OPTIONS-BASED SOURCING STRATEGIES FOR PRODUCERS WITH LIMITED CAPACITY AND VOLATILE DEMAND

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In this study, we investigate the strategy of increasing the production capacity temporarily by means of contingent contractual agreements. These agreements are viewed as capacity options. A continuous material flow production system that supplies materials to meet a random demand that switches randomly between a high level and a low level is considered. The production system does not have enough capacity to meet the demand by production when the demand is high. Therefore, it either produces to stock in advance or uses a subcontractor. The contractual agreement with the subcontractor gives the right to receive additional production capacity when needed. The problem of determining the policies for production and subcontracting is formulated as an optimal control problem and analyzed analytically.

1. Introduction

This study is motivated by the challenges in the retail-apparel-textile channel. The retail-apparel-textile channel is characterized by rapidly changing styles, uncertain customer demand, product proliferation, and long lead times. As a part of the quick response programs, lean retailers demand from manufacturers to supply a higher percentage of its orders within a selling season (Abernathy et. al., 2000).

From a manufacturer's perspective, alternative ways to respond quickly to changes in demand include producing to stock in advance, increasing the production capacity permanently by investing in new production facilities, increasing the production capacity temporarily by using overtime, subcontracting, etc., and combinations of these pure strategies. Making these decisions in the most effective way are of crucial importance for the manufacturer to be competitive.

In this study, we investigate the strategy of increasing the production capacity temporarily by means of contingent contractual agreements with short-cycle manufacturers to manage the risks associated with demand uncertainty. We view all these agreements as *real options*. An *option* is the right, but not obligation, to take an action in the future and a *real option* is the extension of financial option theory on real

(non-financial) assets (Amram and Kulatilaka, 1999). Since the contractual agreement is related to increasing the capacity, we refer this option as a *capacity option*.

More specifically, we consider a production system that supplies products to meet a random demand that switches randomly between a high level and a low level. The production system does not have enough capacity to meet the demand by production when the demand is high. Therefore, it is necessary to produce-to-stock in advance. Alternatively, a contractual agreement with a short-cycle manufacturer can be made. This option gives the right to receive additional production capacity when needed. There is a fixed cost to purchase this option for a period of time and if the option is exercised, there is an additional per unit exercise cost which corresponds to the cost of the goods produced at the short-cycle manufacturer. At the beginning of the planning horizon, the manufacturer decides how much additional capacity will be reserved at the short-cycle manufacturer. Then at a given time, the manufacturer decides how much to produce and how much additional production to be requested from the short-cycle manufacturer.

We formulate the problem as an optimal control problem and analyze it analytically. By comparing the costs between two cases where the agreement with the short-cycle manufacturer is used or not, we determine the price of this option. Furthermore, we investigate the effects of demand variability on this contract.

The contribution of this study is two-fold. First, an analytical model is developed and analyzed thoroughly to investigate capacity options as a way to manage demand uncertainty in production systems. Second, the model is used in numerical experiments to gain some insight when these options are valuable.

Due to the simplicity of the model, the study is geared towards showing the direction of the benefits that can be obtained through options, rather than answering a more specific question of how a company can price such an option to use in day-to-day decision-making.

Organization of the remaining part of this paper is as follows: In §2, a review of the pertinent literature is given. The basic model and its assumptions are given in §3. The production control problem is formulated in §4, and solved in §5. The analysis of the option is provided in §6. Numerical results that investigate the effects of variability are presented in §7. Finally, the concluding remarks are given in §8.

2. Past Work

The studies reviewed on using capacity options are grouped in two areas: inventory management and stochastic modeling of manufacturing. Most of the work on capacity options are found in inventory literature where the main objective is to determine the order quantities.

In the apparel catalog industry, contracts similar to options are used. Eppen and Iyer (1997) present *backup agreements* used in apparel catalog industry. Under this agreement, the buyer makes a firm commitment to purchase a given number of goods at the beginning of the horizon. In the first period, the buyer purchases a certain

percentage of this commitment at a given price. At the second period, if the buyer purchases less than the committed, a penalty is paid for the remaining parts that are not purchased. It is reported that a catalog company Catco uses these contracts with Anne Klein and DKNY (Eppen and Iyer, 1997).

Another contract type is the *quantity flexibility contract* (Bassok and Anupindi, 1997). Under quantity flexibility contract, the buyer provides a forecast of future orders to the supplier. Later, the buyer purchases between a predetermined minimum and the maximum levels within the initial forecasts. That is, a minimum quantity needs to be purchased at the agreed price and there is an option to purchase up to the maximum level at the same price. These contracts are reported to be used in the electronics industry, foe example, by IBM printer division, Sun Microsystems, Solectron, Hewlett Packard, etc. (Anupindi and Bassok, 1998; Tsay and Lovejoy, 1999).

If the capacity is scarce, a buyer may pay an upfront fee to reserve capacity in advance. Agreements called *pay-to-delay capacity reservation* are used in electronics industry (Brown and Lee, 1997). Under this agreement, the buyer makes an agreement with the supplier to purchase a minimum quantity at a given price c_f and pays c_0 per unit to reserve up to a level. These additional units can be purchased at an extra unit cost of c_e.

Van Meighem (1999) and Barnes-Schuster et. al. (2000) present game-theoretic models to the value of the option of subcontracting in two-period, two-stage buyer-supplier system. They also investigate the coordination problems. Barnes-Schuster et. al. (2000) discuss all three agreements explained above as special cases of a general option .

Gershwin (1993) introduces an extension of Flexible Manufacturing System (FMS) scheduling model where the capacity of the FMS can be increased, if necessary. He shows that the optimal policy is similar to an (*s*,*S*) policy where subcontracting is used when the inventory position goes down to a lower hedging level and the parts are produced with the maximum rate until an upper hedging level is reached. A proof of optimality of the hedging point policy for this problem is given by (Huang, et. al., 1999).

3. Model Description

We consider a make-to-stock system with a single manufacturing facility that produces to meet the demand for a single item. The product flow is approximated by a continuous flow. The demand rate at time t is denoted by $d(t)$. The state of the demand at time t is $D(t)$ which is either high (H) or low (L). When the demand is high, the demand rate is μ_H and when the demand is low, the demand rate is μ_L. At time t, the amount of finished goods inventory is $x(t)$.

The times to switch from a high demand state to a low demand state and from a low demand state to a high demand state are assumed to be exponentially distributed random variables with rates p and r. Tan (2000) uses this demand model to extend the unreliable machine-constant demand production control problem to unreliable machine-uncertain demand case. The asymptotic distribution of the total demand generated by this random switching model is normal and the parameters p and r can be chosen to match the mean and variance of a given distribution Tan (1997).

The maximum production rate of the manufacturing facility is μ. The production rate of the manufacturing facility at time t is denoted by $u(t)$. $0 \le u(t) \le \mu$. We assume that the production capacity is sufficient to meet the demand when it is low but insufficient when it is high, i.e., $\mu_L < \mu < \mu_H$. The profit generated through the sales of the goods produced at the plant is L (dollars per unit). The inventory carrying cost is c^+ and the backlog cost is c^- (dollars per unit per time).

An agreement with a subcontractor can be made to receive additional capacity when it is needed. According to this capacity option the company pays an upfront fee of C_O to receive an extra capacity of $0 \le v(t) \le \mu_c$ at time t for a period of T. The exercise cost of the option, i.e., the production cost when it is obtained from the short-cycle manufacturer is above the regular production cost by Δ_c \$/unit. After paying the additional cost, the profit generated through the sales of the goods received from the subcontractor is A (dollars per unit) ($\Delta_c = L - A$).

Since it is uncertain when the demand will be high, the company may consider this option to decrease the need of holding an excessive inventory or investing in capacity expansion. This is also advantageous for the contractor if it has extra capacity not fulfilled with its own demand. Furthermore, the upfront payment will be received regardless of whether the option is exercised or not in the specified time period.

In this study, the effects of this kind of agreement on the performance of the production-inventory system are analyzed. We consider the profit, service level, and the average inventory as the main performance indicators.

4. Production Control Problem

At time t, the manufacturing facility is scheduled to produce at rate $u(t)$, and the subcontractor is requested to supply goods at the rate of $v(t)$ in such a way that the expected profit is maximized.

The profit is the difference between the money generated through sales and the inventory carrying and backlog costs. Then the production control problem is

$$\text{Max } \Pi_1 = E \int_0^T (Lu + Av - c^+x^+ - c^-x^-) dt \tag{1}$$

subject to

$$\frac{d}{dt} x(t) = u(t) + v(t) - d(t) \tag{2}$$

$$0 \le u(t) \le \mu \tag{3}$$

$$0 \le v(t) \le \mu_c$$

$$d(t) = \begin{cases} \mu_H & \text{if } D(t) = H \\ \mu_L & \text{if } D(t) = L \end{cases} \tag{4}$$

Markov dynamics for S(t) with rates p (from H *to* L) *and r (from* D *to* U)

By using the results given for unreliable machine-constant demand subcontracting problem (Gershwin, 1994; Hu, et. al., 1999), it can be shown that the optimal policy for the constant production-uncertain demand problem is also a (Z, S) policy where $Z \geq 0$ is the produce-up-to level, and $S \leq 0$ is the backlog level when it is reached, the subcontractor is requested to supply goods at a rate that keeps the backlog at this level and when the backlog is below this level, the subcontractor supplies with the maximum capacity. This policy drives the backlog/surplus into the region between Z and S. If the subcontractor capacity is sufficient to keep the backlog at this level, i.e., $\mu_c \geq \mu_H - \mu$, $x(t)$ stays bounded between Z and S and $x(t) < S$ is transient.

5. Analysis of the Model

The performance analysis of the model is carried by determining the differential equations that explain the behavior of the system in the interior states and solving these equations subject to some boundary conditions. The complete analysis of the model is reported in (Tan, 2001).

Once the complete solution of the system is available in the derived density functions and the probability masses, the performance measures of interest including average sales per unit time, average inventory and backlog levels and the average rate at which the subcontractor is used can be determined. Since backlog is allowed, the average sales per unit time is equal to the average demand rate. The average finished goods inventory WIP is

$$\text{WIP} = c\left[Ze^{\lambda Z}\left(\frac{\mu_H - \mu_L}{\lambda(\mu - \mu_L)} + \frac{\mu_H - \mu}{r} \right) - e^{\lambda Z}\frac{\mu_H - \mu_L}{\lambda^2(\mu - \mu_L)} + \frac{\mu_H - \mu_L}{\lambda^2(\mu - \mu_L)} \right] \tag{5}$$

where $\lambda = \dfrac{p}{\mu_H - \mu} - \dfrac{r}{\mu - \mu_L}$ $(\lambda \neq 0)$ and

$$c = \left[e^{\lambda Z}\left(\frac{\mu_H - \mu_L}{\lambda(\mu - \mu_L)} + \frac{\mu_H - \mu}{r} \right) - e^{\lambda S}\left(\frac{\mu_H - \mu_L}{\lambda(\mu - \mu_L)} - \frac{\mu_H - \mu}{p} \right) \right]^{-1} \tag{6}$$

Similarly, the average backlog level BG is

$$\text{BG} = c\left[Se^{\lambda S}\left(\frac{\mu_H - \mu_L}{\lambda(\mu - \mu_L)} - \frac{\mu_H - \mu}{r} \right) - e^{\lambda S}\frac{\mu_H - \mu_L}{\lambda^2(\mu - \mu_L)} + \frac{\mu_H - \mu_L}{\lambda^2(\mu - \mu_L)} \right]. \tag{7}$$

The average rate at which the subcontractor supplies goods is

$$\text{TH}_v = c(\mu_H - \mu)(\mu - \mu_L)e^{\lambda S} \tag{8}$$

Finally the profit is

$$\Pi_1 = L \cdot \text{TH} - (L - A) \cdot \text{TH}_v - c^+ \text{WIP} - c^- \text{BG} \tag{9}$$

Once the profits with and without the option are determined excluding the fixed cost, the maximum amount that can be paid for this option for a given time interval can be calculated from the difference. Note that the calculated profits Π^*_1 and Π^*_2 are per unit time. Let T be the expiration time of the option. The expected profit in $[0, T)$ can be approximated with $\Pi^*_1 T$ and $\Pi^*_2 T$ as T approaches infinity. Then the amount that can be paid for this option should not exceed the total additional profit obtained from this option:

$$C_O \leq (\Pi^*_2 - \Pi^*_1)T \qquad\qquad (10)$$

6. Numerical Results

In this section, we present some preliminary results. Figure 1 shows how the terms of the contract can be evaluated by using the relationship between the additional cost, gain in the profit, and the fixed cost and the duration of the option. For example, the figure shows that, it is possible to increase the profits by paying 20% of the expected profit without using the subcontractor during the duration of the option as an upfront payment (option price) and paying less than 120% of the regular production cost as you receive goods from the subcontractor (exercise price of the option).

Figure 2 illustrates the effects of demand variability on a specific option with an upfront payment of 20% of the profit without subcontracting and an exercise price which is 50% above the production cost. As the demand variability, summarized with its coefficient of variation, increases the value of the option increases. However, for low variability cases $cv<0.9$, it is not profitable to use the option.

Figures 3 and 4 depict the case of insufficient subcontractor capacity. Namely, if the maximum rate at which the subcontractor can supply goods is now sufficient to meet the demand when it is high, i.e., $\mu_H > \mu + \mu_c$, it is not possible to keep the backlog level at the lower hedging point S. In this case, S is a switching point. When the demand is high, the backlog decreases with rate $\mu_H - \mu$ above S and with a reduced rate of $\mu_H - \mu - \mu_c$ below S. This case is also analyzed by following a similar methodology.

In the cases, two subcontractors, one with sufficient capacity and another with insufficient capacity are considered. The additional production cost of the insufficient subcontractor is lower than the cost of the sufficient contractor. Both of these cases are also compared to the case no subcontractor is used. For illustrative purposes, the fixed cost of the options with the subcontractors is set to zero.

Figure 3 shows the effect of demand uncertainty on the cost and profit for these three cases. When the demand variability is low, all these cases yield very close results. As the demand uncertainty increases, the option both with the sufficient and insufficient subcontractors give better results than the case where no subcontractor is used. The results for the sufficient and insufficient subcontractors are very close to each other. The insufficient subcontractor yields better results due to its lower additional production cost when the demand uncertainty is high.
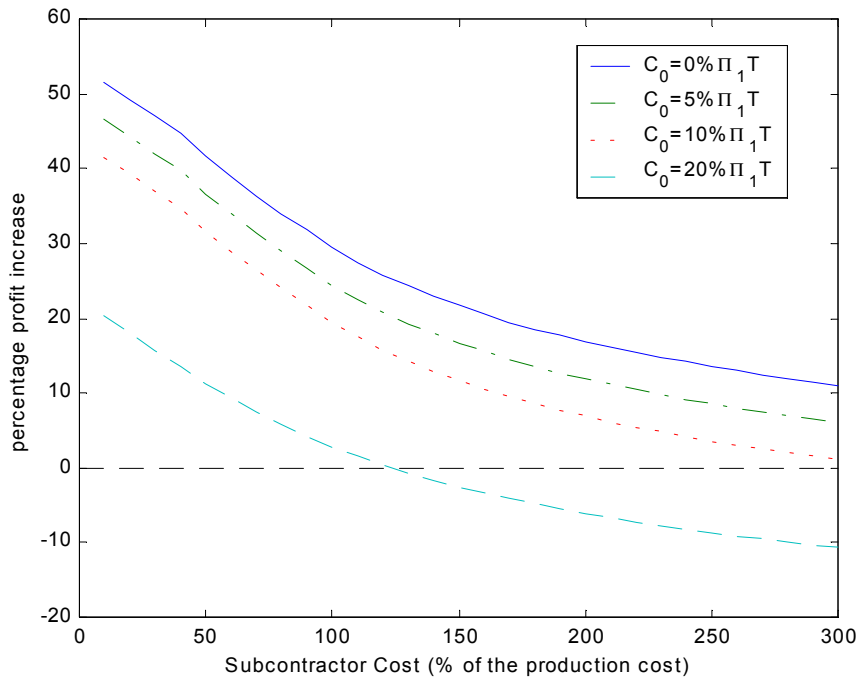
Figure 1. Evaluating the terms of the contract: fixed payment and additional production cost
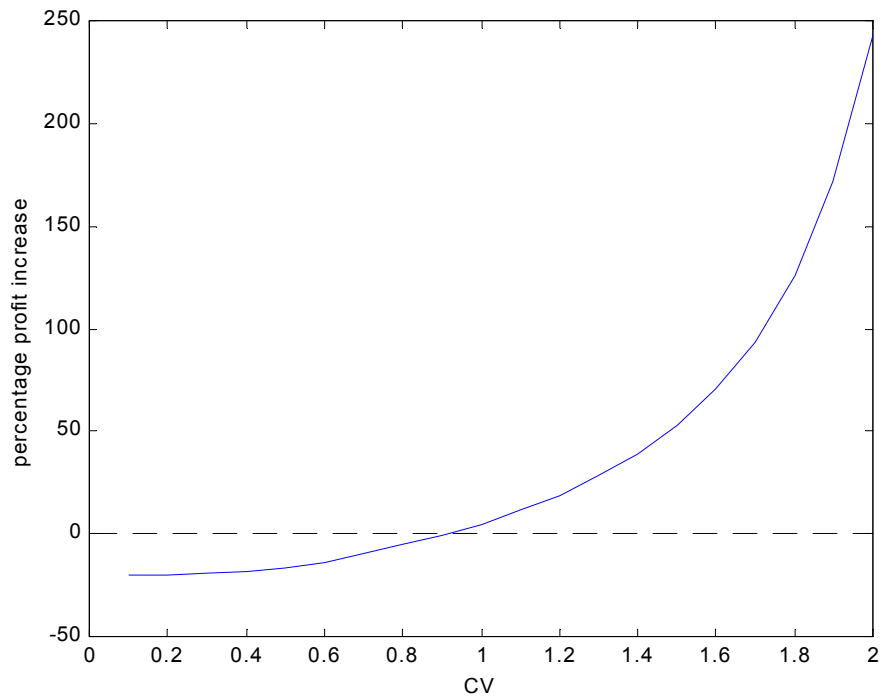

Figure 2. Effects of demand variability on the Additional Profit ($\mu = 1$, $\mu_H = 1.5$, $\mu_L = 0.8$, $\bar{d} = 0.9$, $cv=1$, $c^- = 0.3$, $c^+ = 0.1$, $L=5$, $A=2$, $C_0=20\%\Pi_1T$)
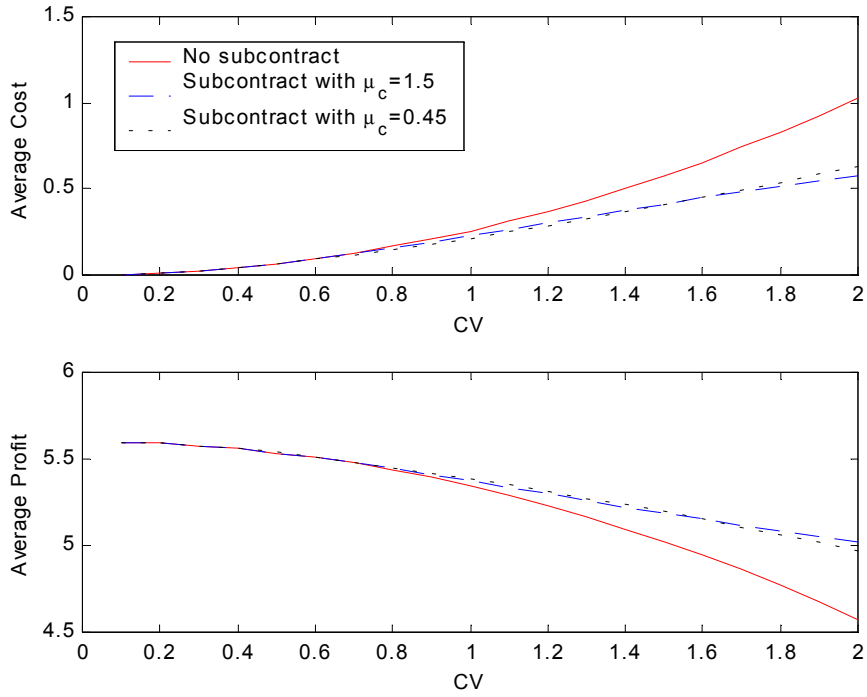
Figure 3. Effects of demand variability on the cost and the profit ($\mu = 1$, $\mu_H = 2.5$, $\mu_L = 0.5$, 7, $cv=0.8$, $c^- = 0.64$, $c^+ = 0.08$, $L=8$, $A=3$, $A'=6$, $\mu_c=1.5$, $\mu'_c=0.45$, $C_0=0$)
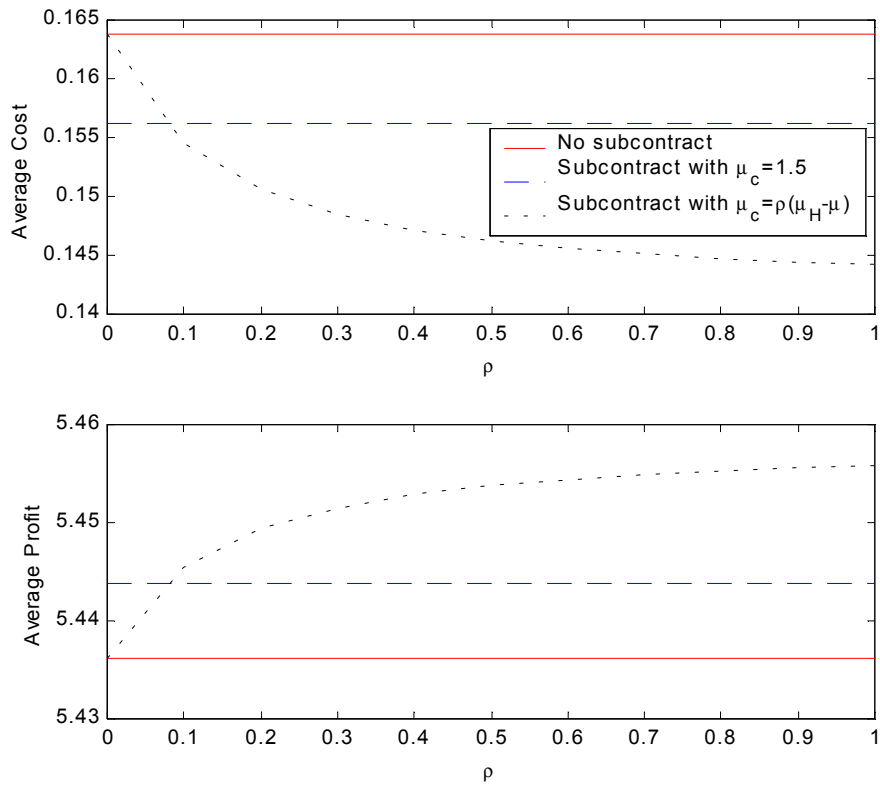

Figure 4. Effects of subcontractor capacity on the cost and the profit ($\mu = 1$, $\mu_H = 2.5$, $\mu_L = 0.5$, 7, $cv=0.8$, $c^- = 0.64$, $c^+ = 0.08$, $L=8$, $A=3$, $A'=6$, $\mu_c=1.5$, $\mu'_c= \rho(\mu_H - \mu)$, $C_0=0$)

Figures 4 shows the effects of the maximum capacity of the insufficient subcontractor on the profit and cost. The maximum production rate of the subcontractor is set to $\rho(\mu_H-\mu)$. When $\rho<1$, the subcontractor has insufficient capacity and when $\rho=1$, it has sufficient capacity to meet the demand when it is high. The additional production cost is the same for all cases. As the subcontractor can provide goods at a higher rate with the same additional production cost, the cost decreases and the profit increases. Furthermore, if the maximum capacity is lower than a specific level (where the curves for the insufficient and sufficient subcontractor cases intersect), it is more advantageous to use the subcontractor with higher capacity and higher additional production cost.

7. Concluding Remarks

The model considered in this study shows that option-type contractual agreements can be used as a strategy to cope with demand variability. The model shows that the value of this strategy increases as the demand uncertainty increases. This result supports the options view that uncertainty creates opportunity. Furthermore the additional costs of sourcing from a short-cycle manufacturer can be justified through increased sales and reduced finished goods inventories. However, this strategy is more advantageous for the producer. Subcontractors should have enough incentives to take part in such agreements. The upfront payment of the option may provide such an incentive. In this case, valuation of the option and deciding how to use this valuation in decision making are important questions that need to be answered.

The simplifying assumptions of the model make it harder to use the results immediately in a corporate setting to determine the price of an option in a daily operation. This requires a more detailed model of the demand, production schedule, etc. However a simulation model and a simulation-based optimization method, such as the ordinal optimization, can be used for this purpose. Even in this case, determining the critical parameters of the model, especially, demand uncertainty and the demand levels is a challenge.

In addition to analyzing a producer with a single subcontractor, the framework can be extended to include multiple subcontractors. Furthermore, the approach can be extended to investigate capacity expansion decisions of the producers and the investment decisions in short-cycle manufacturing. These extensions are left for future research.

Acknowledgements

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