INTEGRATING ADVANCE INFORMATION IN PULL TYPE CONTROL MECHANISMS FOR MULTI-STAGE PRODUCTION

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Abstract

We propose a mechanism for production coordination and control for a multi-stage make-to-stock system which incorporates information on future customer orders. We start from the well known multi-stage base stock system and propose an extension which schedules part release using lead time parameters. To analyze the performance of the proposed system as well as to gain insight into how future demand information affects production control mechanisms, we introduce a single stage analytical model. We provide numerical examples that verify the effectiveness of the proposed mechanism in the single-stage case.

1 Introduction

There has been considerable interest in the development and implementation of pull type production control mechanisms in the recent years. A pull mechanism typically reacts to actual occurrences of demand and uses actual demand information to coordinate production. Although in general a pull control mechanism is considered demand driven, there is no reason why other factors cannot be additional drivers of such a mechanism. We propose here a pull control mechanism driven by future demand information as well as actual demands.

In many interesting practical cases, there is some information available on future demands through advance orders and forecasts. In general, the information is in the form of a mixture of forecasts and firm orders. To facilitate the exact description of the control mechanism, we limit ourselves to the case where the information available is a firm order whose due date is known with certitude. While this restriction is certainly important, this type of advance information structure has become more relevant due to increasing supplier-customer coordination. It is also a convenient way of modeling regular schedules coming from an external material requirement planning system.

Our proposed mechanism is based on a well known multi-stage pull type control mechanism: the base stock. We define the new mechanism by adding a timing mechanism on top of the classical base stock system. The resulting system hence borrows the idea of safety stocks from the base stock system but profits from early information as well through the timing mechanism.

Comparing the performance of different control mechanisms is usually an extremely challenging task even in the absence of early information. To carry out this task, we focus on a special case: the single stage production system. Analyzing this special case, we are nevertheless able to obtain insights on the performance of control mechanisms that use advance information in different ways.

Integrating order information in production and inventory control mechanisms has been studied from several different perspectives in the recent years. Hariharan and Zipkin [7] study the effects of incorporating due date information in inventory control policies. Their results are highly intuitive, early information reduces inventory control costs and this reduction becomes more important as future uncertainty is resolved earlier. Axsater and Rosling [1], [2] show that incorporating a timing mechanism into classical multi-stage inventory control systems (such as installation stock and echelon stock control), one can obtain a new class of control policies that actually dominate the performance of classical systems.

Our perspective is similar to that of the multi-stage PAC (production authorization card) systems described in detail by Buzacott and Shanthikumar [3]. In this framework, the manufacturing stages have limited production capacity and hence the multi-stage production system is modeled as a network of queues. The PAC framework also includes a mechanism for delaying the release of orders. We will discuss the similarities and differences of the PAC system with the proposed system in detail when we define the proposed mechanism.

Buzacott and Shanthikumar [5], analyze an analytical model of a single stage production inventory system with advance demand information. Their model is a make-to-stock queue in continuous time whose analysis provides many interesting insights. The analytical model that we use for the comparing different mechanisms is also a make-to-stock queue but one which evolves in discrete time. The discrete framework enables us not only to provide numerical performance measures for a given mechanism but also to discuss *optimality* issues since it fits conveniently in an optimal control framework.

In section 2 we describe the problem and the modeling approach in detail. Section 3 presents the proposed mechanism, starting from the classical base stock mechanism for multi- stage systems. In section 4, an analytical model for a single stage system with advance information is introduced. Section 5 presents the numerical results and comparisons based on the analytical model. Our conclusions and paths for future research are presented in section 6.

2 Problem

We consider a multi-stage production system, consisting of production stages in series connected with buffers. The system operates in a make-to-stock setting, the output of the last stage of production is placed in the finished goods inventory. Production resources in each stage are capacitated, one can think of machines with a finite processing capacity. (Although the models are valid for uncapacitated production as well, one of the main points of focus is the treatment of production lead times that depend on the load in process at the stage). We assume that setup costs and setup times are negligible so that production lot sizing issues can be avoided.

The usual underlying assumption in a make-to-stock system is that there is no advance information about future demands. This means that demands for finished products are expected to be filled with no delay at the time of their arrivals. In other words, due dates are equal to the arrival instants of the corresponding customer orders. On the other hand, in make-to-stock systems, the only information that may be given is statistical information about the demand process. In the framework considered here, the time at which a demand is claimed may be later than the arrival time of the order. Consider d_n, the nth demand arriving to the system. We associate with d_n a vector of information (t_n, q_n, δ_n). t_n denotes the time of arrival of the order, q_n is the required quantity and $\delta_n \geq t_n$ is the required due date of the order. It is useful define $l_n = \delta_n - t_n$ for each demand. l_n is the difference between the due date and the arrival date of an order. As in Hariharan and Zipkin [7] we call l_n the *demand lead time* of the nth demand.

We assume that orders that are not fulfilled at their required due dates are backlogged (for batch orders, we may assume that the orders may be partially fulfilled and that the unsatisfied part of an order is backlogged, but the order fulfillment policy is not essential to the description). The production control problem is to find a part release policy (i.e. to decide when to release parts form one stage of production to the subsequent stage) to minimize the total inventory and backorder costs. Since the exact solution of this problem is too complicated, our main interest in this paper is to develop a reasonable control mechanism that provides a good approximate solution.

3 The Proposed Mechanism

In this section, we define the proposed mechanism that integrates advance information on demand to the well-known base stock mechanism. Our vision of the base stock mechanism is the one presented in Dallery and Liberopoulos [6]. This vision emphasizes the demand transmission aspect of base stock and is slightly different than the classical representation of base stock (but the dynamics under the two representations are completely equivalent.) As in [6], the base stock mechanism leads to a queueing network representation with certain synchronization restrictions. Figure 1 displays this representation of a two stage Base Stock control system.

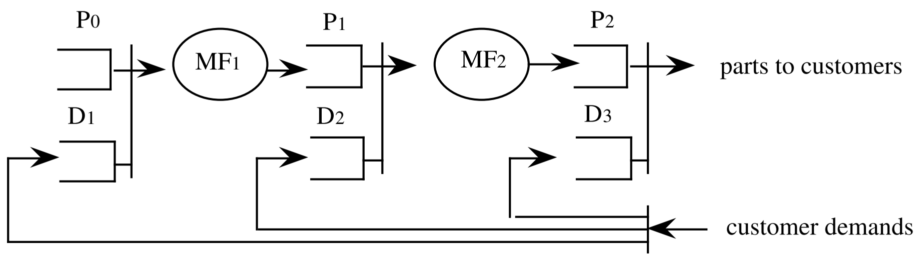

Figure 1: The multi-stage base stock system

In figure 1, the buffers P_1 and P_2 correspond to the outputs of stages 1 and 2 respectively. P_0 is the raw parts buffer. Buffers D_i correspond to demands that request parts. The multi-stage base stock mechanism functions in the following manner: Initially there are S_i parts (which have been processed by stage i) in buffer P_i ($i = 1, 2$), when a demand arrives it is immediately transmitted to all demand buffers. The arrival of demand to buffer D_i triggers the release of parts from P_{i-1} to the ith manufacturing stage if there are parts available in

P_{i-1}, otherwise the demand waits in buffer D_i for the arrival of parts to buffer P_{i-1} for a release into the manufacturing stage.

To incorporate the advance information on future demands in the base stock policy, associate with each stage of production a lead time parameter L_i as well as a base stock level S_i. The parameter L_i is, intuitively, related to the average waiting (including production) time in stage i and is a parameter of the mechanism to be determined. Consider the last stage of production, I at the instance of arrival of the nth demand d_n to the system. The parameter L_I now plays the role of timing the release of parts into stage I. Ideally a demand arrives with a sufficient demand lead time, i.e. $l_n \geq L_I$. Since releasing a part too early in the production stage is costly, we delay this release by $l_n - L_I$ units of time (so that the part is released exactly L_I units of time in advance). On the other hand, it may also be the case that the demand lead time of the part is shorter than its production lead time (i.e $l_n < L_I$). In this case, the part is released immediately into stage I without a further delay. The timing of the part release into stage I is entirely determined by the respective values of L_I and l_n. The requested release time of a part to the stage is, hence, $t_n + \max(0, l_n - L_I)$. If, furthermore, at this instance there is a finished part of stage $I-1$ available, the release time of this part to stage I is exactly $t_n + \max(0, l_n - L_I)$.

To complete the description of the multi-stage mechanism, note that the requested release time at stage I corresponding to the nth demand, $t_n + \max(0, l_n - L_I)$ is precisely the instance the preceding production stage $I-1$ should have the nth part ready. However, to have the part ready, without using its safety stock, at $t_n + \max(0, l_n - L_I)$, it should be released to stage $I-1$ L_2 units of time in advance of this time whenever possible. It is reasonable then that if $l_n \geq L_I + L_{I-1}$, a part should be requested for release to the stage $I-1$ at time at $t_n + l_n - L_I - L_{I-1}$ and if if $l_n < L_I + L_{I-1}$, a part should be requested for release to stage $I-1$ at time t_n. Thus, the request tine at stage $I-1$ should be $t_n + \max(0, l_n - (L_I + L_{I-1}))$.

Figure 2 represents an implementation of the proposed policy for a two stage system. Initially, there are S_1 and S_2 parts in the buffers P_1 and P_2 respectively while all other buffers are empty (except for P_0 that contains an infinite number of raw parts). As in the single stage system the nth demand joins the buffer at time $t_n + l_n$. The demand signal is transmitted to the buffer D_i with a delay of $l_n - \sum_{k=i}^{I} L_i$ if the required production lead time $l_n \geq \sum_{k=i}^{n} L_i$, whereas if $l_n < \sum_{k=i}^{n} L_i$ the demand is transmitted instantaneously.

The proposed mechanism is entirely described by two parameters, L_i and S_i for each stage i. Note that, when all demand lead times are zero (i.e $t_n = \delta_n$, for all n), the mechanism reduces to the classical base stock mechanism described by the safety stocks S_i

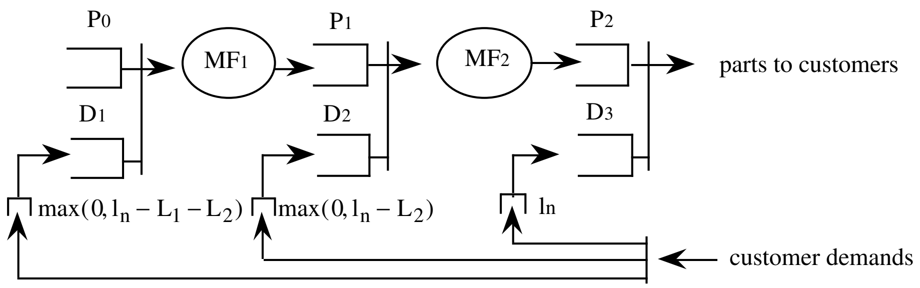

Figure 2: The multi-stage base stock system with lead time parameters

for stage i.

In the special case where all demand lead times are constant (i.e. $l_n = l$), the above policy reduces to the MRP interpretation of the PAC system as described in detail by Buzacott and Shanthikumar [3]. In the PAC system arriving demand signals are delayed by an amount of τ_i (where $\tau_i \geq 0$) at stage i. Recall that in our system, demand signals are delayed by an amount of $\max\{l_n - \sum_{k=i}^{n} L_i, 0\}$ for stage i. When $l_n = l$, the PAC delay parameters can be obtained by the relation $\tau_i = \max\{l - \sum_{k=i}^{n} L_i, 0\}$.

4 An Analytical Model

In this section, we introduce an analytical model that enables us to analyze some trade-offs between different control mechanisms when advance information is available. For tractability of analysis, we model a single stage production system operating in discrete time and assume unit production and demand quantities. We would like to extract two interesting trade-offs from the analysis of this system. We would like to, first, quantify the value of advance information on future demands and second, compare the performance of the proposed mechanism to the best mechanism possible within the same information availability.

Let $X(t)$ $(t = 1, 2, ..)$ denote the finished inventory level (where negative values of $X(t)$ represent backlogs. At each period, the probability of an order arrival is q (note that orders arrive in single units) and the probability of a production completion is p (whenever the facility is allowed to produce). We assume that production decisions are taken before the demand arrival in the same period and that each order spends at least one unit of time in the production stage. At time t we assume that we have access to the demand information from period $t+1$ through period $t+H$. This information will be summarized by the vector $D(t) = (D_1(t), D_2(t), ..., D_H(t))$ where an element $D_h(t)$ of the vector gives the number of demands that will claim finished items at time $t + h$. We will further assume that all demand lead times equal H to simplify the transition structure. With this assumption:

$$D(t+1) = \begin{cases} (D_2(t), D_3(t), ..., D_H(t), 1) & \text{with probability } q \\ (D_2(t), D_3(t), ..., D_H(t), 0) & \text{with probability } 1-q \end{cases}$$

Furthermore, when the facility is producing at time t

$$X(t+1) = \begin{cases} X(t) - D_1(t) + 1 & \text{with probability } p \\ X(t) - D_1(t) & \text{with probability } 1-p \end{cases}$$

Within this markovian framework, we analyze three systems: first a system that does not use any advance information (the vector $D(t)$), a second system that uses the proposed mechanism, and finally a system using advance information in an *optimal* way. Each system will be analyzed in one of the following subsections.

4.1 The system that does not use advance information

The analysis of the system that does not use advance information is parallel to that of the continuous version of the single product make-to-stock queue. Note that the optimality equation for the dynamic programming formulation which minimizes expected holding and backorders costs per unit time is given by:

$$\begin{aligned} V(x) + g &= c(x) + \min\{q(1-p)V(x-1) + p(1-q)V(x+1) \\ &\quad +(pq + (1-p)(1-q))V(x), qV(x-1) + (1-q)V(x)\} \end{aligned}$$

where x denotes the inventory level at the beginning of the period and $c(x) = hx$ when $x \geq 0$ and $c(x) = -bx$ when $x < 0$.

A closer examination of the of the above equation reveals that the optimal production policy is of the base stock type. The facility should produce when the inventory level is below the base stock level S and should stop otherwise. Under a base stock policy with base stock level S (≥ 0), the process $S - X$ is a Geometric/Geometric/1 queue in discrete time. With the assumption of late arrivals, we obtain (see Pujolle, Claude and Seret [10] for example):

$$\begin{aligned} f(S) &= 1 - (q/p) \\ f(i) &= \left[1 - \tfrac{q}{p}\right] \tfrac{1}{1-p} \left[\tfrac{q(1-p)}{(1-q)p}\right]^{(S-i)} \quad i \leq S-1 \end{aligned}$$

Defining the constants:

$$\rho = \frac{q(1-p)}{(1-q)p} \text{ and } \kappa = \left[1 - \frac{q}{p}\right] \frac{1}{1-p}$$

we obtain the optimal base stock level as:

$$S^* = \left\lfloor \frac{\ln\left[\frac{h}{(h+b)} \frac{(1-\rho)}{\kappa}\right]}{\ln[\rho]} \right\rfloor$$

4.2 Analysis of the Proposed Mechanism

The single stage model assumes that, at time t, complete demand information is available from period t through period $t+H$. Translated in terms of due dates, all orders arriving at time t require due dates of $t+H$. It is then possible to have a two parameter (S, L) release policy with $0 \leq L \leq H$. Under the assumption of constant lead times, the resulting model is similar to the corresponding continuous time model of Buzacott and Shanthikumar [5]. The analysis thus follows that in [5]. Let $I(t)$ and $B(t)$,$I(t), B(t) \geq 0$,denote the physical inventory and backlog processes. Then it follows that:

$$I(t) = S + B(t) - N^*(t)$$

where $N^*(t)$ is the corresponding queueing process. $N^*(t)$ is closely related to the queueing process, $N(t)$, in the Geometric/Geometric/1 queue with the identical arrival and service parameters. It turns out that:

$$P\{N^*(t) = n\} = \sum_{k=n}^{\infty} P\{\text{exactly } (k-n) \text{ production completions in } \tau\} P\{N(t-\tau) = k\}$$

when $n \geq 1$.

The above relationship enables us to relate the distribution of $N^*(t)$ to that of $N(t)$ which consequently yields the average amount of physical inventory and the average number of backlogs. It is then possible to calculate the expected cost per unit time of a given (L, S) mechanism.

4.3 Advance information exploited optimally

As explained in the beginning of this section, the one period transition structure of a system evolving according to a production control policy that depends on the state $(X(t), D(t))$ can be represented by a discrete time markov chain. An optimal control problem to decide whether to produce or not at a particular state $(x, (d_1, d_2, .., d_h))$ can then be formulated. The optimality equation of dynamic programming for this problem can be expressed as:

$$\begin{aligned} V(x, d_1, d_2, ..., d_k) + g \quad = \quad & c(x) + \min\{pqV(x - d_1 + 1, d_2, d_3, ..., 1) \\ & + p(1-q)V(x - d_1 + 1, d_2, d_3, ..., 0) \\ & + (1-p)qV(x - d_1, d_2, d_3, ..., 1) \\ & + (1-p)(1-q)V(x - d_1, d_2, d_3, ..., 0), \\ & qV(x - d_1, d_2, d_3, ..., 1) + (1-q)V(x - d_1, d_2, d_3, ..., 0)\} \end{aligned}$$

Although difficult to analyze exactly, the optimal control policy corresponding to the above equation of optimality can easily be obtained numerically. We summarize some of the results in the next section.

5 Numerical Examples

We present a short summary of the numerical results obtained from the analysis of the single stage model. The first issue is the value of utilizing advance information. We compare the optimal cost performance of two systems: one using no advance information (but setting the optimal inventory level in the absence of information) and the other using the information in an optimal way. We fix the production completion probability p and vary the arrival probability q. The cost parameters are $h = 1$ and $b = 10$. Figure 3 displays the percentage improvement in system costs as a result of using advance information. The improvement is very significant for low levels of capacity utilization but diminishes with increasing capacity utilization.

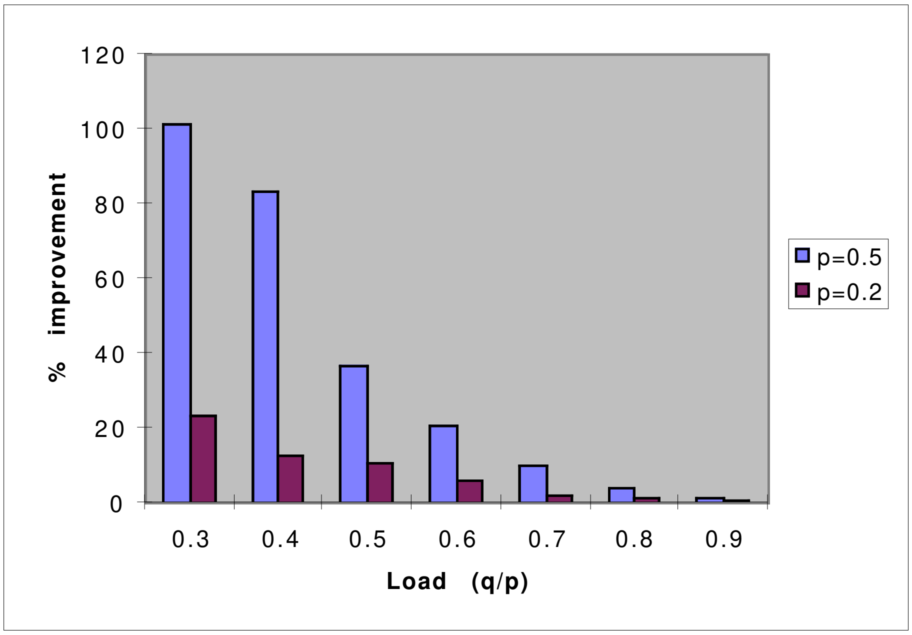

Figure 3: The improvement in performance due to advance information

The second important issue is the performance of the proposed mechanism with the respect to the optimal production control mechanism. It is found that, in general, the proposed mechanism (with the optimized parameters S and L) exactly coincides with the optimal control mechanism. To see the difference in cost, we take an example with a service completion probability of $p = 0.9$ (so that the production system is very *reliable*). We then vary the parameter q (with $h = 1$ and $b = 10$). Figure 4 displays the percentage relative error (in cost) due to using the proposed mechanism. It is seen that the relative error is zero when the capacity utilization is low, it becomes more significant for medium to high capacity utilization but diminishes in high capacity utilization levels).

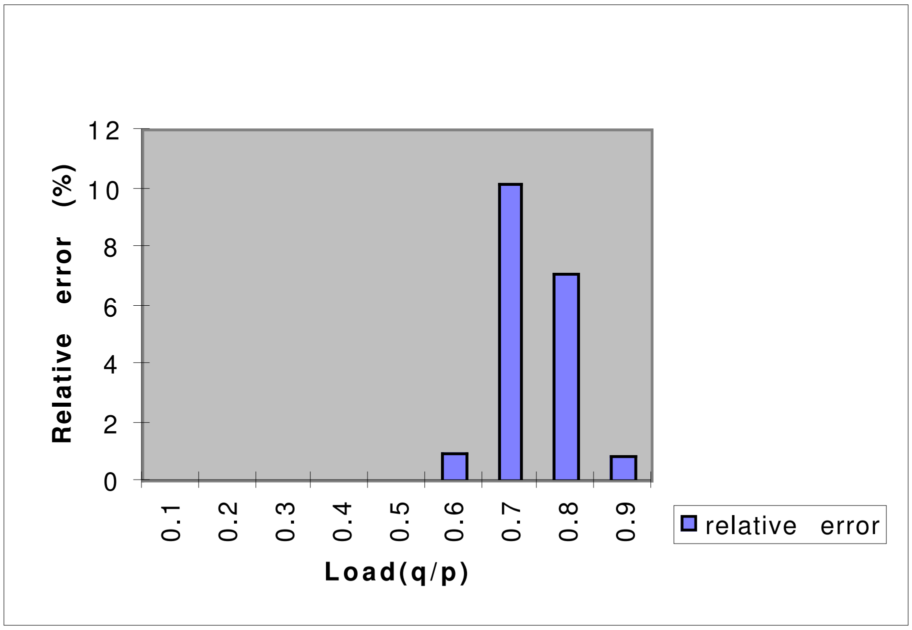

Figure 4: The relative percentage error of the proposed mechanism with respect to the optimal mechanism

6 Conclusions and Future Research

We have proposed a production coordination mechanism which integrates advance demand information for the timing of part release between stages. Several simplifying assumptions were made to introduce the general framework. It is of significant practical interest to relax some of these assumptions particularly on the advance information structure.

The proposed two parameter per stage control mechanism obtained from the base stock system seems to perform remarkably well in the special case of a single production stage. Multi-stage systems bring additional complications that cannot always be handled within the class of base stock policies. Most importantly, multi-stage systems usually require better control of work-in-process inventories. This is a major disadvantage of the base stock system which does not control work-in-process. It would be interesting to consider the integration of advance information in multi-stage coordination mechanisms that can control work in process inventories.

References

[1] Axsater, S. and K. Rosling, "Installation vs. Echelon Stock Policies for Multilevel Inventory Control", *Management Science*, Vol. 39, pp. 1274-1280, 1993.

[2] Axsater, S. and K. Rosling, "Multi-level Production-Inventory Control: Material Requirements Planning or Reorder Point Policies", *European Journal of Operational Research*, Vol. 75, pp. 405-412, 1994.

[3] Buzacott, J.A., and J.G. Shanthikumar, "A General Approach for Coordinating Production in Multiple-Cell Manufacturing Systems", *Production and Operations Management*, Vol. 1, pp. 34-52, 1992.

[4] Buzacott, J.A. and J.G. Shanthikumar, *Stochastic Models of Manufacturing Systems*, Prentice-Hall, 1993.

[5] Buzacott, J.A. and J.G. Shanthikumar, "Safety Stock versus Safety Time in MRP Controlled Production Systems", *Management Science*, Vol. 40, pp. 1678-1689, 1994.

[6] Dallery, Y. and G. Liberopoulos, "Extended Kanban Control System: A New Kanban Type Pull Control Mechanism for Multi-Stage Manufacturing", *Working paper*, 1997.

[7] Hariharan, R. and P. Zipkin, "Customer-order Information, Leadtimes and Inventories", *Management Science*, Vol. 41, pp. 1599-1607, 1995.

[8] Lambrecht, M.R., J.A. Muckstadt and R. Luyten, "Protective Stocks in Multi-stage Manufacturing Systems", *International Journal of Production Research*, Vol. 22,pp. 1001-1025, 1984.

[9] Liberopoulos, G. and Y. Dallery, "A Unified Framework for Pull Control Mechanisms in Multi -Stage Manufacturing Systems", *Working paper*, 1997.

[10] Pujolle G., J.P. Claude and D. Seret, "A Discrete Queueing System with a Product Form Solution", in *Computer Networking and Performance Evaluation*,ed. T. Hasegawa, H. Takagi and Y. Takahashi, Elsevier Science Publishers, 1986.