Closed Finite Queueing Network Models with General Service Time Distributions

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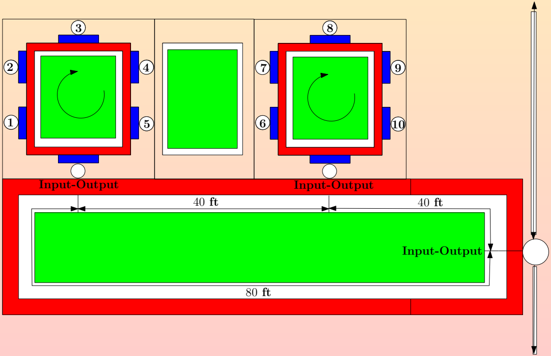
Outline of Lecture Topics

- *A.* **Motivation**
- *B.* **Background**
- *C.* **Literature Review**
- *D.* **Mathematical Model**
- *E.* **Performance Algorithm**
- *F.* **Experimental Results**
- *G.* **Summary & Conclusions**



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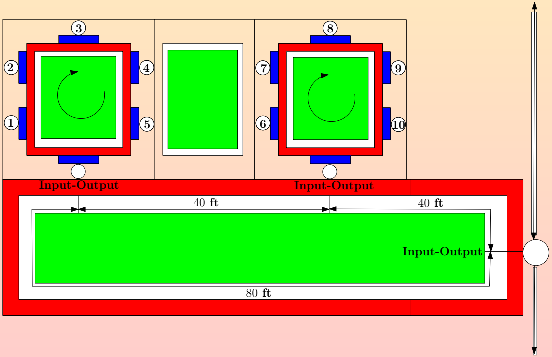

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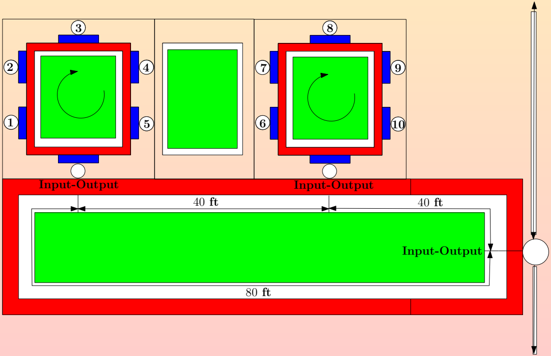

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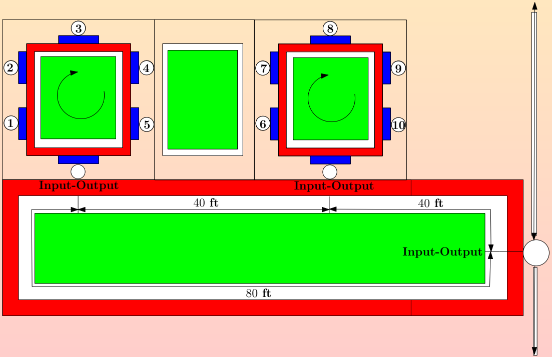

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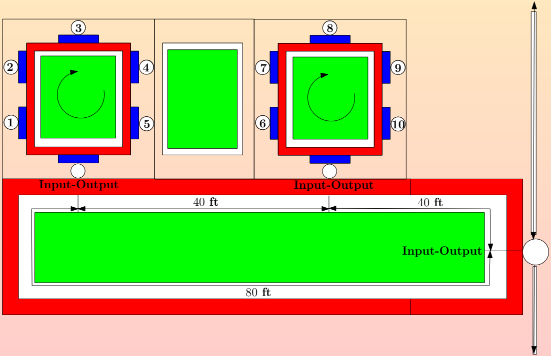

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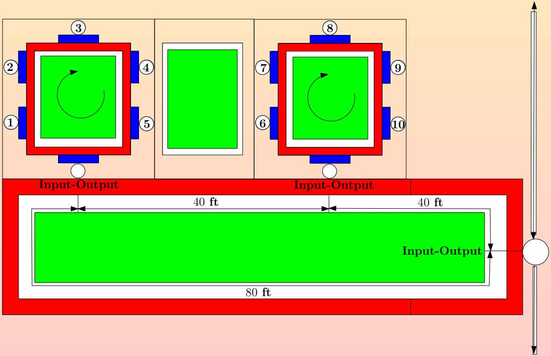

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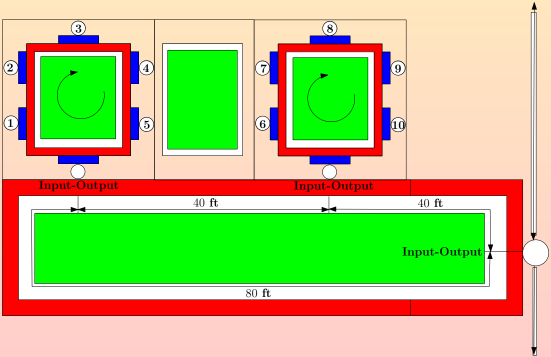

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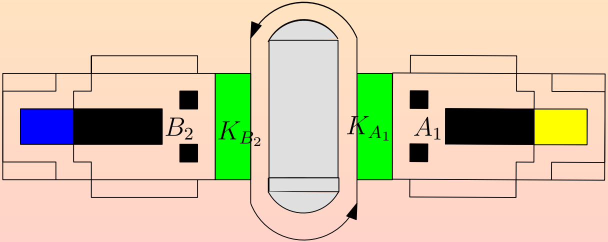
- Assumptions
  - Unpaced, asynchronous Flow line or FMS
  - Finite Buffers & Production Blocking
  - Closed Network Models, finite population, single-servers
- **Approximate Mean Value Analysis (MVA) Model**
  - Two-Moment General Service Model
  - Two-Moment Blocking Probability
- **Integrated Material Handling System**



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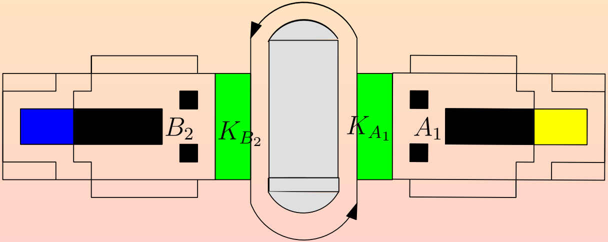

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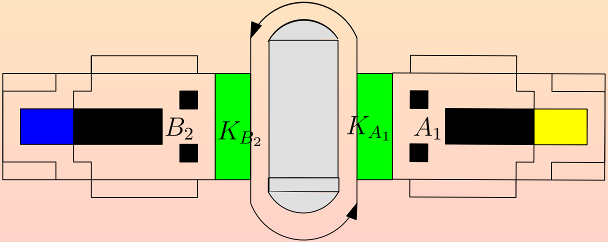

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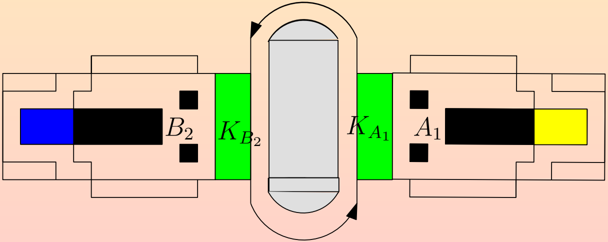

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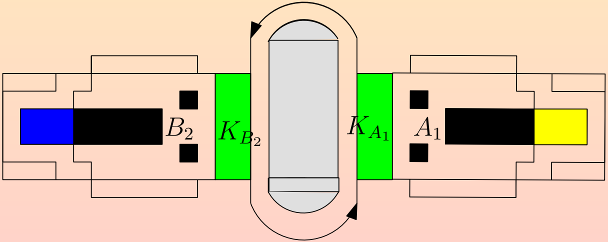

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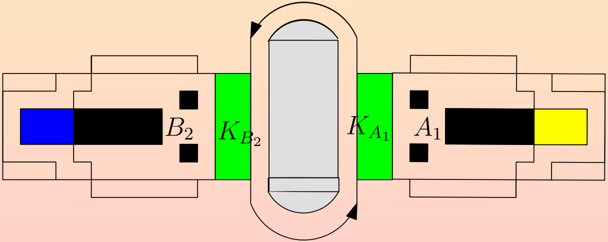

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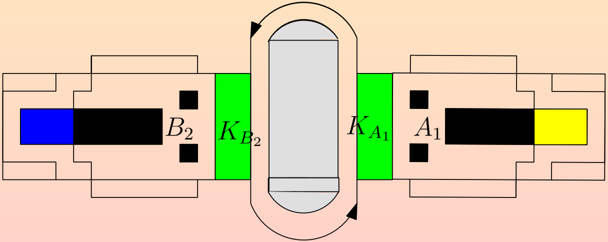

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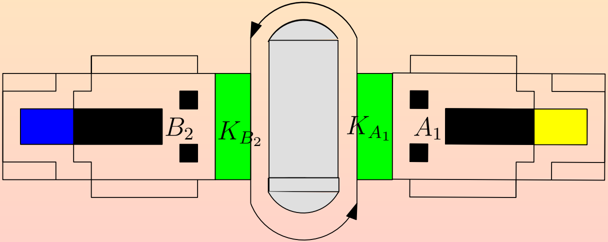

Literature Review

- *CQN*
  - **Infinite Buffers**
    - Diffusion Approximation
      - Gaver and Shdler, 1973
      - Gelenbe, 1975
      - Tijms, 1992
      - Kimura, 1996
    - Marie's Method
      - Marie, 1979
    - Two-moment Approximations
      - Buzacott et.al., 1993
      - Curry and Feldman, 2008
  - **Finite Buffers**
    - Marie's Method
      - Akyildiz, 1988
      - Dayar and Meri, 2008
    - Linear Programming
      - Matta and Chefson, 2005
      - Helber, et. al., 2011
    - Open Network Approximation
      - Bouhchouch, 1996
      - Lagershausen, et.al., 2013

Closed General Finite Queueing Methodology

Basic Issues:

- **Developing a closed network approximation for generally distributed finite blocking processes?**
- **Accounting for blocking from General distributions?**
- **Incorporating General probability distribution service information (moments and variability)?**
- **Creating an efficient running time algorithm?**

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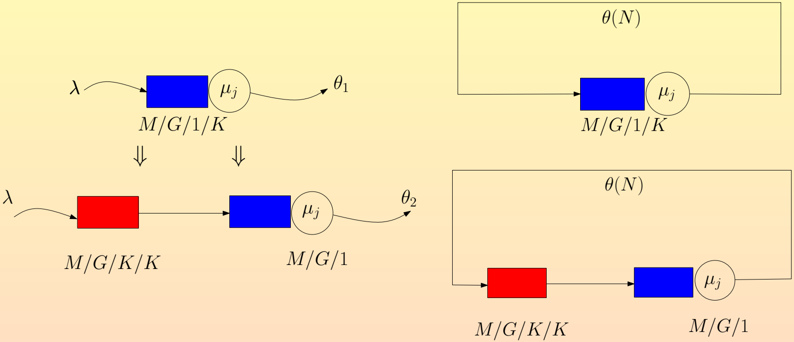
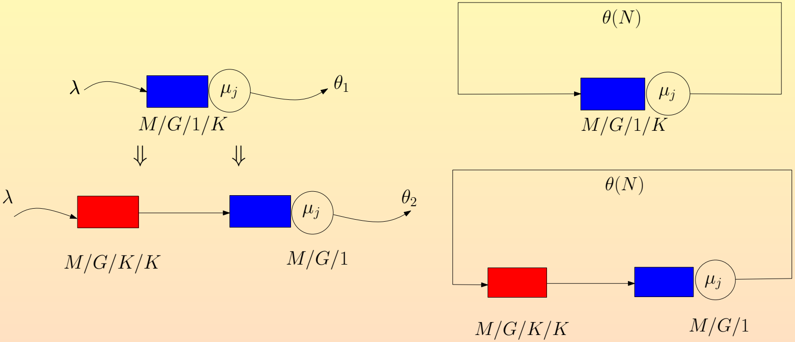
Queue Decomposition Mathematical Models



- Underlying logic behind Queue Decomposition idea:
  - $M/G/K/K$ queue acts as a holding node for the parts.
  - As the population increases, the congestion (blocking) increases as a function of the # of parts within the system.
  - Effective service rates decay as a function of the blocking in the system.

Queue Decomposition Mathematical Models



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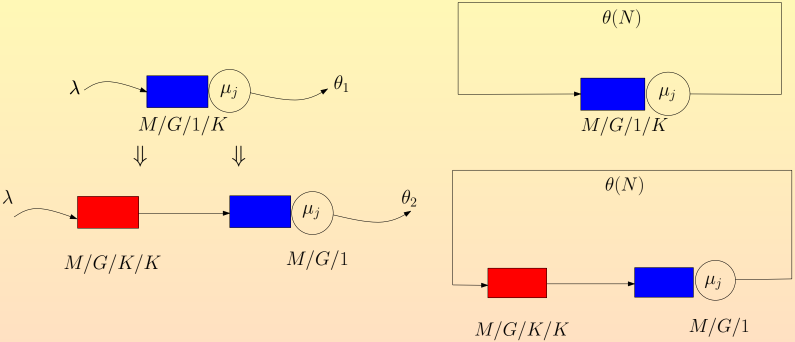

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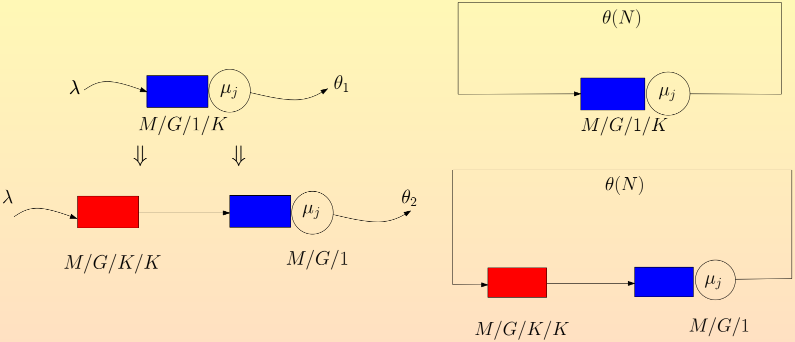

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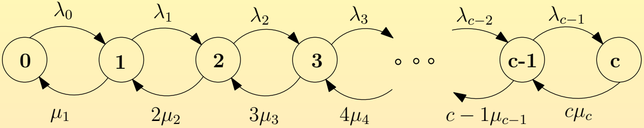
M/G/c/c State Dependent Probability Distribution



- For the $M/G/c/c$ probability distribution, we have for the idle time probability:

$$p_0 = \left(1 + \sum_{n=1}^{C} \lambda^n \left(\prod_{j=1}^{n} j V_1 \, e^{-\left(\frac{j-1}{\beta}\right)^g} L^{-1}\right)^{-1}\right)^{-1} \qquad (1)$$

- Finally, for the rest of the distribution:

$$p_n = n \mapsto \lambda^n \left(\prod_{j=1}^{n} j V_1 \, e^{-\left(\frac{j-1}{\beta}\right)^g} L^{-1}\right)^{-1} \left(1 + \sum_{n=1}^{C} \lambda^n \left(\prod_{j=1}^{n} j V_1 \, e^{-\left(\frac{j-1}{\beta}\right)^g} L^{-1}\right)^{-1}\right)^{-1} \qquad (2)$$

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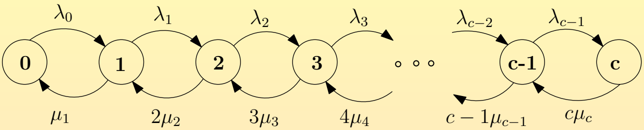

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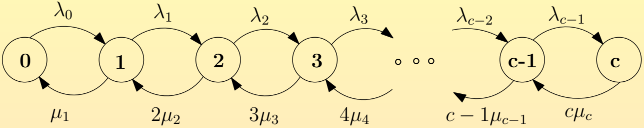

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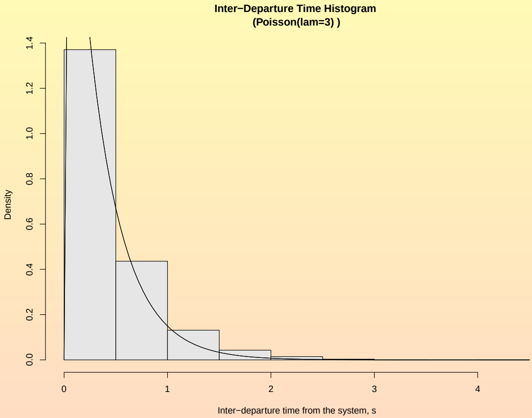
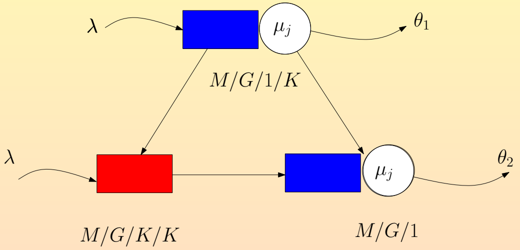
Quasi-Reversibility Property



Proposition (Quasi-Reversibility Cheah and Smith, 94)

In the M/G/c/c state dependent model, the departure process (including both customers completing service and those that are lost) is a Poisson process at rate λ.

Erlang Loss Flow Process Decomposition Property



Proposition (Upper Bound on Decomposition Throughput)

As $\mu_n \to \infty$, $\theta(N) \leq \theta(\infty)$ i.e. the throughput of the queue decomposition is bounded above by the throughput of an infinite capacity system.

Blocking Probability (Two moment estimation)

If one fixes the number of servers, one can solve for the blocking probability of the $M/M/1/K$ system.

$$\mathbf{p_K} = \frac{(\mathbf{1}-\rho)\rho^{\mathbf{K}}}{\mathbf{1}-\rho^{\mathbf{K+1}}} \Rightarrow \mathbf{K} = \left\lceil \frac{\ln(p_K/(1-\rho+p_K\rho))}{\ln(\rho)} \right\rceil \tag{3}$$

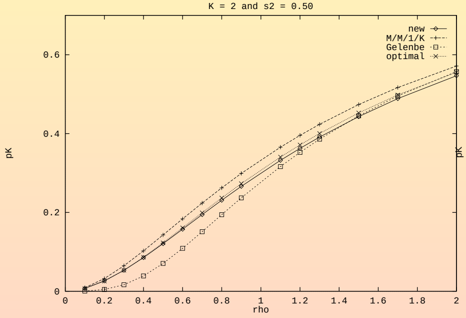
$$B = \frac{\left(\ln\left(\frac{p_K}{1-\rho+p_K\rho}\right) - \ln(\rho)\right)\left(2 + \sqrt{\frac{\rho}{e^{s^2}}}s^2 - \sqrt{\frac{\rho}{e^{s^2}}}\right)}{2\ln(\rho)} \tag{4}$$

In the case of $c = 1$, the following expression is obtained for the blocking probability:

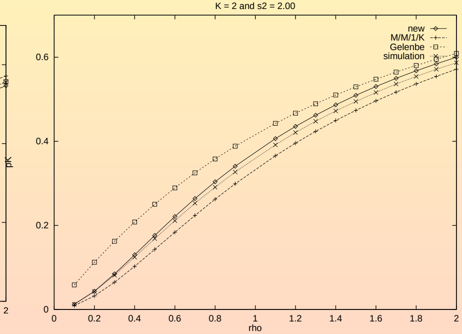
$$p_K = \frac{\rho^{\frac{\sqrt{\rho}s^2 - \sqrt{\rho} + 2K}{2+\sqrt{\rho}s^2-\sqrt{\rho}}}\left(\rho - 1\right)}{\left(\rho^{2\frac{1+\sqrt{\rho}s^2-\sqrt{\rho}+K}{2+\sqrt{\rho}s^2-\sqrt{\rho}}} - 1\right)} \tag{5}$$

Blocking Probability (Two moment estimation)

P_K Comparisons M/G/1/2 $s^2 = \frac{1}{2}$



P_K Comparisons M/G/1/2 $s^2 = 2$



General Service Time Approximation

The standard Equation 6 in the MVA for the expected delay time at a queue is based upon the PASTA property that

$$w_\ell(N) = \tau_\ell[1 + n_\ell(N-1)] \tag{6}$$

Accounting for the remaining service time which is a function of the utilization of the queue, the full service time of the number of customers in the queue, and **the full service time of the arriving customer**:

$$w_\ell(N) = \rho_\ell(N-1)\frac{\tau_\ell(1+s^2)}{2} + (n_\ell(N-1) - \rho_\ell(N-1))\tau_\ell + \tau_\ell \tag{7}$$

Mean Value Analysis (MVA) Algorithm

- Reiser and Lavenberg's modified property of product-form networks to estimate the delay or residence time at the queue:

$$w_\ell(N) = \rho_\ell(N-1)\frac{\tau_\ell(1+s^2)}{2} + (n_\ell(N-1) - \rho_\ell(N-1))\tau_\ell + \tau_\ell \tag{8}$$

- Little's equation for product chains:

$$\lambda_\ell(N) = \frac{N}{[\sum_{\ell=1}^{m} w_\ell(N)\alpha_\ell]} \tag{9}$$

- Little's equation for queues:

$$n_\ell(N) = \lambda_\ell(N) w_\ell(N) \tag{10}$$

V_1 Lower Bound

$$\rho_\ell = \frac{n_\ell}{K_\ell} \approx \rho_\ell = \frac{\theta_\ell}{\mu_\ell}$$

- [Step 1.0:] Solve for θ^U in the infinite buffer network with a given squared coefficient of variation s^2.
- [Step 2.0:] Find the bottleneck queues ℓ^β in this topology with maximum $\rho^\beta = \frac{\theta_\ell}{\mu_\ell}$.
- [Step 3.0:] Set up a finite buffer network with N, K_ℓ, s_ℓ^2 and find the total utilization rate across the network:
  - [Step 3.1:] $\rho_T = \sum_{\ell=1}^{M} \rho_\ell$.
  - [Step 3.2:] Set the lower bound velocity to:

$$V_1^\ell = V_1\left(\frac{1-\rho_\ell}{\rho_T}\right)$$

- This lower bound value will be useful in the general algorithm.

$\mathbf{V_1}$ Service Rate Adjustment

$$V_1 = V_1 \exp^{-\rho * s^2}$$

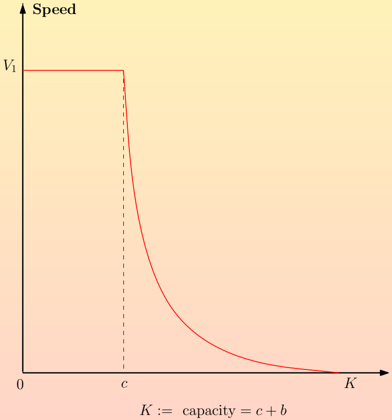


Figure: Shifted Exponential Distribution

Throughput Buffer Envelope

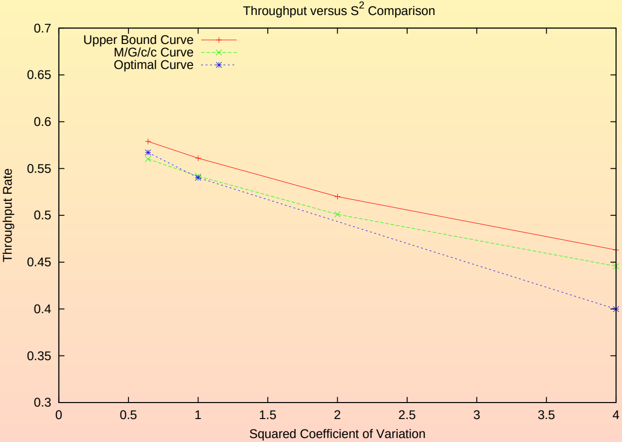
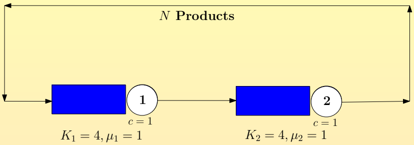
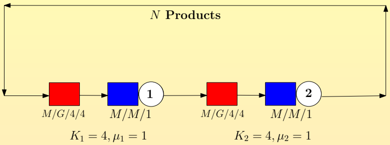


Figure: Upper and Lower Bounds θ vs. s^2

Two-Stage Series Experiments

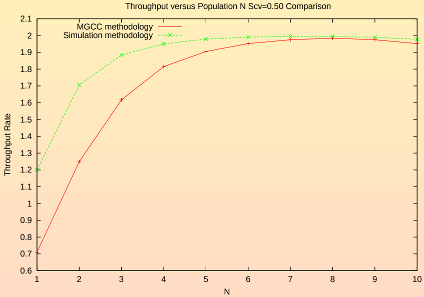




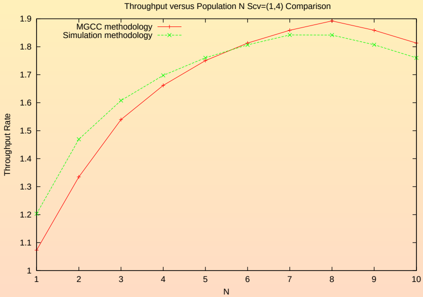
s_1^2	s_2^2	$\theta(10)_e$ [a]	$\theta(10)_s$ [b]	$\theta(10)_a$ [c]	$\theta(10)_m$ [d]	% dev.	$\theta(10)_l$ [e]
0.50	0.50	1.9790	1.9780	1.9761	1.9522	1.35	1.892
0.50	1.00	1.9469	1.9460	1.9469	1.9281	0.50	1.838
0.50	4.00	1.7769	1.7916	1.7992	1.7897	0.72	1.572
0.50	9.00	1.6632	1.6609	1.6906	1.7231	3.60	1.318
1.00	0.50	1.9402	1.9388	1.9402	1.8593	4.17	1.836
1.00	1.00	1.9038	1.9026	1.9038	1.8712	1.71	1.780
1.00	4.00	1.7510	1.7602	1.7510	1.8113	3.44	1.528
1.00	9.00	1.6497	1.6409	1.6497	1.6700	1.23	1.292

[a] Exact method
[b] Simulation with Arena
[c] Akyildiz's method
[d] $M/G/c/c$ method
[e] Lagershausen's et.al. open network model

Throughput Curves $s^2 = \{0.50, 0.50\}; s^2 = \{1.00, 4.00\}$

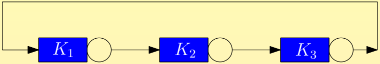


$s^2 = \{0.50, 0.50\}$



$s^2 = \{1.00, 4.00\}$

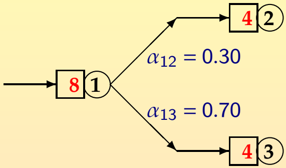
Three-Stage Experiments



s^2	K_i	N^*	$\theta(N)_e$ [a]	$\theta(N)_s$ [b]	$\theta(N)_m$ [c]	% dev.
1.00	2	4	0.4393	0.4390	0.4552	2.94
1.00	3	6	0.4844	0.4842	0.5064	4.54
1.00	4	7	0.5116	0.5115	0.5234	2.31
1.00	5	9	0.5322	0.5321	0.5421	1.86
1.00	6	10	0.5463	0.5465	0.5403	1.10
0.64	2	4	0.4795	0.4782	0.4823	0.58
0.64	3	6	0.5233	0.5228	0.5326	1.78
0.64	4	7	0.5467	0.5463	0.5481	0.26
0.64	5	9	0.5629	0.5627	0.5500	2.20
0.64	6	11	0.5730	0.5728	0.5771	0.72
4.00	2	4	0.3502	0.3442	0.3209	8.37
4.00	3	6	0.3789	0.3763	0.3548	6.36
4.00	4	7	0.4000	0.4001	0.3950	1.25
4.00	5	9	0.4184	0.4207	0.4256	1.72
4.00	6	10	0.4335	0.4379	0.4384	1.13

[a] CTMC Markov Process method
[b] Simulation
[c] $M/G/c/c$ method

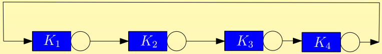
Three-Stage Split Experiments



s_1^2	s_2^2	s_3^2	$\theta(N)_s$ [a]	$\theta(N)_m$ [b]	% dev.
0.50	1	1	1.8751	1.8461	1.55
0.50	1	2	1.8089	1.8342	1.40
0.50	1	4	1.7256	1.8239	5.70
0.50	1	9	1.6022	1.6735	4.45
1	1	1	1.8363	1.8073	1.58
1	1	2	1.7743	1.7882	0.78
1	1	4	1.6994	1.7962	5.70
1	1	9	1.5815	1.6406	3.74

[a] Simulation
[b] $M/G/c/c$ method

Four Stage Tandem Experiments

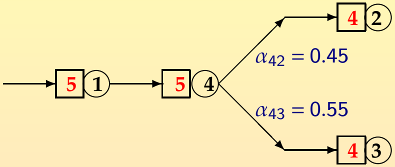


s^2	K_i	N^*	$\theta(N)_e$ [a]	$\theta(N)_s$ [b]	$\theta(N)_m$ [c]	% dev.
1.00	2	6	0.4166	.4206	0.4561	9.48
1.00	3	8	0.4652	.4653	0.4956	6.53
1.00	4	10	0.4971	.4959	0.5178	4.16
1.00	5	12	0.5196	.5174	0.5354	3.04
1.00	6	14	0.5361	.5339	0.5464	1.92
0.64	2	6	0.4617	.4652	0.4801	3.99
0.64	3	8	0.5085	.5096	0.5153	1.34
0.64	4	10	0.5365	.5369	0.5390	0.47
0.64	5	12	0.5547	.5548	0.5515	0.58
0.64	6	14	0.5671	.5672	0.5680	0.16
4.00	2	5	0.3092	.3078	0.3017	2.43
4.00	3	7	0.3408	.3423	0.3397	0.32
4.00	4	9	0.3659	.3692	0.3853	5.30
4.00	5	11	0.3865	.3914	0.4107	6.26
4.00	6	13	0.4042	.4106	0.4316	6.78

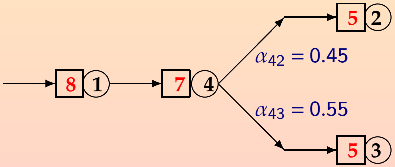
[a] CTMC Markov Process method
[b] Simulation
[c] $M/G/c/c$ method

Figure: 4-stage Comparison Results

Four-stage Split



N	$\theta(N)_s$	$\theta(N)_m$	% dev.	W_s	W_m	% dev.
10	0.7097	0.7221	1.75	14.090	13.848	1.72



N	$\theta(N)_s$	$\theta(N)_m$	% dev.	W_s	W_m	% dev.
13	0.7337	0.7475	1.85	17.717	17.391	1.84

Five-Stage Split Topology

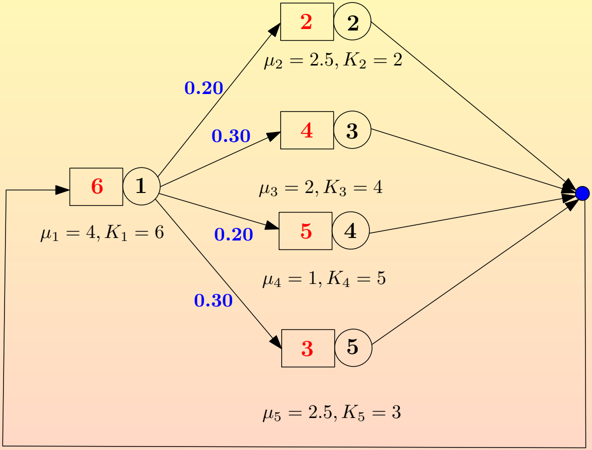


Figure: Five-stage Split-Merge Topology Network

Five-Stage Split Throughput Curve

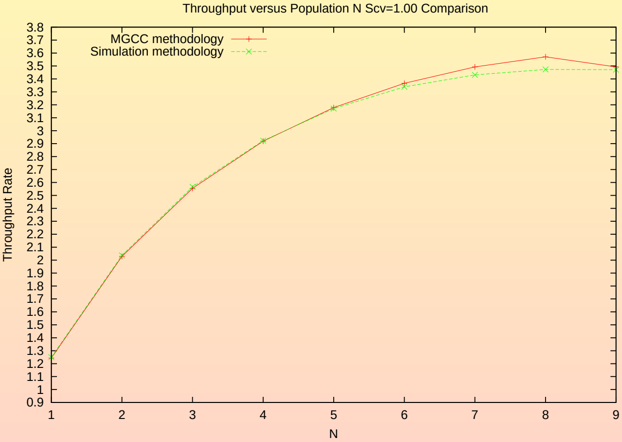


Figure: Five Node Split Throughput Curve $s^2 = 1$

Five-Stage Split Throughput Curve

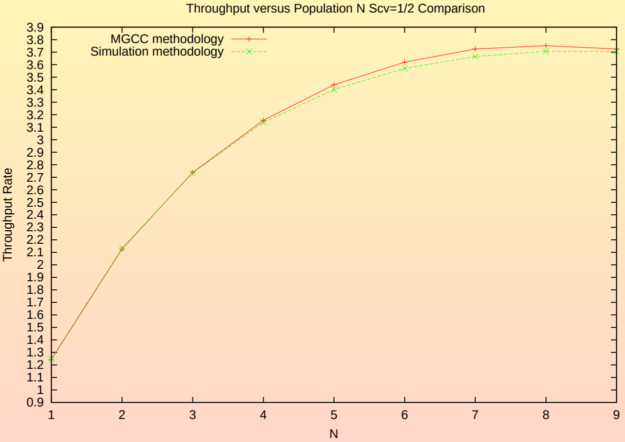


Figure: 5-stage Split Throughput Curve $s^2 = 1/2$

Five-Stage Split Throughput Curve

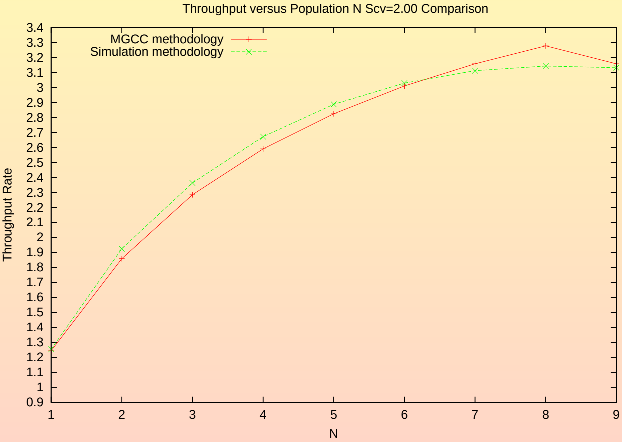
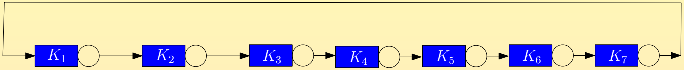


Figure: 5-stage Split Throughput Curve $s^2 = 2$

Seven-Stage Balanced Tandem Line

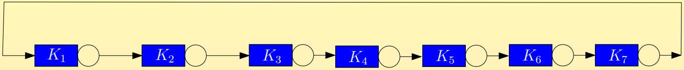


#	s^2	$\theta(N)_s$[a]	$\theta(N)_b$[b]	%	$\theta(N)_\ell$[c]	%	$\theta(N)_m$[d]	%
1	1.00	0.7664	0.7486	2.3	0.7640	0.2	0.8063	5.21
2	0.50	0.8567	0.8464	1.2	0.8561	0.1	0.8455	1.31
3	2.00	0.6482	0.6282	3.1	0.6470	0.1	0.6929	6.90
4	5.00	0.4936	0.4570	7.4	0.4791	2.9	0.5570	12.84

[a] Simulation
[b] Bouhchouchm, Frein, and Dallery
[c] Lagershausen's et.al. Open Network model
[d] $M/G/c/c$ model

Figure: Seven stage Tandem Line Comparison

Unbalanced Seven-Stage Line Throughput Curves



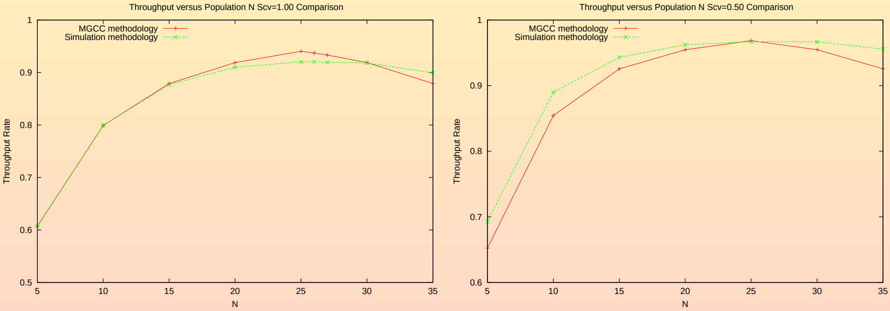
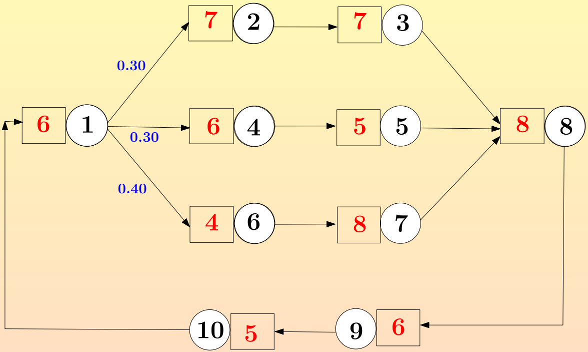


Figure: Throughput Curve 7-stage $s^2 = 1, 1/2$

Ten Stage Topology



Service Rates

$\mu_1 = 8$	$\mu_2 = 2$	$\mu_3 = 2$	$\mu_4 = 2.5$	$\mu_5 = 2.5$
$\mu_6 = 4$	$\mu_7 = 2.5$	$\mu_8 = 10$	$\mu_9 = 8$	$\mu_{10} = 10$

Buffer Values

$K_1 = 6$	$K_2 = 7$	$K_3 = 7$	$K_4 = 6$	$K_5 = 5$
$K_6 = 4$	$K_7 = 8$	$K_8 = 6$	$K_9 = 6$	$K_{10} = 5$

Table: 10-stage Parameters

Ten Stage Topology Throughput Curves

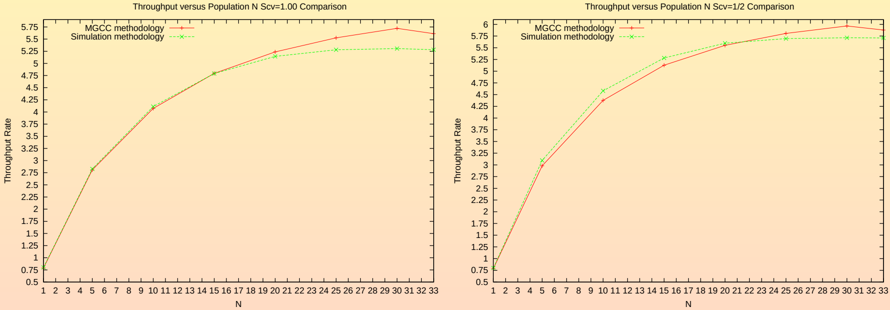
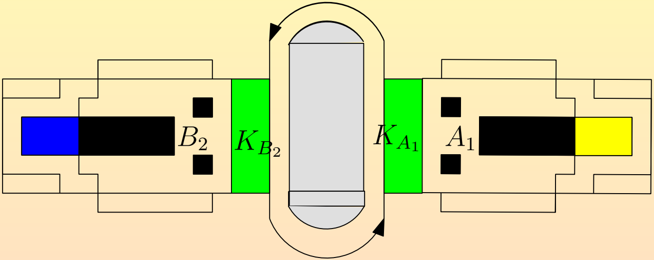


Figure: Throughput Curve 10-stage $s^2 = 1, 1/2$

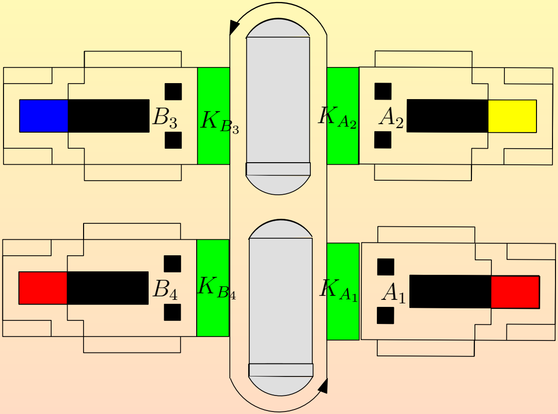
Material Handling Systems



s^2: $\theta(N_\alpha)$	$\theta(N_s)$	%	W_α	W_s	%	Wip_α	Wip_s	%
1 : 0.872	0.846	3.07	10.317	10.382	0.63	0.872	0.846	3.07
1/2: 0.925	0.922	0.33	9.734	9.758	0.25	0.925	0.899	2.89
2 : 0.802	0.798	0.50	11.221	11.280	0.52	0.802	0.778	3.08

Table: 2-stage MHS model

Four-Stage MHS



s^2:$\theta(N_\alpha)$	$\theta(N_s)$	%	W_α	W_s	%	Wip_α	Wip_s	%
1 : 0.825	0.801	3.00	20.605	21.227	2.93	3.614	3.300	9.52
1/2:0.868	0.879	1.25	19.585	19.331	1.31	3.573	3.633	1.65
2: 0.764	0.702	8.83	22.252	24.222	8.13	3.674	3.666	0.22

Table: 4-stage MHS model

Four-Stage MHS Throughput Curve

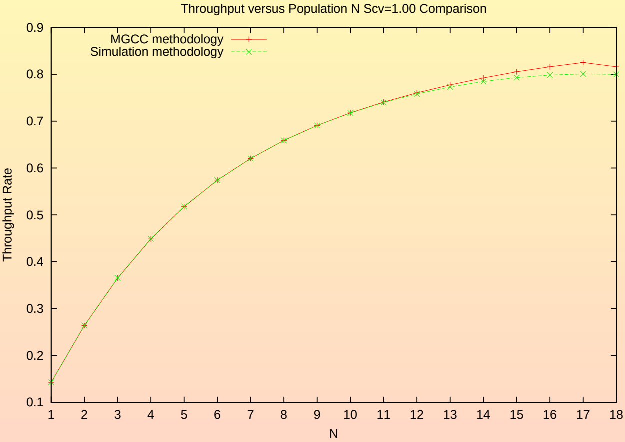
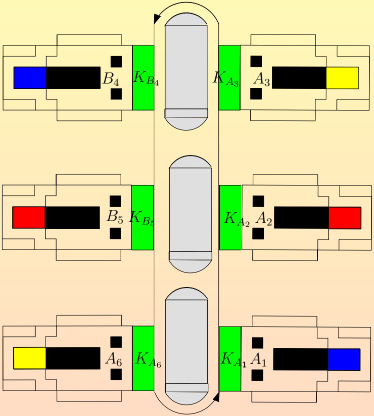


Figure: Throughput Curve $s^2 = 1$ Comparison

Figure: 4-stage MHS model

Six-Stage MHS



s^2:$\theta(N_\alpha)$	$\theta(N_s)$	%	W_α	W_s	%	Wip_α	Wip_s	%
1 : 0.817	0.783	4.34	31.836	33.203	4.12	3.772	3.400	10.94
1/2:0.857	0.868	1.27	30.352	29.952	1.19	3.736	3.067	21.81
2: 0.759	0.673	12.78	34.255	38.633	11.33	3.824	3.065	24.76

Table: 6-stage MHS model

Six-Stage Split MHS System

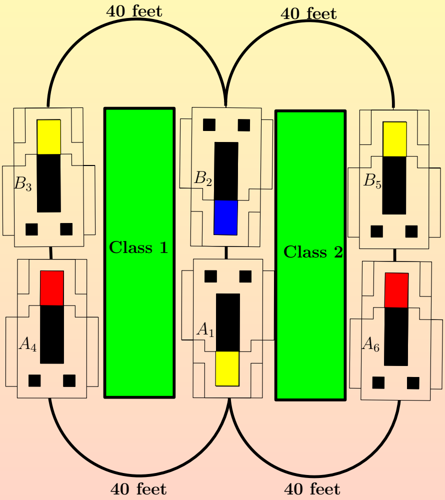
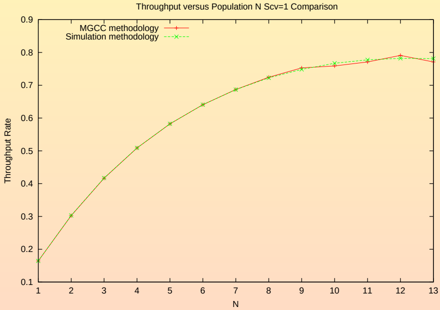


Figure: Six-stage Split Topology

Six-Stage Split Throughput Curves



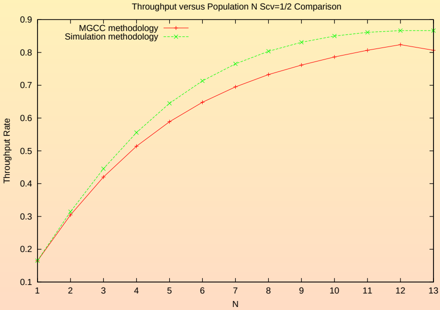
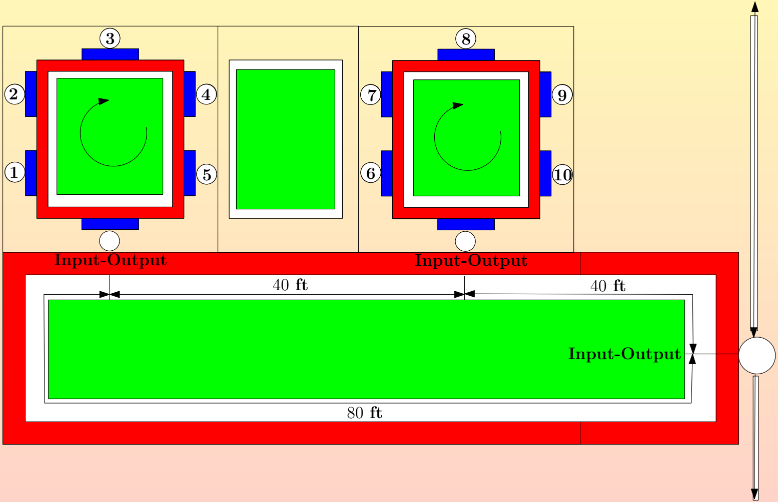
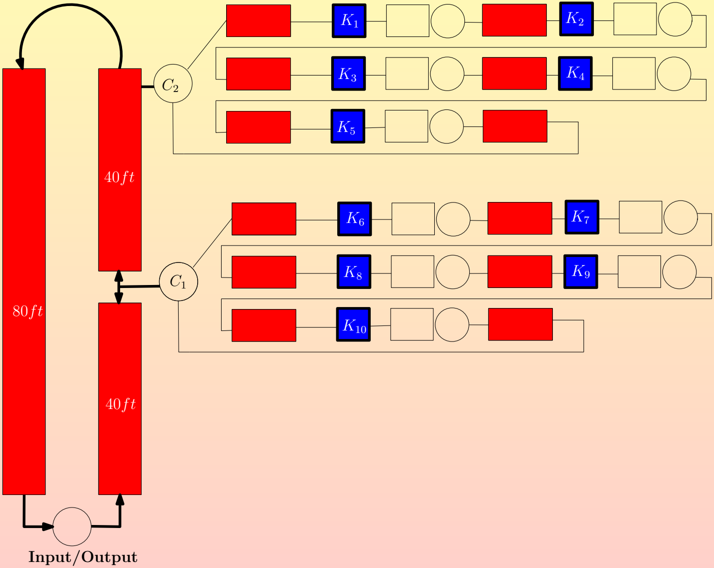


Figure: Throughput Curve 6-stage split $s^2 = 1, 1/2$

Multi-Chain MHS Layout Queueing Network



Multi-Chain MHS Layout Queueing Network



Multi-Chain MHS Layout (continued)

- There are a total of thirty-eight nodes with single-server and material handling conveyors.
- Finite buffers of $K = 3$ at each station and $\mu = 2$ except at the input-output stations which are infinite server nodes.
- Thus, this is an example of an Engset queueing network topology.
- This is also a multi-chain system with two chains and varying populations. So this is a very complex closed queueing network topology.

Arena Simulation Model

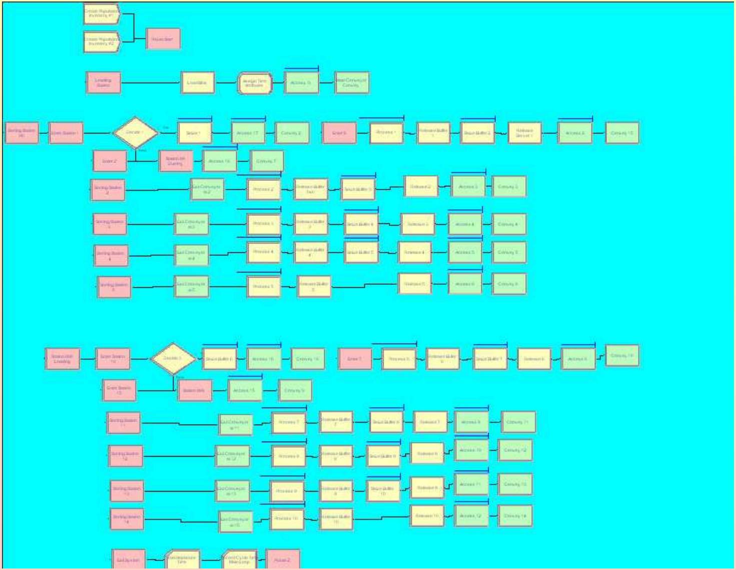


Figure: Multi-Chain Layout Arena Simulation Model

Multi-Chain MHS Layout Throughput Curves

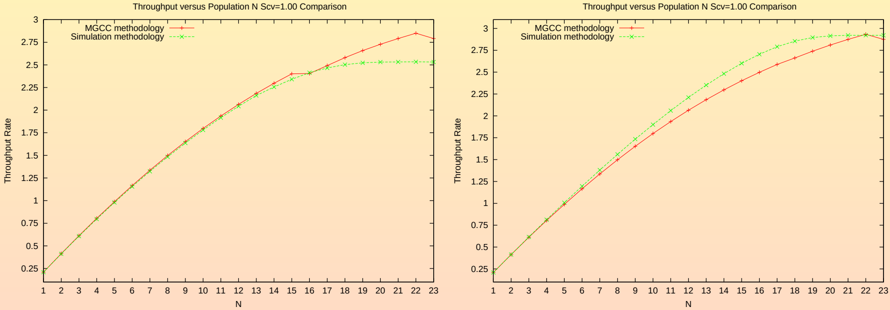
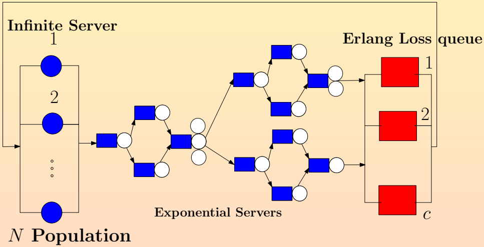


Figure: Layout Throughput Curve $s^2 = 1, 1/2$

Summary & Conclusions and Open Questions

- **Closed Finite Queueing Network Models**
  - Queue Decomposition concept
  - Performance & Optimization Problems
  - General Service Times
  - Including the material handling system.
- **Open Questions & Extensions**
  - V_1 parameter refinements
  - Optimization Models $\{\mathbf{K_i}, \mathbf{N}, \mu_i, \mathbf{c_i}\}$ & Layout Topologies
  - Open Network Models
  - General Multiple Servers



Generalized Engset Networks

Summary & Conclusions and Open Questions

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  - Queue Decomposition concept
  - Performance & Optimization Problems
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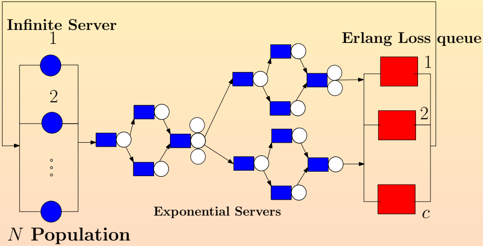

Generalized Engset Networks