 ± 0.012	0.932 ± 0.013	0.457 ± 0.007	1.54 ± 0.13
					EDDP	0.97 ± 0.013	0.968 ± 0.013	0.968 ± 0.013	124.97 ± 2.28
					EDDP-IR	0.995 ± 0.001	0.835 ± 0.012	0.32 ± 0.008	196.45 ± 3.35
2	3	1000	[100, 600, 300]	0.8	D-CPP	0.975 ± 0.009	0.921 ± 0.009	0.224 ± 0.002	1.96 ± 0.12
					EDDP	0.925 ± 0.017	0.925 ± 0.015	0.924 ± 0.015	77.86 ± 2.31
					EDDP-IR	0.995 ± 0.001	0.773 ± 0.014	0.135 ± 0.003	170 ± 3
3	3	1000	[100, 600, 300]	0.9	D-CPP	0.958 ± 0.014	0.918 ± 0.014	0.147 ± 0.003	2.35 ± 0.17
					EDDP	0.915 ± 0.017	0.891 ± 0.016	0.862 ± 0.015	28.029 ± 0.67
					EDDP-IR	0.994 ± 0.002	0.718 ± 0.013	0.114 ± 0.004	125.09 ± 2.79
4	5	1000	[50, 200, 600, 50, 100]	0.9	D-CPP	0.947 ± 0.017	0.921 ± 0.017	0.453 ± 0.005	2.42 ± 0.18
					EDDP	0.927 ± 0.018	0.884 ± 0.017	0.898 ± 0.016	52.45 ± 01.2
					EDDP-IR	0.997 ± 0.001	0.885 ± 0.009	0.189 ± 0.005	161 ± 2.6

- The proposed D-CPP always provides the target Service Level for the highest priority part, also for high saturation cases ( $l=0.9$ ).
- D-CPP always performs better than the EDDP and worse than the EDDP-IR in terms of part type 1 Service Level.
- However, the D-CPP always performs better than the EDDP-IR for lower priority parts.
- The D-CPP outperforms both policies in terms of WIP (savings in the range **-92%,-99%**).

# Dynamic CPP – application scenario

Production of shafts in the aeronautics sector.



## Problem:

- Multiple shaft types, very expensive parts.
- Several machining-measurement-rework loops cause stochastic processing times and routing.
- Long lead times, large inventory and operational costs.

# Conclusions

## Major findings:

- Formulation of a new dynamic, state-based Control Point Policy.
- Derivation of the lead time distribution conditional to the loading policy.
- Analysis of the impact of the proposed policy on the system performance in simple cases.

## Future Research:

- Wider experimental analysis to prove the benefits of the policy.
- Extension to multiple distributed Control Points.
- Extension to non-linear systems and systems with part type dependent routings.
- Analytical derivation of the system behavior operating under the proposed state-dependent Control Point Policy.

---

# Dynamic Lead Time Based Control Point Policy for Multi Stage Manufacturing Systems

**Marcello Colledani<sup>(1)</sup>, Stanley B. Gershwin<sup>(2)</sup>**

---

11<sup>th</sup> Conference on Stochastic Models of Manufacturing and  
Service Operations, SMMSO2017, Acaya, Lecce, June 5<sup>th</sup>.

(1) Department of Mechanical Engineering  
Politecnico di Milano

(2) Laboratory for Manufacturing and Productivity  
Massachusetts Institute of Technology

---

**SMMSO 2017**