A HEURISTIC ALGORITHM FOR THE BUFFER ALLOCATION IN SHORT UNRELIABLE PRODUCTION LINES

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This paper investigates the buffer allocation problem (BAP) in short unbalanced and unreliable production lines consisting of up to six machines. Times to failure are assumed exponential whereas service and repair times are allowed to follow any Erlang-k distribution (with $k \geq 1$). A heuristic algorithm that is based on the sectioning (segmentation) approach was developed which solves the buffer allocation problem. This, in conjunction with a method that determines a `good' initial solution for the buffer allocation constitutes the main contribution of the present work. The accuracy of the proposed heuristic algorithm is remarkably good and its convergence is fast making it a promising tool that can be implemented in conjunction with a fast decomposition method to solve the buffer allocation problem in large production lines.

1. Introduction and literature review

One of the key questions that the designers face in a serial production line is the buffer allocation problem (BAP), i.e., how much buffer storage to allow and where to place it within the line. This is an important question because buffers can have a great impact on the efficiency of the production line. They compensate for the blocking and the starving of the line's stations. Unfortunately, buffer storage is expensive both due to its direct cost and due to the increase of the work-in-process (WIP) inventories. Moreover, the requirement to limit the buffer storage can also be a result of space limitations in the shop floor.

In Papadopoulos *et al.* (1993) both evaluative and generative (optimization) models are given for modelling the various types of manufacturing systems. The former are concerned with the evaluation of the various performance measures of the systems whereas the latter try to optimize these measures by determining the optimal values of the decision variables involved. This work falls into the second category.

The literature on this problem is vast. A systematic classification of the research work in this area is given in Singh and MacGregor Smith (1997) and Papadopoulos *et al.* (1993). The works are splitted according to the method used to solve the buffer allocation problem. To our knowledge, three are the basic optimization methods: (1) Search methods, (2) Dynamic programming methods and (3) Simulation methods.

Another classification of the research work relevant to the buffer allocation problem is based on whether the lines under study are balanced or unbalanced. A line is called balanced (or unbalanced) if the mean processing times at each station are equal (or unequal). Powell (1994) provided a literature review according to this scheme. The majority of the existing papers deal with reliable lines, i.e., the machines at each workstation are assumed to be perfectly reliable and therefore don't fail.

Only a few papers consider the assumption of unreliable stations. The reason is simple. Only a few algorithms have been developed to calculate the performance measures of unreliable production lines (see Glassey and Hong, 1983, Choong and Gershwin, 1987, Gershwin, 1989 and Heavey, Papadopoulos and Browne, 1993). Hillier and So (1991) and Seong, Chang and Hong (1995) have been dealt with the buffer allocation problem in unreliable production lines.

Only a few researchers have been dealt with the buffer allocation problem in production lines with multiple (parallel) machines at each workstation (see Hillier and So, 1995, Magazine and Stecke, 1996 and Singh and MacGregor Smith, 1997).

A last classification scheme is given in the next section where the buffer allocation problem is expressed in mathematical terms. This scheme is based on the type of the objective function that is used.

The present work deals with the buffer allocation problem in unreliable unbalanced short production lines with single-machine stations. A heuristic search algorithm is proposed that starts with a `good' initial buffer allocation. With this buffer allocation which was found to be close enough to the optimal buffer allocation (determined by enumeration), it was observed that the convergence of the algorithm was very fast. This search algorithm was used in conjunction with the evaluative algorithm of Heavey *et al.* (1993) which can handle not only exponential lines but also lines with the service and repair times following the Erlang-k distribution with any $k \geq 1$ number of phases. However, this evaluative algorithm can solve only short lines in contrast to the evaluative decomposition algorithm of Glassey and Hong (1983) used by Seong *et al.* which solves large production lines very fast but at the expense of the accuracy. The accuracy of the proposed search algorithm is remarkably good and its convergence is very fast. This algorithm could be applied in conjunction with a decomposition algorithm to solve the buffer allocation problem in large production lines fast and accurately.

The remainder of the paper is organized as follows. In section 2, the production line model and the problem of our interest are described. In section 3, the solution approach and the algorithm are given. In section 4, numerical results are provided which show the accuracy of the proposed algorithm for solving the buffer allocation problem. Finally, section 5 concludes the paper and some future research directions are suggested.

2. The model and the buffer allocation problem

An asynchronous line is one in which each workstation can pass parts on when its processing is complete as long as a buffer space is available. This type of line is subject to manufacturing blocking and starving. Material is not `pulled' by demand. Instead, models are operated in a `push' mode, i.e., it is assumed that a part is always available when needed at the first workstation and space is always available after the last workstation to dispose of a completed part (in other words, it is assumed that the first station is never starved and the last station is never blocked).

A K-station line consists of K machines in series, labelled M_1, M_2, ... , M_K and K-1 locations for buffers, labelled B_2, B_3,..... . , B_K, in Figure 1.

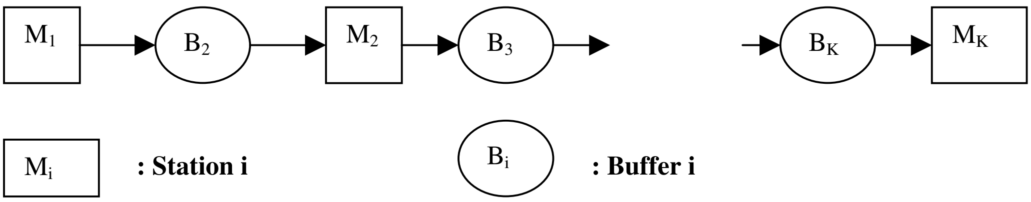

Figure 1: A K-station production line with K-1 intermediate buffers

Other typical assumptions of our model are: All the random variables (processing or service times, uptimes, downtimes) are independent random variables (r.v.). The transfer through the buffers takes zero time. The failures are single-machine failures (as opposed to total line failures) and operation dependent failures (for the definition of operation/time dependent failures see Papadopoulos *et al.*, 1993, among others). When a failure occurs, the part stays on the machine and it is reworked when the machine is up again, that is, there is no scrapping of parts and the work resumes exactly at the point it stops. Times to failure are assumed to be exponentially distributed whereas processing times and times to repair are allowed to follow the Erlang-k (k $\geq$1) distribution, in general.

The basic quantities of our model are:

- S_i is a r.v. representing service (processing) time of station i with μ_i the average service (processing) rate or speed.
- $MTTF_i$ is the average time to failure of station i and β_i is its average failure rate. $MTTR_i$ is the average time to repair of station i and r_i is its average repair rate.
- By A_i is denoted the availability of station i. This is the average fraction of the time that station i is operational if it is operated in isolation, i.e., never starved or blocked. This quantity is also referred to as the isolated efficiency of station i and is also denoted by e_i. For a reliable station $A_i = 1$ and for an unreliable station $A_i < 1$.
- T_i is a r.v. representing the effective service (processing) time of station i. This equals the sum of the service time and the repair times (if more than one failures occur during one service period; this number is assumed to be a random variable following the Poisson distribution).
- ρ_i is the average effective service (processing) rate or production rate of station i in isolation. The expected value of r.v. W in general, is denoted by $E[W]$. Finally, X_K, denotes the throughput or mean production rate of the K-station line. The availability and the mean effective service rate of a station i is given by:

$$A_i = e_i = \frac{MTTF_i}{MTTF_i + MTTR_i} = \frac{r_i}{\beta_i + r_i} \cdots\cdots (1)$$

$$\rho_i = \mu_i A_i = \frac{\mu_i r_i}{\beta_i + r_i} \cdots\cdots (2)$$

A production line is completely characterized by the three 1xK vectors : $\boldsymbol{\mu} = (\mu_1,...,\mu_K)$, $\boldsymbol{r} = (r_1,...,r_K)$, and $\boldsymbol{\beta} = (\beta_1,.....,\beta_K)$. Depending on the values these three vectors can take on the production lines are classified into the following categories :

1. Reliable production lines, with $\boldsymbol{r} = \boldsymbol{\beta} = \boldsymbol{0} = (0,...,0)$. These are futher classified into two sub-classes: (1.1) balanced lines with $\mu_1 = \cdots = \mu_K$, and (1.2) unbalanced lines with $\mu_j \neq \mu_i$. for some $i,j \in \{1,\ldots,K\}$ (It is assumed that all stations are identically distributed.)

2. Unreliable production lines with $\boldsymbol{r}, \boldsymbol{\beta} \neq \boldsymbol{0}$. These again are further classified into two sub-classes: (2.1) balanced lines with $\rho_1 = \cdots = \rho_K$ (same effective rate at all workstations. *Note*: The case where $\mu_1 = \cdots = \mu_K$, $\beta_1 = \cdots = \beta_K$ and $r_1 = \cdots = r_K$ is a subset of the set of balanced unreliable production lines) and (2.2) unbalanced lines where all three vectors $\boldsymbol{\mu}, \boldsymbol{r}, \boldsymbol{\beta}$ take any values from the positive section of R^K, i.e., $R^{+,K}$, and similarly to case 1.2, for at least two stations $i, j \in \{1,...,K\}$, the corresponding values of these three parameters are different.

The object of our study is the optimal buffering of the unreliable unbalanced production lines with asynchronous transfer of discrete parts. This is the more general class 2.2 of production lines, given above.

The optimal buffer allocation (OBA) problem: In mathematical terms, our problem may be stated as a non-linear integer programming problem with linear constraints, as follows:

Find

$$\mathbf{B}^N = (B_2^N, B_3^N, ..., B_K^N)$$

so as to $\max X_K(\mathbf{B^N})$

subject to :

$$\sum_{i=2}^{K} B_i^N = N$$

and B_i^N nonnegative integers ($i = 2,3,...,K$),

where: N is a fixed nonnegative integer denoting the total buffer space available in the system which has to be allocated among the *K-1* buffer locations so as to maximize the throughput of the production line. The case where there are buffer capacity limitations such as physical limitations between stations can be easily handled by the algorithm by setting upper limits for each component of the buffer vector.

This type of buffer allocation problem is termed in Gerhswin and Schor (2000) as *dual* problem as opposed to the *primal* problem where the objective is the minimization of the total buffer space provided a desired throughput level is achieved. With this primal problem Gershwin and Schor (2000), Yamashita and Altiok (1997) and So (1997) have been dealt, among others.

The optimal allocation of N buffer slots among the K-1 buffer locations of an unreliable unbalanced production line depends on the values of the parameters μ_i and r_i, β_i (or equivalently, the mean effective service rates, ρ_i of the unreliable stations). All these parameters may take on any real value. This generality and lack of information to form some design rules for the optimal buffer allocation in unreliable unbalanced lines creates another difficulty in our trial to solve this significant design problem.

The concavity property of the throughput must hold to guarantee the global optimum to the OBA problem. Meester and Shanthikumar (1990) and Anantharam and Tsoucas (1990) have shown the stochastic concavity of the throughput in tandem reliable queueing systems with single-server stations and exponential service times. Altiok and Stidham (1983) have pointed out that the form of the throughput offers empirical support for the conjecture that the objective function is concave when machine failures are taken into account. In this work, similar to Seong *et al.* (1995), the concavity of the throughput function is also assumed. This property was experimentally shown to hold in all cases treated in our analysis (we plotted the throughput of the production lines for various cases and we had an empirical support for the concavity assumption of this function). The same property was assumed in Gershwin and Schor (2000) (see their Figure 2).

3. The solution approach

The objective is to solve the linearly constrained integer nonlinear buffer allocation problem given in the previous section. Unfortunately, as it has been already mentioned, an analytical solution is not attainable since the nonlinear throughput function cannot be expressed explicitly. This dictates the search for a numerical solution only. For the calculation of the mean production rate of the production lines considered in this study the analytical method and the software developed by Heavey, Papadopoulos and Browne (1993) were used. This gives the exact throughput of any K-station production line with exponential times to failure and Erlang-k (for any k ≥ 1) service and repair times.

In our general case of unreliable unbalanced production lines the reversibility property is valid but cannot help reduce the search space. This means that in order to determine the optimal allocation of a given total number of N buffer slots we were obliged to scan all the solution search space. Primarily, a complete enumeration was used to evaluate the throughput of the systems. However, the number of iterations we had to run for obtaining the OBA were huge. The number of distinct ways for allocating N buffer slots in K-1 buffer locations in a K-station line is equal to

$$\frac{(N+1)(N+2)...(N+K-2)}{(K-2)!} = \binom{N+K-2}{K-2}$$

For example, there are 10,626 distinct ways of allocating N=20 buffer slots into the 5 buffers of a six-station line.

At a second stage, after the complete enumeration, we exploited the knowledge acquired from the thorough investigation of the systems and we succeeded to reduce greatly the number of iterations needed to find the OBA. The amount of reduction increases for large values of N and K, approaching even to 99 %. For example, for K=7 and N=30, the number of iterations required to find the OBA by the improved enumeration (IE) are 3,458 as compared with the 324,632 iterations needed by the complete enumeration (CE).

The disadvantage of the improved enumeration is that this method works by induction. That is, to find the OBA for $N=m$, one has to determine the OBAs for $N=1,2,...,m-1$, first. To overcome this difficulty, the ideal would be to apply a direct search method that would determine the OBA very fast. We applied a search method that is based on the sectioning (segmentation) approach in conjunction with a method, not previously reported, that determines a `good' initial solution for the buffer vector. This constitutes the main contribution of the present work.

In the sequence, the steps of the proposed algorithm are described:

3.1 The steps of the algorithm

The steps of the proposed algorithm for finding the OBA in any general unbalanced unreliable production line may be summarized as follows.

Step 1: Preparation for a 'good' initial buffer allocation

Order the stations $M_1, M_2, .. ,M_K$ of the production line from the slower to the faster based on the value of the mean effective service rate, ρ_i, given by formula (2). A similar idea was proposed by Kalir and Arzi (1997). Let this arrangement be the $M_1', M_2', ..., M_K'$.

Step 2: Determination of a 'good' initial buffer allocation

To determine a `good' initial buffer allocation vector, apply the following linear buffer allocation scheme.

- (i) The buffer that is located toward the center of the line and next to station M_j' is assigned 2(K + 1 - j) points;
- (ii) The buffer that is located toward the end of the line and next to station M_j' is assigned 2(K + 1 - j) - 1 points;
- (iii) The central buffer is assigned K points and in case there are two central buffers these are assigned equally K/2 points each.

Step 3: Search for the OBA via the use of a sectioning method

Starting with the `good' initial buffer vector determined in step 2, call the sectioning (segmentation) routine to find the optimal (or near optimal) buffer vector (the reader is addressed to Ravindran *et al.*, 1987, Chapter 11, pp. 511-512, for a description of the sectioning or segmentation method). More specifically:

(3.1) [Search loop] Increase and decrease by one unit each one of the K-1 initial buffer decision variables; find the throughput of the line for the corresponding buffer allocation; select the buffer allocation that gives the maximum throughput. This will be the new initial buffer allocation for the next cycle of searches.

(3.2) [Stopping criterion] Continue the search procedure up to a certain number of cycles of iterations or until there is no further improvement of the throughput of the line.

4. Numerical results

In this section, some representative numerical results are given concerned with the OBA of unreliable unbalanced production lines which were obtained from the implementation of the algorithm proposed in the previous section. Unfortunately, only a few papers (Seong *et al.*, 1995 and Altiok and Stidham, 1983) from the international literature give some numerical results for unreliable unbalanced lines to compare them with ours. For this reason, we followed this procedure. We chose various unreliable and unbalanced lines with K=3,...,6 stations. For each one of these systems and for a certain range, 1 - N, of the total number of buffer slots we determined the optimal (or near-optimal) buffer allocation both by enumeration (complete and improved) and by the proposed search algorithm. Then we compared the numerical results and the accuracy of the new method. Due to space problem these results are not given here. For the few cases available from Seong *et al.* (1995) we compared the accuracy of the results derived by their and our algorithms. Table 1 tabulates 20 cases considered in this study.

Table 1: Various systems analysed in this study : 20 cases

Case #	K	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	r_1	r_2	r_3	r_4	r_5	r_6	β_1	β_2	β_3	β_4	β_5	β_6
1	3	1	1.1	1.2				0.09	0.09	0.09				0.01	0.02	0.03			
2	3	1.1	1	1				0.1	0.08	0.1				0.02	0.01	0.01			
3	3	1.5	1	1				0.1	0.08	0.1				0.05	0.01	0.01			
4	3	1	1	0.9				0.08	0.2	0.5				0.01	0.02	0.05			
5	4	1	1.1	1.1	1			0.08	0.09	0.09	0.08			0.01	0.02	0.02	0.01		
6	4	1	1	0.9	1			0.09	0.07	0.1	0.08			0.01	0.01	0.01	0.01		
7	4	1.1	1	1	0.9			0.08	0.07	0.09	0.05			0.02	0.01	0.01	0.01		
8	5	1	1.1	1	1.1	1		0.09	0.09	0.09	0.09	0.09		0.01	0.02	0.01	0.02	0.01	
9	5	1	0.9	1.1	1.2	1		0.09	0.09	0.09	0.09	0.09		0.01	0.005	0.012	0.015	0.01	
10	4	3.7	1.5	1.1	3			0.17	0.37	0.78	0.5			0.07	0.11	0.49	0.19		
11	4	3	1	2	3.6			0.45	0.55	0.5	0.4			0.38	0.3	0.35	0.45		
12	5	1.2	1	3	2	1.8		0.3	0.5	0.2	0.3	0.1		0.1	0.3	0.5	0.4	0.2	
13	5	2.8	1.7	2.5	3.4	1.9		0.64	0.83	0.75	0.85	0.74		0.3	0.4	0.45	0.35	0.1	
14	5	2.1	3	3.4	4.7	1.5		0.4	0.4	0.6	0.5	0.3		0.08	0.24	0.2	0.17	0.1	
15	6	3	1	1.2	1.8	1.5	2	0.2	0.5	0.3	0.1	0.2	0.3	0.3	0.5	0.1	0.2	0.3	0.4
16	3	0.25	0.2	0.3				0.1	0.3	0.5				0.01	0.02	0.04			
17	4	3.7	1.5	2	3.6			0.17	0.37	0.5	0.4			0.07	0.11	0.35	0.45		
18	4	1.1	3	3	1			0.78	0.5	0.45	0.55			0.49	0.19	0.38	0.3		
19	5	1.2	1	1.8	1.4	1.2		0.3	0.5	0.75	0.85	0.74		0.1	0.3	0.45	0.35	0.1	
20	4	1	1.1	0.9	1			0.08	0.09	0.1	0.08			0.01	0.02	0.01	0.01		

In several cases (15 out of 20 or more precisely, in 364 experiments out of 373 if we count the different scenarios we ran for the various values of N and K) the algorithm gives the OBA and in the remaining 5 cases or 9 experiments only, the buffer allocation is near-optimal. The algorithm fails to give the OBA due to the fact that the initial buffer allocation is not so good and via the use of the sectioning method, the algorithmic procedure cannot exit the search loop to reach the optimal solution. However, the solution is very close to the optimal.

Unfortunately, as it has been already mentioned, there are not enough numerical results to compare the accuracy of our algorithm.

Table 2 gives a few comparative numerical results concerned with the optimal buffer allocation. These results, derived by Seong *et al.'s* heuristic algorithm (called `*SEVA*') and by our algorithm (called `*New*') are compared against the exact results obtained by the complete enumeration method (CE). In three out of four cases, the *SEVA* algorithm attains a near optimal solution, whereas our algorithm reaches the OBA in all cases.

Table 2: Some comparative results obtained by the proposed algorithm ("New") and "SEVA" (by Seong *et al.*)

			BUFFER ALLOCATION				No. of iterations	
Case #	K	N	Initial (CE)	Optimal	"New" algorithm	SEVA algorithm	New	SEVA
10	4	10	(3-3-4)	(0-7-3)	(0-7-3)	(0-7-3)	21	26
10	4	10	(2-5-3)	(0-7-3)	(0-7-3)		14	
11	4	30	(10-10-10)	(11-15-4)	(11-15-4)	(11-16-3)	28	53
11	4	30	(9-15-6)	(11-15-4)	(11-15-4)		14	
12	5	10	(2-2-2-4)	(1-3-3-3)	(1-3-3-3)	(1-3-4-2)	9	31
12	5	10	(2-3-2-3)	(1-3-3-3)	(1-3-3-3)		9	
13	5	15	(3-3-3-6)	(4-7-3-1)	(4-7-3-1)	(4-6-3-2)	45	74
13	5	15	(4-6-4-1)	(4-7-3-1)	(4-7-3-1)	(4-6-3-2)	9	

Table 3 gives some sample data from the cases examined in this study (given in Table 1) which show that the initial buffer allocation obtained from the proposed scheme gets close enough to the optimal buffer allocation.

Table 3 : Some comparative results concerning the initial and optimal buffer allocations

		BUFFER ALLOCATION				BUFFER ALLOCATION	
Case #	N	Initial	Optimal	Case #	N	Initial	Optimal
1	11	(6-5)	(6-5)	10	15	(2-8-5)	(1-10-4)
1	16	(9-7)	(8-8)	10	19	(3-10-6)	(1-13-5)
5	10	(3-4-3)	(3-4-3)	10	23	(4-12-7)	(1-15-7)
5	12	(4-4-4)	(4-4-4)	10	29	(5-15-9)	(2-19-8)
5	20	(6-8-6)	(6-8-6)	12	10	(2-3-2-3)	(1-3-3-3)
7	10	(3-4-3)	(3-4-3)	12	15	(2-4-4-5)	(2-4-4-5)
7	15	(4-6-5)	(4-6-5)	12	20	(4-5-5-6)	(2-5-6-7)
7	17	(5-7-5)	(5-6-6)	12	24	(4-6-6-8)	(3-6-7-8)
7	20	(6-8-6)	(6-7-7)	12	26	(5-7-6-8)	(3-7-7-9)
7	29	(8-12-9)	(8-11-10)	19	8	(3-2-2-1)	(3-3-1-1)

5 Conclusions and further research

In this work a heuristic algorithm was presented for solving the buffer allocation problem in short unreliable unbalanced production lines. The main contribution relies on the determination of a `good' initial buffer vector which is found by exploiting the information about the parameters of the production line, that is, the mean service rates, μ_i' s, the mean repair rates, r_i 's and the mean failure rates, β_i 's. Then based on this information, the stations of the line are ordered according to the value of the mean effective service rates, ρ_i's, starting with the bottleneck station and ending with the faster one. Some priority rules are utilized and the initial buffer capacities are determined based on a linear buffer allocation scheme. After the determination of the initial buffer vector, a sectioning search method is called to search for the OBA. This method searches for each variable in two directions, one buffer slot up and one down of a certain value of the decision variables which are the buffer capacities. The optimal buffer allocation was found in almost all cases examined in this study (above 97 % of the 373 experiments we have tried) and in the cases where the heuristic algorithm fails to find the OBA, the solution was near optimal.

The proposed idea of using a `good' initial buffer allocation can speed up the solution to the buffer allocation problem in a production line of any size. It is the evaluation tool that restricts the applicability of the proposed heuristic algorithm and not the search algorithm itself. It is proposed use of the new search algorithm in conjunction with a decomposition algorithm to solve the buffer allocation problem in longer production lines.

Another area for further improvement would be to focus more on finding an even better initial solution by further exploiting the knowledge about the criticality of the unreliable stations of the line.

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