COMBINING MAKE TO STOCK AND MAKE TO ORDER IN A MULTI-PRODUCT MANUFACTURING SYSTEM

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SUMMARY : *In this paper, we address the problem of optimal selection of a mechanism for production control in the case of a company producing multiple heterogeneous products at a shared manufacturing facility. We consider there are two families of products : the first class encompasses a few number of high demand rate products, while the second class corresponds to a large number of products with low demand rates. The problem is motivated by flexible manufacturing facilities which are simultaneously confronted to very different market segments. Demands for the products are independent Poisson processes with different arrival rate. The costs of managing the production-inventory system include inventory holding, while the service level is modeled via a specific constraint. We address the question of optimal selection of scheduling rules for the global production system : the classical First-in First Out (FIFO) rule or a preemptive priority (PP) rule, giving priority to the low volume products, each time coupled with the selection of the associate optimal production strategy (namely Make-to-Stock (MTS) or Make-to-Order (MTO)). We model the joint production-inventory control problem in the context of a simple multi-class queueing system to gain insight into how scheduling rules and optimal selection of MTO or MTS policies do affect the operating costs for the whole system.*

KEY-WORDS : *Make-to-Order (MTO); Make-to-Stock (MTS); Base-stock policy; Scheduling rule; First-In-First-Out; Preemptive Priority; Heterogeneous multi-product queueing network.*

1. INTRODUCTION

Competitive pressure and a need for continuous improvement led many firms to review and optimize their production-inventory practices, traditionally, divided in Make-to-Stock (MTS) systems, Make-to-Order (MTO) systems and Assemble-to-Order (ATO). The characteristics of these three production-inventory strategies can be summarized as follows :

- Make to stock : the items are produced in anticipation of demands and immediately available when the orders arrive. The critical issues relate to forecasting future demands and maintaining inventory levels that meet customer service goals. MTS strategy is efficient in case of high volume, standard and stable demands.
- Make to order : the items are produced and delivered in response to customer order. Note that our designation of MTO indicates stockless production and does not refer to an engineered-to-order system. Such an approach minimize the inventory level of finished products, but makes critical the time required to make to order. MTO corresponds typically to batch production, with low volume and fluctuating demands.
- Assemble to order (ATO) : in this case, items are produced in standard modules to which options are added according to customer specifications. Components are thus made-to-stock, and then assembled-to-order after the customer order has been received. This strategy permits to minimize the inventory level of standard components and simultaneously offer short delivery lead times for the finished products.

Until recently, it has been frequent that firms make all their products in accordance to a unique system (MTO, MTS or ATO). So, naturally, performance analysis and optimization of mechanisms for inventory-production coordination and control, in a given unique system has represented a significant area for research (see [1] and [2] for exhaustive references on MTO and MTS systems, and [8,9] for ATO systems). But more and more firms follow a middle ground where two (or three) inventory-production strategies are used simultaneously for different product families. For example, some items are made to stock while others are made to order. Such an approach seems appropriate as a production facility rarely produce a single item for sale to a unique customer. Typically, a manufacturing unit produce a variety of different items for a variety of market segments. In each market segment, the customers may have different requirements (in term of service level, demand rate, lead-time constraints, …). A classical example could be the case of a supplier whose products sales for a first market segment are regular high volume - low margin sales, but specifies large penalties if the products are not available when required, and for a second market segment, corresponding to highly irregular - high margin sales, with lower required service level. In such a case, it seems efficient to combine MTO and MTS strategies with specific priority rules.

Recent works have focused on performance evaluation of MTO/MTS systems as well as on the problem of how a manufacturing company can decide whether to operate in a MTS or MTO mode. In [7], fluid and diffusion approximations are developed for the performance of MTO/MTS system with FIFO service discipline. In [4], the framework of cyclic polling system with setups is used to address the question of how many products to make to order and how many to make to stock. In [1], the optimality of MTO versus MTS policies is studied, in the case of a company producing multiple heterogeneous products at a shared manufacturing facility. These authors derived explicit optimality conditions, as a function of the induced costs, distribution of manufacturing times and demand variety. In [3], the authors consider the problem of admission control and sequencing in a combined MTO/MTS system. As a conclusion, their study permits to get insight into the size of the annual quantity of orders to commit when signing a contract for MTS products, in the framework of a combined MTO/MTS system.

In the present paper, we consider a combined MTO/MTS inventory-production system, producing multiple heterogeneous products. These products correspond to two specific different product families. The first family is related to a small number of products with a high demand rate and is denoted as HV (High Volume). The second family encompasses a large number of finished goods with low demand rate and is denoted as LV (Low Volume). Classically, a MTS mode should be preferred for the family of high volume items, while a MTO system is more usual in the case of low volume items.

The other element that defines the production-inventory policy is the scheduling rule: if several products are waiting to be manufactured, which one should be produced first? The first rule to consider is the First In First Out (FIFO) rule, which is very common in practice due to its very simple implementation. However, some recent works suggest that other scheduling rules may outperform FIFO (see [11]). Along these lines, we consider the following priority rule: a preemptive priority (PP) rule is considered, where the highest priority is given to the LV product family. It means that if an order for a HV product is being served when a new order for a LV product enters the system, the current HV product is preempted (ejected back in the queue) and the LV order is produced in priority. Clearly, this PP rule results in lower lead-times for the LV products, but makes necessary higher inventory levels for HV products.

The purpose of the paper is to address the two issues in a combined way: 1) should the production of the HV and LV products be done in a MTO or MTS way; how should production capacity be allocated between the two product families. In particular, the following issues will be analyzed: is the PP rule more efficient than the FIFO rule with respect to the costs involved in the model; how can the optimal parameters of the associated inventory-production systems be computed and what is the link between the optimal strategy for each product, in the global multi-product framework, namely MTO and MTS, and both scheduling rules?

The rest of this paper is organized as follows. In section 2, we introduce the assumptions and formulation of the problem. In section 3, we describe optimization procedure for the parameters of the inventory-production systems under the two considered scheduling rules, namely FIFO and PP. In section 4, we apply these procedures to a numerical example and we exhibit some interesting properties. Section 5 concludes the paper.

2. THE PROBLEM

2.1. Assumptions

The main assumptions are as follows :

- We consider a single-stage production system, controlled by a classical Base Stock Control System (BSCS). The single-stage BSCS is a control mechanism that decides when to release raw parts into the manufacturing process, based on customer demands for finished products. Figure 1 shows the queueing network model this single-stage BSCS.

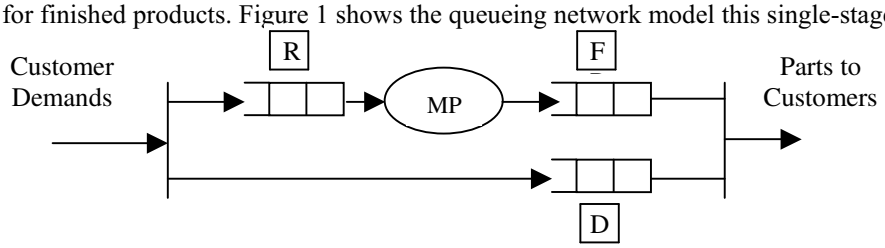


Figure 1: Queueing network model of a single-stage BSCS

Mp represents the considered manufacturing process. It contains parts that are receiving processing, which are referred to as work-in-process. Queue R contains the raw parts waiting for processing on MP. Queue F represents the output buffer of the system and contains parts that have completed processing and are ready for delivery to customers. Queue D contains demand for finished part delivery to the customers.

Initially, before any customer demands arrive to the system, F contains a base stock of *s* parts, R, MP and D are empty. The base stock s is the control parameter to optimize. When a customer demand arrives to the system, it is split into a demand for the delivery of a finished part to the customer and a demand for the production of a new part by MP. The first demand is stored in queue D, while the new raw part is stored in R. Queues F and D are linked into a synchronization station: if there is a customer demand in D and an available part in F, this part is immediately delivered to the

customer and the demand in D is satisfied. If there is no available part in F, the customer demand remains waiting in queue D, until the arrival of a new finished product in queue F. For an extensive description of such models, see [6].

We consider a model with k types of products. The first k_{HV} types (i.e. i=1, .., k_{HV}) correspond to high demand rates products, while the $k_{LV} = k - k_{HV}$ remaining types (i.e. i= k_{HV} + 1, …, k) are associated to low demand rate products.

- The arrivals of demands occur according to independent Poisson processes with rates λ_i for product type i (i = 1, .., k).
- The production rate of any type product are independent and exponentially distributed with parameter μ
- The buffers in the queueing network have unlimited capacity (which means also that the finished product inventories are unlimited)
- No setup is required to switch from producing one type of product to another.
- There is no failure mechanisms perturbating the manufacturing process

The present paper is deals with a situation corresponding to the following assumptions :

- We assume that the aggregate workloads for the HV family and for the LV family are the same. Namely, if we denote by ρ_{HV} the utilization factor of the manufacturing facility by HV products and ρ_{LV} the utilization factor of the manufacturing facility by LV products, we have

$$\rho_{HV} = \rho_{LV} \Leftrightarrow \sum_{i=1}^{k_{HV}} \frac{\lambda_i}{\mu} = \sum_{i=k_{HV}+1}^{k} \frac{\lambda_i}{\mu}$$

As by definition of the problem we have : $\lambda_j << \lambda_i$ for i corresponding to a HV product type and j to a LV product type, we also have $k_{LV} >> k_{HV}$ which implies that the number of HV products is much lower the number of LV products. Such a feature is realistic and can be associated to numerous firms which are confronted to two very different market segment. A first one concentrated on a few number of references which are consumed in high volume. This segment constitutes the basic activity of the firm. And then another segment encompassing a large number of specific low volumes intermittent demands. From an intuitive point of view, the case where $\rho_{HV} \sim \rho_{LV;}$ is the most interesting. As a matter of fact, if where $\rho_{HV} >> \rho_{LV}$, (resp. $\rho_{HV} << \rho_{LV}$) we will be mainly confronted to a MTS (resp. MTO) system and the underlying capacity sharing problem would be of minor interest.

- We assume that orders related to product types within the same product family (HV or LV) are scheduled according to the FIFO rule. On the other hand, orders related to different product family (HV or LV) are scheduled according to a preemptive priority (PP) rule, where the highest priority is given to the LV products family. It means that if an order for a HV product is being served when a new order for a LV product enters the system, the current HV is preempted (ejected back in the queue) and the LV order is produced in priority. The reasons inducing the choice of a PP rule are twofold. First, under FIFO and PP, it can be shown that the multi-product queueing problem can be decomposed into independent single-products problems. This decomposition appears to be the basis of the whole solving approach of this paper. On the other hand, the idea of the paper consists of comparing performances of scheduling rules as much different as possible, in order to get insight into the MTO/MTS mode selection problem.

2.2. Notations

We adopt the following notations :

μ : the average production rate of the manufacturing facility

ρ : the utilization factor of the manufacturing process

k : the number of product types, with k_{HV} the number of high volume product types and k_{LV} the number of low volume product types (with $k_{HV} + k_{LV} = k$)

Z : the average holding cost rate for the whole system

For each product type i, (i = 1,2…, k), we define :

s_i : the order upto level for product type i (i=1, .., k)

λ_i : the average demand rate for product type i

X_i : the random variable associated to the number of type i parts in the finished product inventory

N_i : the random variable associated to the number of type i parts in the system (i.e. in the inventory or in the work-in-process)

Y_i : the random variable associated to the number of backordered demands of type i products

D_i : the random variable associated to the backorder time for demand of type i products

h_i : the holding cost (per period and per item) for type i products

$\overline{x_i}$: the average number of type i parts in the finished product inventory

$\overline{y_i}$: the average number backordered demands of type i products

$\overline{d_i}$: the average backorder time for demand of type i products

2.3. Problem formulation

Numerous formulations are available for the general problem of optimizing inventory-production system with respect to cost rates, eventually under some additional service constraints. Some equivalence properties between these formulation are analyzed in [10]. In this paper, the optimization problem consists of finding the optimal values for the order upto levels for the LV products, i.e. (s_1^*, s_2^*,.....,s_{kHV}^*), which minimize the average holding cost of running the production-inventory, under the constraint that for each product type the average backorder time is lower than a specified critical value.

$$\begin{cases} \text{Min } Z(s_1,s_2,.....,s_k) = \sum_{i=1}^{k} h_i \ \overline{x_i}\ (s_1,s_2,.....,s_k) \\ \text{s.t. : } \overline{d_i} \le d_i^c \text{ , i = 1, 2……k} \\ \quad s_i \ge 0, \text{ , i = 1, 2……k} \end{cases}$$

3. THE CONSIDERED SOLVING APPROACH

In order to compare the performances associated to the two considered scheduling rules, FIFO and PP, the optimal parameters (s_1^*, s_2^*,.....,s_k^*) have to be computed and then the corresponding average cost $Z^*(s_1^*, s_2^*,.....,s_k^*)$ is to be evaluated. In sake of simplicity, instead of representing each cost, we will present the difference between the costs induced by the scheduling rules, via the gain function defined as :

$$G\% = \frac{Z_{FIFO}^* - Z_{PRE}^*}{Z_{FIFO}^*} * 100$$

with Z_{FIFO}^* the optimal cost for the FIFO rule and Z_{PRE}^* the optimal cost for the PP rule.

3.1. Analysis of the production-inventory model under the *FIFO* scheduling rule

Buzacott and Shanthikumar, in [2], have analyzed the FIFO scheduling rule in the present framework. They have proved that this multi-product problem can de decomposed into independent single-product models. Via these results our FIFO problem can be reformulated as :

$$\begin{cases} \text{Min } Z(s_1,s_2,.....,s_k) = \sum_{i=1}^{k} h_i \ \overline{x_i}\ (s_1,s_2,.....,s_k) \\ \text{s.t. : } \overline{d_i} \le d_i^c \text{ , i = 1,2……k} \\ \quad s_i \ge 0, \text{ , i = 1,2……k} \end{cases} \Longleftrightarrow \begin{cases} \text{Min } Z_i(s_i) = h_i \ \overline{x_i}\ (s_i) \\ \text{s.t. : } \overline{d_i} \le d_i^c \\ \quad s_i \ge 0 \\ \text{for i = 1, 2……k} \end{cases}$$

With specific parameters for each subproblem. Each subproblem can be solved analytically, which is a very appealing feature of this decomposition. In particular, expressions for the stationary probability distributions of the random variables X_i, Y_i and N_i.are given in [2]. From these results, $\overline{x_i}(s_i)$ and $\overline{y_i}(s_i)$ can be analytically reformulated as:

$$\begin{cases} \overline{x_i} = s_i - \dfrac{\hat{\rho}_i}{1-\hat{\rho}_i}\left(1-\hat{\rho}_i^{s_i}\right) \\ \overline{y_i} = \dfrac{\hat{\rho}_i^{s_i+1}}{1-\hat{\rho}_i} \end{cases}$$

It is proven in [5], that for each subproblem, the optimal order upto level s_i* (i = 1,2…, k) can be analytically characterized by the following expression :

$$s_i^* = \text{MAX}\left(\left\lceil \frac{\ln\left(\lambda_i d_i^c (1-\hat{\rho}_i)\right)}{\ln \hat{\rho}_i} - 1 \right\rceil, 0\right)$$

It is easily seen that one has, for i = 1,2…, k ,

$$\overline{x_i} = s_i^* - \frac{\hat{\rho}_i}{1-\hat{\rho}_i}\left(1-\hat{\rho}_i^{s_i^*}\right)$$

in such a way that the optimal cost for each product type can be expressed as :

$$Z_i(s_i^*) = h_i\left(s_i^* - \frac{\hat{\rho}_i}{1-\hat{\rho}_i}\left(1-\hat{\rho}_i^{s_i^*}\right)\right)$$

The optimal aggregate average cost rate for the whole system, under a FIFO scheduling rule, is then given by

$$Z_{FIFO}^* = \sum_{i=1}^{k} Z_i(s_i^*) .$$

3.2. Analysis of the production-inventory model under the *PP* scheduling rule

In the case of a preemptive priority rule, there is no known analytical expression for the aggregate optimal average cost Z_{PRE}^*. We had thus recourse to an optimization-by-simulation approach to estimate the optimal order upto levels and the associate optimal costs.

It can be shown that the problem exhibits a particular structure, which permits to find these values from a single simulation with arbitrarily chosen initial values for (s_1, $s_{2,\ldots,}$s). As matter of fact, consider for a given product type the value $\Pr\{X_i = j \mid s_i = s\}$ which is the probability that the number of type i parts in the finished product inventory is equal to j, knowing that the order upto level for product i is equal to *s*. Let's define also $\Pr\{Y_i = j \mid s_i = s\}$ the probability that the number of backordered demands of type i products is equal to j, with an order upto level for product i equal to *s*.

Via a sample path analysis, it is proved in [5] that the multi-product problem can be decomposed into independent single-product problems and compute the optimal order upto level for each product independently. From a computational point of view this property is very interesting as the optimal value for the order upto level s* can be evaluation from the probability distribution estimated via a single simulation run. The single part optimal cost is then given by

$$Z_i(s_i^*) = h_i\ \overline{x_i}(s_i^*)$$

And the aggregate cost is expressed as:

$$Z_{PRE} = \sum_{i=1}^{k} Z_i(s_i^*) .$$

The knowledge, for a given value of the parameter $\underline{s}$, of the probability distributions,

$\Pr\{X_i = j \mid s_i = \underline{s}\}$ for j = 0, 1, 2, …and $\Pr\{Y_i = j \mid s_i = \underline{s}\}$ for j = 0, 1, 2, …

permits to evaluate the average on hand inventory $\bar{x}(s)$ and the average backorder $\bar{y}(s)$ for any other value s of the order upto level. The exact formulas are the following :

$$\bar{x}(\tilde{s}+a) = \left[\nabla(a).\Pr\{X=0, Y=0 \mid s=\tilde{s}\}\right] + \sum_{i=-\infty}^{-1}\nabla(i+a)\Pr\{Y=i \mid s=\tilde{s}\} + \sum_{i=1}^{+\infty}\nabla(i+a)\Pr\{X=i \mid s=\tilde{s}\}$$

$$y(\tilde{s}+a) = \left[\nabla(-a)\Pr\{X=0, Y=0 \mid s=\tilde{s}\}\right] + \sum_{i=-\infty}^{-1}\nabla(i-a)\Pr\{X=i \mid s=\tilde{s}\} + \sum_{i=1}^{+\infty}\nabla(i-a)\Pr\{Y=i \mid s=\tilde{s}\}$$

with $\nabla(x) = x$ for $x > 0$ and $\nabla(x) = 0$ otherwise.

Via Little law, we also directly have : $\bar{d}(s) = \frac{1}{\lambda}\bar{y}(s)$.

4. NUMERICAL STUDY

In this section, we apply the method proposed in this paper to numerical examples. The simulations results were obtained using the Arena simulation package.

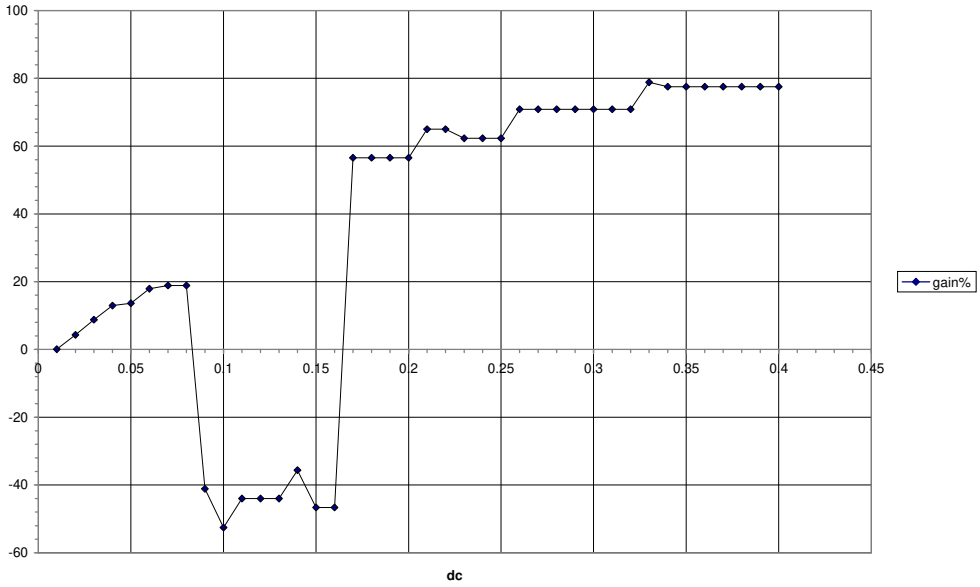
4.1. Data

We present here an example of combined MTO/MTS inventory-production mode with 11 products types (i.e. k = 11), corresponding to a single high volume product (k_{HV} = 1) and 10 low volume products (k_{LV} = 10). As an average production rate of the manufacturing facility, we choose μ = 10. The average demand arrival rates are chosen as λ_1 = 4, for the high volume demand and λ_i=0.4 , i =2, 3...11, for the low volume demands. This corresponds to a utilization factor of the manufacturing process given by ρ = 0.8, with $\rho_{HV} = \rho_{LV}$ = 0.4. The holding cost rate is given by h_i = 1, i=1,…,11.

4.2. Numerical results

4.2.1. Structure of the gain curve

The following figure depicts the gain G versus the critical lead-time parameter d^c, assumed to be identical for all the product types, used in the service level constraint.



We recall that following the definition of the gain function, when the gain is positive, it means that the PP scheduling rule is more efficient that the FIFO priority rule. A negative gain is then associated with the opposite. This simple example exhibits the complexity of the optimal strategy and shows that the PP rule does not systematically overtake the simple FIFO rule for any parameter in the problem. However, it worth noting that the gain magnitude can be significant :from -50% to +80% in the considered example. Now, we have to get insight in this phenomenon.

4.2.2. Optimal order upto levels

Figure 3 below exhibits the values of the optimal order upto levels $s^*_{HV_FIFO}$, $s^*_{LV_FIFO}$, $s^*_{HV_PRE}$ and $s^*_{LV_PRE}$, associated to the two products types HV and LV and to the scheduling rules FIFO and PP, versus the critical lead-time parameter d^c, assumed to be identical for all the product types, used in the service level constraint.

First, we can remark that

$$s^*_{HV_FIFO} \leq s^*_{HV_PRE}$$

i.e. the optimal high volume product type order upto level is lower with the FIFO rule than with the PP rule. This is in coherence with the fact that the switching from FIFO to PP increase the variance (or standard deviation) of the characteristic random variables associated with the HV family (namely X_1, N_1 and D_1). Such an increase of the variance is well known to induce an increase of the associate optimal order upto level (see [11]). This complementary effect applies to the LV product types. In this case, the PP policy reduces variances of the associated random variables; X_i, N_i and D_i, for i = 2, .., k, decreasing the optimal order upto levels, in such a way that

$$s^*_{LV_FIFO} \geq s^*_{LV_PRE}$$

An interesting issue is now the whole system performance evolution when switching from FIFO rule to PP rule (in the above described framework). A priori, no straightforward result could be expected due to the non-linear relationship between critical distribution variances and associate optimal order upto levels (see [11]), combined with the discrete value phenomenon potentially associated with the LV class.

As a matter of fact, let's first consider the particular value

$$d^c = 0.35$$

In this case, we can see in figure 3 that the switching from FIFO to PP induces a decrease of one unit of the optimal order upto levels for each LV products, i.e.

$$s^*_{LV_FIFO} = 1,\ s^*_{LV_PRE} = 0.$$

As in the here considered example, we have 10 LV products, the global decrease amounts to 10 units. In the same time, switching from FIFO to PP, increases the optimal order upto level for the HV class by 3 units, i.e.

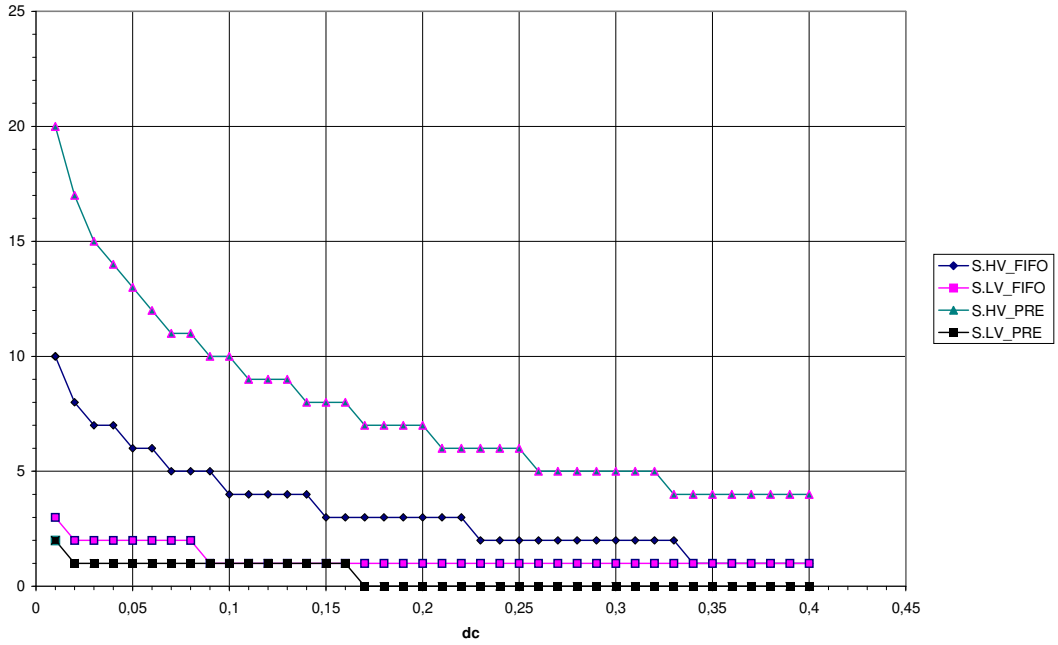
$$s^*_{HV_FIFO} = 1,\ s^*_{HV_PRE} = 4.$$

In such a way that for the whole system, the resulting gain is positive.

If we consider now the value $d^c = 0.12$,

we observe that the priority rule switching has no effect on the optimal order upto levels for the LV products, i.e.

$$s^*_{LV_FIFO} = s^*_{LV_PRE} = 1,$$



while it has increased the optimal order upto level of the HV class, as

$$s^*_{HV_FIFO} = 4,\ s^*_{HV_PRE} = 9.$$

In this case, for the whole system, the resulting gain is negative. Again, the explanation lies into the combination of the nonlinearities involved in the link between the underlying probability distributions and the fact that the level must be integer. We could expect that by relaxing this lasrt constraint, we would $s_{LV_FIFO} > s_{LV_PRE}$ for any value of d^c.

4.2.3. Impact of the service constraint

An analysis of the evolution of the optimal order upto levels versus the required bound on the delivery lead-time, d^c, provides the following insights.

When the average delivery lead-time is strongly constrained, i.e. for low values of d^c, the optimal order upto levels are strictly positive, for any product type, in such way that we have

$$s^*_{HV\text{-}FIFO} > 0,\ s^*_{HV\text{-}PRE} > 0,\ s^*_{LV\text{-}FIFO} > 0 \text{ et } s^*_{LV\text{-}PRE} > 0$$

When increasing the d^c values, we first observe that $s^*_{LV\text{-}PRE} = 0$, while the other order upto levels remains positive. Then, at the following stage, one has $s^*_{HV\text{-}FIFO} > 0$, $s^*_{HV\text{-}PRE} > 0$, $s^*_{LV\text{-}FIFO} > 0$. Then, for higher values of d^c, one finally finds

$$s^*_{HV\text{-}FIFO} = s^*_{HV\text{-}PRE} = s^*_{LV\text{-}FIFO} = s^*_{LV\text{-}PRE} = 0$$

As matter of fact, let's denote by $d^c_{0,HV\text{-}FIFO}$, $d^c_{0,HV\text{-}PRE}$, $d^c_{0,LV\text{-}FIFO}$ and $d^c_{0,LV\text{-}PRE}$ the "switching to zero "d^c values for $s^*_{HV\text{-}FIFO}$, $s^*_{HV\text{-}PRE}$, $s^*_{LV\text{-}FIFO}$ and $s^*_{LV\text{-}PRE}$.

It is proved in [5] that

$$d^c_{0,LV\text{-}PRE} < d^c_{0,LV\text{-}FIFO} \text{ and } d^c_{0,LV\text{-}FIFO} = d^c_{0,HV\text{-}FIFO}$$

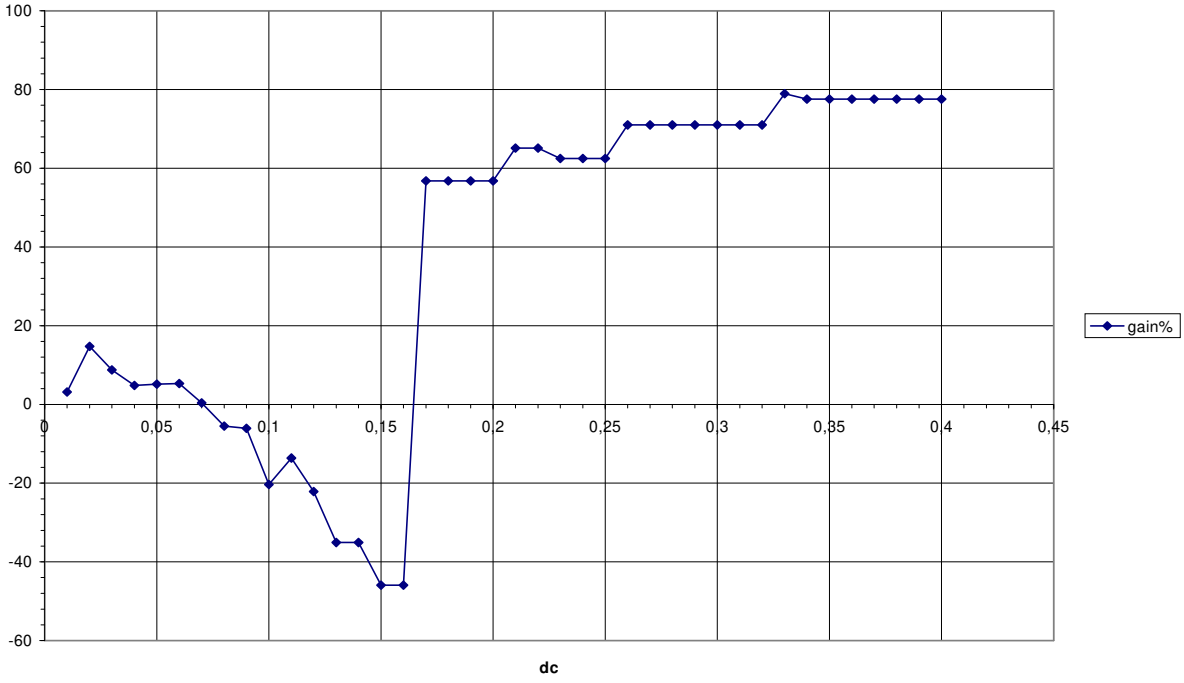
The effect of the service constraint (w.r.t. the d^c values) can be summarized in the following table

0 → d^c → ∞			
Range 1	Range 2	Range 3	Range 4
$s^*_{LV\text{-}PRE} > 0$ $s^*_{LV\text{-}FIFO} > 0$ $s^*_{HV\text{-}FIFO} > 0$ $s^*_{HV\text{-}PRE} > 0$	$s^*_{LV\text{-}PRE} = 0$ $s^*_{LV\text{-}FIFO} > 0$ $s^*_{HV\text{-}FIFO} > 0$ $s^*_{HV\text{-}PRE} > 0$	$s^*_{LV\text{-}PRE} = 0$ $s^*_{LV\text{-}FIFO} = 0$ $s^*_{HV\text{-}FIFO} = 0$ $s^*_{HV\text{-}PRE} > 0$	$s^*_{LV\text{-}PRE} = 0$ $s^*_{LV\text{-}FIFO} = 0$ $s^*_{HV\text{-}FIFO} = 0$ $s^*_{HV\text{-}PRE} = 0$
MTS	MTS → MTO	MTS → MTO $G < 0$	MTO

Range 1 (and respectively Range 4) corresponds to unique inventory-production systems : MTS (and resp. MTO), independently of the scheduling policy.
In range 3, the optimal cost associated to the FIFO policy is equal to 0 (as the optimal order upto levels are 0), while this cost is strictly positive with the PP rule (as $s^*_{HV\text{-}PRE} > 0$). The gain G is thus strictly negative in this range and the FIFO rule outperform the PP rule. From an intuitive point of view, it seems straightforward that it is useless to give more priority to the LV products, via a PP rule, while one already has $s^*_{LV\text{-}FIFO} = 0$.
Range 2 is the most interesting case : in this range of d^c values, it is necessary to keep inventories for the LV products when using the FIFO rule, while one has $s^*_{LV\text{-}PRE} = 0$, when switching to the PP rule. In this cased, the gain G appears to be significantly positive.

4.2.4. LV arrival rates dispersion effect

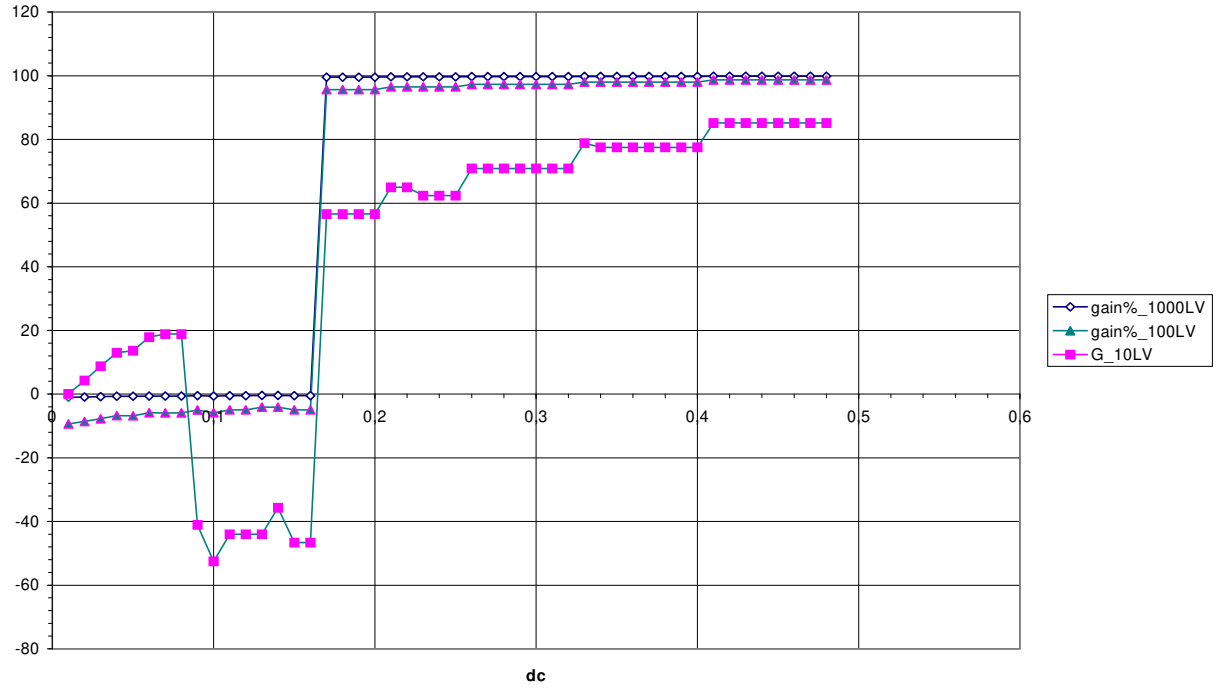
The discontinuities in the gain curve corresponds to the discrete jumps of the order upto levels s*. If the parameters are the same for several product types, these jumps occur simultaneously, producing an important discontinuity in the aggregate gain function. This phenomenon can be seen in figure 3, where for every LV product one has $\lambda_i = 0.4$. In order to exhibit this "discontinuity effect", we consider another example with the following data :
$\lambda 1 = 4$, $\lambda 2 = 0.1$, $\lambda 3 = 0.166$, $\lambda 4 = 0.233$, $\lambda 5 = 0.3$, $\lambda 6 = 0.366$, $\lambda 7 = 0.433$, $\lambda 8 = 0.5$, $\lambda 9 = 0.566$, $\lambda 10 = 0.633$ and $\lambda 11 = 0.7$. We can notice that the LV average arrival rate is still equal to 0.4.



In this case, the unique important jump of the gain function correspond to the switch from range 2 to range 3. Furthermore, it is worth to note that the value of the gain in range 3 is greater or equal to the gain in the previous example : this range (for which $s^*_{HV\text{-}FIFO} > 0$, $s^*_{HV\text{-}PRE} > 0$, $s^*_{LV\text{-}FIFO} > 0$, $s^*_{LV\text{-}PRE} = 0$) is independent from the dispersion of the arrival rates.

4.2.5. Influence of the number of LV classes

Let us consider again the situation where all LV classes have the same demand rates. We are interested in seeing the effect of an increase of the number of LV classes with respect to the number of HV classes. We again assume that there is a single HV class and consider three different cases for the number of LV classes: 10, 100 and 1000. The following graph shows the gain for these three situations.



5. CONCLUSION

This paper analyzes and compares performances of two scheduling rules (First-In-First-Out and Preemptive preemption) in the framework of a finite capacity production-inventory system combining Make-to-order and Make-to-stock products. Under the considered assumptions, such multi-products systems can be decomposed into single-product models. These models are then analyzed and optimized, analytically in the case of FIFO scheduling rule and by a simulation approach in the PP case.

We deduce, as a function of the problem parameters, optimal the optimal pairs, i.e. the optimal control rule (MTO or MTS) and the associate optimal scheduling rule (FIFO or PP). In each case, the optimal order upto levels and the corresponding holding costs are computed.

We present numerical examples, which shows that type structure of the optimal pairs is not straightforward : the switching from FIFO to PP can have opposite effects on the performance criterion, as a function of the parameters of the problem. It is worth noting that if the sign of this effect is a complex issue, the size of the consequence of such a switching is significant.

It is important to recall that numerous assumptions support the presented results. At first, no inventory fixed cost, associated to a non-zero inventory, is included in the cost criterion. Such costs could be expected to reinforce optimality of the PP scheduling rule, in the sense that this rule tends to render optimal MTO production-inventory mode for the LV products, avoiding by the way such fixed non-zero inventory costs.

At the other hand, we consider a perfectly flexible manufacturing system : there are no setup costs or changeover times (and hence no need for long production runs to exploit some economies of scale mechanism), nor notion of admissible lead-time for the customers (i.e. a lead-time delivery which induces no penalty costs). Effects of these features, as well as analytical analysis of the basic phenomena described in this paper are still subjects of ongoing research.

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