Strategic customer behavior in queueing systems with mixed/delayed observation structure

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joint work with Maria Grigoriou (mixed o.s.)
and Apostolos Burnetas and George Vasiliadis (delayed o.s.)

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Model I:

The $M/M/1$ queue with delayed observation structure

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  8. Customers may decide to renege at any time.

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  They observe the system every $\text{Exp}(\theta)$ time units.

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- Because of the exponentiallity assumptions, the customers may renege only at announcement instants.
- Because of the FCFS discipline & the full observation, they may renege only after the first announcement.
- A customer stays after the first announcement, if his position n at the system is such that $R - C\frac{n}{\mu} > 0$.
- The best strategy of a customer taking into account the reaction of the others is to stay if his position n at the first announcement is such that

$$n \leq n_e,$$

with

$$n_e = \lfloor \frac{\mu R}{C} \rfloor \text{ (Naor's threshold).}$$

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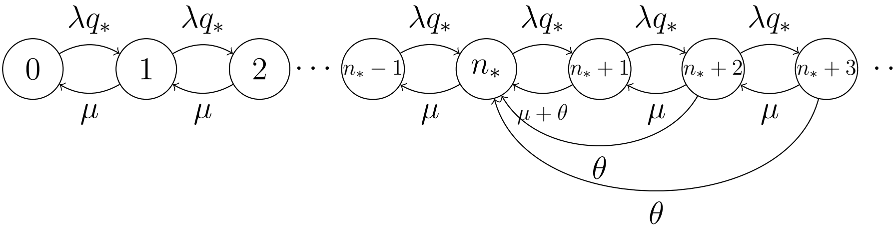

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where

$$\rho_{*1} = \frac{\lambda q_*}{\mu}, \ \rho_{*2} = \frac{\lambda q_* + \mu + \theta - \sqrt{(\lambda q_* + \mu + \theta)^2 - 4\lambda q_* \mu}}{2\mu}$$

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$$B_* = \frac{(1-\rho_{*1})(1-\rho_{*2})}{1-\rho_{*2}-\rho_{*1}^{n_*+1}+\rho_{*1}^{n_*}\rho_{*2}}.$$

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Proposition (continued)

The mean stationary number of customers in system is

$$\mathbb{E}_{(n_*,q_*)}(N) = \frac{(1-\rho_{*2})[(n_*-1)\rho_{*1}^{n_*+1} - n_*\rho_{*1}^{n_*} + \rho_{*1}]}{(1-\rho_{*1})[1-\rho_{*2}-\rho_{*1}^{n_*+1}+\rho_{*1}^{n_*}\rho_{*2}]} + \frac{(1-\rho_{*1})[n_*\rho_{*1}^{n_*} - (n_*-1)\rho_{*1}^{n_*}\rho_{*2}]}{(1-\rho_{*2})[1-\rho_{*2}-\rho_{*1}^{n_*+1}+\rho_{*1}^{n_*}\rho_{*2}]}.$$

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  Solving the balance equations using difference equations or generating functions.

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  Computing geometric sums for the normalization constant and the mean.

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$$\mathcal{U}(n|n_*) = \begin{cases} R - \frac{C(n+1)}{\mu} & \text{if } n < n_*, \\ \left(R - \frac{Cn_*}{\mu} + \frac{C}{\theta}\right)\left(\frac{\mu}{\mu+\theta}\right)^{n-n_*+1} - \frac{C}{\theta}, & \text{if } n \geq n_*. \end{cases}$$

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- $\mathcal{U}(n|n_*)$ does not depend on λ nor on q_*.

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$$\begin{aligned}\mathcal{U}(n_*, q_*) &= B_* \left(R - \frac{C}{\mu} \right) \frac{1 - \rho_{*1}^{n_*}}{1 - \rho_{*1}} \\ &- B_* \frac{C}{\mu} \frac{(n_* - 1)\rho_{*1}^{n_*+1} - n_* \rho_{*1}^{n_*} + \rho_{*1}}{(1 - \rho_{*1})^2} \\ &+ B_* \left(R - \frac{C n_*}{\mu} + \frac{C}{\theta} \right) \frac{\mu \rho_{*1}^{n_*}}{\mu + \theta - \mu \rho_{*2}} \\ &- B_* \frac{C}{\theta} \frac{\rho_{*1}^{n_*}}{1 - \rho_{*2}}.\end{aligned}$$

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- $\mathcal{U}(n_*, q_*)$ is decreasing in q_* for any fixed n_*.

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$$\mathcal{U}(n_e, q) = 0$$

in $(0, 1)$, *with respect to* q.

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Case II: $\mathcal{U}(n_e, 1) < 0 < \mathcal{U}(n_e, 0)$. *The unique equilibrium is* (n_e, q_e), *where* q_e *is the unique solution of the equation*

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in $(0, 1)$, *with respect to* q.

Case III: $\mathcal{U}(n_e, 1) \geq 0$. *The unique equilibrium is* $(n_e, 1)$.

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Model II:

The $M/M/1$ queue with mixed observation structure

The model with mixed observation structure

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  1. Poisson(λ) arrival process.

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- The best strategy of a customer against any strategy of the others is to join, if his position n given that he joins is such that

$$n \leq n_e,$$

with

$$n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \text{ (Naor's threshold).}$$

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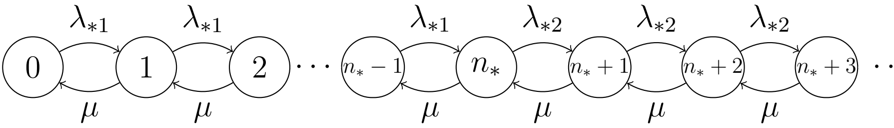

where $\lambda_{*1} = \lambda p_o + \lambda p_u q_*$, $\lambda_{*2} = \lambda p_u q_*$.

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- $\mathcal{U}(n_*, q_*)$ is a decreasing function of q_* for any fixed n_* (a coupling argument shows that N is stochastically increasing in q_*.

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- It is too complicated to reduce it in closed form and to maximize.
- For each $n_* = 0, 1, 2, \ldots, n_e$ we find q_* that maximizes $S(n_*, q_*)$ and then choose the one that gives the overall maximum, namely (n_{soc}, q_{soc}).

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But there are cases where the graph of PoA shows peculiar behavior with very abrupt changes

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Questions?