A DECOMPOSITION METHOD FOR THE PERFOMANCE EVALUATION OF PRODUCTION LINES WITH RANDOM PROCESSING TIMES, MULTIPLE FAILURE MODES AND FINITE BUFFER CAPACITY

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Abstract. *A new set of decomposition equations for the analysis of transfer lines with unreliable machines subject to different failure modes and random processing times is developed. The behaviour of the n-machine line is approximated by the behaviour of n-1 two-machine lines (building blocks). The improvement of the method over existing techniques is given by the separation of virtual failures from real failures in each building block in which the line is decomposed. While real failures are directly connected with the reliability of the machines of the original line, virtual failures are introduced to take into account blocking and starvation phenomena. Simulation and numerical experiments are carried out to show the performance of the new method in comparison with existing ones. The method can be extended to assembly/disassembly networks.*

Keywords: *perfomance evaluation, exponential lines, decomposition, multiple failures*

1. Introduction

In this paper we develop an efficient analytical method for the approximate perfomance evaluation of production lines with random processing times, multiple failure modes and finite buffer capacity.

The proposed method is based on the decomposition technique proposed by Gershwin [3], that is on the approximation of the behaviour of the n-machine line by the behaviour of $(n-1)$ two-machine lines; the difference and the major improvement is given by the possibility of taking into account machines with different failure modes. The proposed method represents an extension to the case of exponential lines of the multiple failures approach by Tolio and Matta for deterministic lines [7].

In fact, in real systems machines fail in different ways with more or less severe consequences: for example, an assembly station is able to complete a certain operation by using different devices (feeders, pick & place manipulators) that have different reliability parameters and can cause the failure of the entire station. All these kinds of failures happen with different frequencies and interrupt the station for periods of different lenght: therefore, in order to evaluate the perfomance of an n machine line, it is very important to be able to distinguish among different failure modes; with existing techniques all the different failure modes of a station/machine must be grouped and considered as an *average*

failure leading to a not neglegible error in performance evaluation. The proposed method overcomes this limitation by considering multiple failure modes at each machine.

2. Model Description and Assumptions

The focus of this study are transfer lines with unreliable machines separated by buffers of finite capacity. Raw parts come from outside the system to be processed on the first machine, then move to the first buffer, then visit the other machines and buffers in sequence until they reach the last machine and leave the system.

Each machine requires a random amount of time to process a unit and, when operational, it is subject to a series of random failures (i.e. we are assuming operation dependent failures, *ODFs*): if the machine fails in a certain mode, immediately starts the repair that also requires a random amount of time. Therefore, each machine is characterized by a series of random variables exponentially distributed: the processing time and, for each failure mode, the mean time between failures and the mean time to repair.

When a machine finishes to process a part and the downstream buffer is full the machine is said to be *"blocked"* while if the upstream buffer is empty the machine is said to be *"starved"*. Starvation and blocking mainly occur because of failures in the portion of the line upstream or downstream the machine under examination but they may also happen due to the difference in the processing times of the various machines: indeed, when a certain machine inside the line takes a long time to process a workpiece, the upstream buffer tends to become *full* while the downstream tends to become *empty*. If the phenomenon persists, blocking propagates upstream while starvation propagates downstream.

Finally, we assume there are always raw parts available at the input of the system (i.e. the first machines is never starved) and space available at the output of the system (i.e. the last machine is never blocked).

3. Machine Behaviour

We consider a transfer line L with K machines M_i and $K-1$ buffer B_i of size C_i in which each machine M_i can fail in F_i different modes.

A machine M_i can be operational or failed in a certain mode: when a machine is operational, it processes a workpiece if it is not starved or blocked and it continues working until either it completes the piece or one of the different failures occurs, whichever happen first. Either events can take place during the time interval $(t, t+\delta t)$ with probability approximately $\mu_i \delta t$ or $p_{i,f}\delta t$, for small δt. If a machine is failed in mode f at time t, it can be repaired during the time interval $(t, t+\delta t)$ with probability approximately $r_{i,f}\delta t$.

Since blocked or starved machines are not operating on a piece, they are not subject to failures (i.e. *ODFs*). For the same reason, an already failed machine cannot fail in a different mode.

4. Performance Measures

Given the failure and the repair rates for each failure mode of machine M_i it is possible to obtain the *efficiency in isolation*, that is the fraction of time the machine would be productive if it was not subject to blocking and starvation [7]:

$$e_i = \frac{1}{1 + \sum_{i=1}^{F_i} \frac{p_{i,f}}{r_{i,f}}} \tag{1}$$

The *efficiency of the machine inside the line*, that is the fraction of time in which the

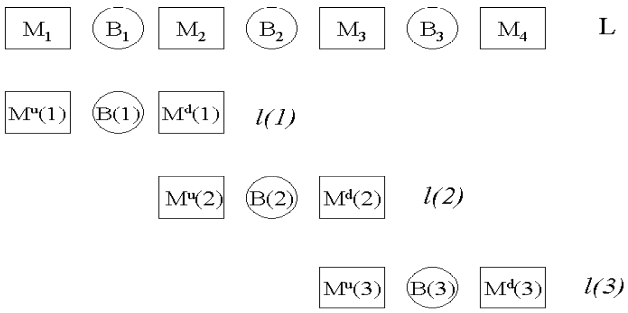

Figure 1: Decomposed 4-machine Line

machine produces pieces is given by:

$$E_i = e_i \left[1 - Ps_i - Pb_i\right] \tag{2}$$

that takes in account the losses caused by blocking (Pb_i) and starvation (Ps_i).

This equation is known in literature as *flow rate-idle time* and it represents an approximate relationship because it is based on the assumption that a machine cannot be starved and blocked at the same time. On the basis of the efficiency it is possible to evaluate the production rate by means of the relation:

$$P_i = \mu_i E_i \tag{3}$$

5. Outline of the Method

In the following, integer subscripts refer to machine and buffer of the original line L while symbols followed by an integer in parenthesis refer to the two-machine line indicated by the parenthesis.

The original transfer line L composed of K machines is decomposed, as shown in Fig.1, into a set of two machine lines $l(i)$, for $i = 1, ..., K-1$. Each two-machine line is characterized by a buffer $B\,(i)$ of the same size of the corresponding buffer B_i of the original line, an upstream pseudo machine $M^u(i)$ and a downstream pseudo machine $M^d(i)$. Pseudo machines model the portion of the original line upstream and downstream buffer B_i. In other words, the upstream machine $M^u(i)$ models how workpieces are transferred into buffer B_i while the downstream machine $M^d(i)$ models how workpieces are taken from buffer B_i.

The flow of parts into a buffer of the original line can be interrupted because the upstream machine fails in a certain mode or it takes a long time to process a part or because it becomes starved (this is due, in turn, to a failure or to a long processing time of one of the upstream machines). Conversely, the upstream machine of the corresponding two machine line cannot be starved being the first machine of the line: therefore, to represent the effect of starvation of the upstream machine some *"virtual"* failures are introduced on this machine.

Similarly, the need to represent the effect of blocking on the downstream machine of a two machine line leads to the introduction of some *"virtual"* failures also on the downstream machine.

The key to the decomposition method is to find the right values for the unknown parameters of these additional failures so that the flow of parts into and out of the buffer of two-machine line closely matches the flow of parts in the corresponding buffer of the original line.

6. Notation and Definitions

The failure modes of the upstream pseudo machine $M^u(i)$ of the two-machine line $l(i)$, that model the interruption of flow into buffer B_i, are distinguished in:

- *real failure modes* corresponding to the failures of the physical machine M_i, with failure and repair rates respectively equal to $p_{i,f}$ and $r_{i,f}$, for $f = 1, ..., F_i$.
- *virtual failure modes* corresponding to the starvation of the physical machine M_i due to a failure of one of the upstream machines, with failure and repair rates respectively indicated by $p^u_{j,f}\ (i)$ and $r^u_{j,f}(i)$, for $j = 1, ..., i-1$ and $f = 1, ..., F_j$.
- *virtual failure modes* corresponding to the starvation of the physical machine M_i due to a long processing time of one of the upstream machines, with failure and repair rates respectively referred as $p^u_{j,0}\ (i)$ and $r^u_{j,0}(i)$, for $j = 1, ..., i-1$.

Similarly, the failure modes of the downstream pseudo machine $M^d(i)$ of the two machine line $l(i)$, that model the interruption of flow out buffer B_i, are distinguished in:

- *real failure modes* corresponding to the stops of the physical machine M_{i+1}, with failure and repair rates respectively equal to $p_{i+1,f}$ and $r_{i+1,f}$, for $f = 1, ..., F_{i+1}$.
- *virtual failure modes* corresponding to the blocking of the physical machine M_i due to a failure of one of the downstream machines, with failure and repair rates respectively indicated by $p^d_{j,f}\ (i)$ and $r^d_{j,f}(i)$, for $j = i+2, ..., K$ and $f = 1, ..., F_j$ (the convention for blocking is $N_i = C_i + 2$, where C_i is the capacity of buffer B_i, for $i = 1, ..., K-1$).
- *virtual failure modes* corresponding to the blocking of the physical machine M_i due to a long processing time of one of the downstream machines, with failure and repair rates respectively referred as $p^d_{j,0}\ (i)$ and $r^d_{j,0}(i)$, for $j = i+2, ..., K$.

Given a two-machine line $l(i)$, $P(i)$ represents the average production rate of the line, $S_{j,f}(i)$ is the steady state probability of starvation of the downstream pseudo machine due to failure mode j, f, for $j = 1, ..., i$ and $f = 0, ..., F_j$, $B_{j,f}(i)$ is the steady state probability of blocking of the upstream pseudo machine due to failure mode j, f, for $j = i+1, ..., K$ and $f = 0, ..., F_j$. Finally, the total probability of starvation of the downstream pseudo machine (i.e. the probability of starvation regardless of the cause) is given by $Ps\,(i)$ while the total probability of blocking of the upstream pseudo machine is given by $Pb\,(i)$.

7. Pseudo Machine States

The upstream pseudo machine $M^u\,(i)$ can be in any of the following states:

- working, that is processing a workpiece: $W^u(i)$.
- down in a real mode: $D^u_f(i)$, for $f = 1, ..., F_i$.
- down in a virtual mode j, f: $V^u_{j,f}(i)$, for $j = 1, ..., i-1$ and $f = 1, ..., F_j$.
- down in a virtual mode $j, 0$: $V^u_{j,0}(i)$, for $j = 1, ..., i-1$.
- blocked due to failure mode j, f of $M^d\,(i)$: $B_{j,f}\,(i)$, for $j = i+1, ..., K$ and $f = 1, ..., F_j$.
- blocked due to failure mode $j, 0$ of $M^d\,(i)$: $B_{j,0}\,(i)$, for $j = i+1, ..., K$.

Similarly, $M^d\,(i-1)$ can be in any of the following states:

- working, that is processing a workpiece: $W^d(i-1)$.
- down in a real mode: $D^d_f(i-1)$, for $f = 1, ..., F_i$.
- down in a virtual mode j, f: $V^d_{j,f}(i)$, for $j = i+1, ..., K$ and $f = 1, ..., F_j$.

- down in a virtual mode $j,0$: $V_{j,0}^{d}(i)$, for $j = i+1, ..., K$.
- starved due to failure mode j,f of $M^{u}\,(i-1)$: $S_{j,f}\,(i)$, for $j = 1, ..., i-1$ and $f = 1, ..., F_j$.
- starved due to failure mode $j,0$ of $M^{u}\,(i-1)$: $S_{j,0}\,(i)$, for $j = 1, ..., i-1$.

8. Decomposition Equations

Since it is assumed that the pseudo machines of each two-machine line are characterized by a series of random variables exponentially distributed, that are the processing time and, for each failure mode, the mean time between failures and the mean time to repair, the next step is to find the parameters of these distributions so that the flow of parts into and out of the buffer of the two-machine line closely matches the flow of parts in the corresponding buffer of the original line.

The processing rates and the parameters of the real failures of the pseudo machines are known, since they are exactly the same of the corresponding machines in the original line; on the other hand for the parameters of the virtual failures decomposition equations must be developed.

8.1 Resumption of Flow Equations

Since the period in which a machine is under repair is not affected by the states of other machines or buffers, the repair rates of virtual failures j,f of pseudo-machines $M^{u}\,(i)$ and $M^{d}\,(i-1)$ are exactly the same of the corresponding machine of the original line, that is $r_{j,f}$. On the other hand, the repair rates of virtual failures $j,0$ of pseudo-machines $M^{u}\,(i)$ and $M^{d}\,(i-1)$ are just the processing rates of the corresponding machine of the original line, that is μ_j: indeed, the probability that machine M_i exits from a condition of starvation (blocking) due to a long processing time on the upstream (downstream) machine M_j is equal to the probability that M_j completes the part it is processing.

8.2 Interruption of Flow Equations

8.2.1 Preliminary Remarks

Since failure frequency must equal repair frequency for each failure mode (real or virtual), we can write for $M^{u}\,(i)$ the following equations:

$$p_{i,f} \cdot W^{u}\,(i) = r_{i,f} \cdot D_{f}^{u}(i),\ f = 1, ..., F_i \tag{4}$$

$$p_{j,f}^{u}\,(i) \cdot W^{u}\,(i) = r_{j,f} \cdot V_{j,f}^{u}(i),\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{5}$$

$$p_{j,0}^{u}\,(i) \cdot W^{u}\,(i) = \mu_j \cdot V_{j,0}^{u}(i),\ j = 1, ..., i-1 \tag{6}$$

Similarly, for $M^{d}\,(i-1)$ the following equations hold:

$$p_{i,f} \cdot W^{d}\,(i-1) = r_{i,f} \cdot D_{f}^{d}(i-1),\ f = 1, ..., F_i \tag{7}$$

$$p_{j,f}^{d}\,(i-1) \cdot W^{d}\,(i-1) = r_{j,f} \cdot V_{j,f}^{d}(i-1),\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{8}$$

$$p_{j,0}^{d}\,(i-1) \cdot W^{d}\,(i-1) = \mu_j \cdot V_{j,0}^{d}(i-1),\ j = i+1, ..., K \tag{9}$$

Since the sum on all possible states of $M^{u}\,(i)$ must be equal to 1, we can write:

$$W^{u}\,(i) + \sum_{f=1}^{Fi} D_{f}^{u}(i) + \sum_{j=1}^{i-1}\sum_{f=1}^{F_j} V_{j,f}^{u}(i) + \sum_{j=1}^{i-1} V_{j,0}^{u}(i) + \sum_{j=i+1}^{K}\sum_{f=1}^{F_j} B_{j,f}(i) + \sum_{j=i+1}^{K} B_{j,0}(i) = 1 \tag{10}$$

Equation (1) which defines the efficiency in isolation of a machine can be extended to

consider also virtual failures thus obtaining for the upstream pseudo machine $M^u\,(i)$ of the two machine line $l(i)$:

$$e^u(i) = \frac{1}{1 + \sum_{f=1}^{Fi} \frac{p_{i,f}}{r_{i,f}} + \sum_{j=1}^{i-1} \sum_{f=1}^{F_j} \frac{p^u_{j,f}(i)}{r_{j,f}} + \sum_{j=1}^{i-1} \frac{p^u_{j,0}(i)}{\mu_j}} \tag{11}$$

Substituting equations (4), (5), (6) and (11) into equation (10), after some manipulations we obtain:

$$W^u\,(i) = e^u(i)\,[1 - Pb\,(i)] \tag{12}$$

which corresponds to the flow rate-idle time equation (2), since $M^u\,(i)$, being the first machine of the line $l(i)$, cannot be starved. Therefore, we can conclude:

$$W^u\,(i) = E^u(i) \tag{13}$$

Similarly, since the sum on all possible states of $M^d\,(i-1)$ must be equal to 1, we can write:

$$\begin{gathered} W^d\,(i-1) + \sum_{f=1}^{Fi} D^d_f(i-1) + \sum_{j=i+1}^{K} \sum_{f=1}^{F_j} V^d_{j,f}(i-1) + \\ + \sum_{j=i+1}^{K} V^d_{j,0}(i-1) + \sum_{j=1}^{i-1} \sum_{f=1}^{F_j} S_{j,f}(i-1) + \sum_{j=1}^{i-1} S_{j,0}(i-1) = 0 \end{gathered} \tag{14}$$

Introducing the efficiency in isolation for the pseudo downstream machine $M^d\,(i-1)$:

$$e^d(i-1) = \frac{1}{\sum_{f=1}^{Fi} \frac{p_{i,f}}{r_{i,f}} + \sum_{j=i+1}^{K} \sum_{f=1}^{F_j} \frac{p^d_{j,f}(i-1)}{r_{j,f}} + \sum_{j=i+1}^{K} \frac{p^d_{j,0}(i-1)}{\mu_j}} \tag{15}$$

Substituting equations (7), (8), (9) and (15) into equation (14), after some manipulations we obtain:

$$W^d\,(i-1) = e^d(i-1)\,[1 - Ps\,(i-1)] \tag{16}$$

which represents the flow rate-idle time equation (2), since $M^d\,(i-1)$, being the last machine of the line $l(i)$, cannot be blocked. Therefore, we can conclude:

$$W^d\,(i-1) = E^d(i-1) \tag{17}$$

8.2.2 Solution Methodology

Since the virtual failures of $M^u(i)$ have been introduced to mimic the different causes of starvation of the physical machine M_i, the steady state probability of $M^u(i)$ being in a virtual failure state must equal the steady state probability of starvation of the previous two-machine line $l(i-1)$:

$$V^u_{j,f}(i) = S_{j,f}(i-1),\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{18}$$

$$V^u_{j,0}\,(i) = S_{j,0}\,(i-1)\,,\ j = 1, ..., i-1 \tag{19}$$

Similarly, the virtual failures of $M^d(i-1)$ have been introduced to mimic the different causes of blocking of the physical machine M_i. Therefore, the steady state probability of $M^d(i-1)$ being in a virtual failure state must equal the steady state probability of blocking of the following two-machine line $l(i+1)$:

$$V^d_{j,f}(i-1) = B_{j,f}(i),\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{20}$$

$$V^d_{j,0}\,(i-1) = B_{j,0}\,(i)\,,\ j = i+1, ..., K \tag{21}$$

Therefore, substituting equations (18) and (19), respectively, into equations (5) and (6), and taking in account equation (13), we obtain the following expressions for virtual failure rates of $M^u(i)$:

$$p^u_{j,f}\,(i) = \frac{S_{j,f}(i-1)}{E^u\,(i)} \cdot r_{j,f},\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{22}$$

$$p_{j,0}^{u}(i) = \frac{S_{j,0}(i-1)}{E^{u}(i)} \cdot \mu_j,\ j = 1, ..., i-1 \tag{23}$$

while substituting equations (20) and (21), respectively, into equations (8) and (9), and taking in account equation (17), we obtain the following expressions for virtual failure rates of $M^d(i-1)$:

$$p_{j,f}^{d}(i-1) = \frac{B_{j,f}(i)}{E^{d}(i-1)} \cdot r_{j,f},\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{24}$$

$$p_{j,0}^{d}(i-1) = \frac{B_{j,0}(i)(i)}{E^{d}(i-1)} \cdot \mu_j,\ j = i+1, ..., K \tag{25}$$

8.2.3 Conservation of Flow

Considering that the real failure modes of $M^u(i)$ are the same as the real failure modes of $M^d(i-1)$ (both pseudo machines refer to the same physical machine M_i), we can write:

$$D_f^u(i) = D_f^d(i-1),\ f = 1, ..., F_i \tag{26}$$

Substituting equations (18), (19), (20), (21) and (26) into the normalization equations (10) and (14), we can conclude:

$$E^{u}(i) = E^{d}(i-1) \tag{27}$$

For the same reason we have:

$$\mu^{u}(i) = \mu^{d}(i-1) = \mu_i \tag{28}$$

Therefore, using these last two equations, we can write:

$$\mu^{u}(i) E^{u}(i) = \mu^{d}(i-1) E^{d}(i-1) \tag{29}$$

which corresponds to the conservation of flow relationship:

$$P(i) = P(i-1) \tag{30}$$

9. Algorithm

On the basis of the analysis carried out, the algorithm to solve the decomposition equations (similar to the DDX algorithm [2]) consists of the following steps:

1. Initialization: for each two-machine line $l(i)$ in which the original line L is decomposed, the known parameters are introduced and an initial guess for the unknown parameters is made. In the following we define α this initial guess (for istance α may be set to a value like $0,01$):

$$\mu^{u}(i) = \mu_i \tag{31}$$

$$p_{i,f}^{u}(i) = p_{i,f},\ f = 1, ..., F_i \tag{32}$$

$$r_{i,f}^{u}(i) = r_{i,f},\ f = 1, ..., F_i \tag{33}$$

$$p_{j,f}^{u}(i) = \alpha,\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{34}$$

$$r_{j,f}^{u}(i) = r_{j,f},\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{35}$$

$$p_{j,0}^{u}(i) = \alpha,\ j = 1, ..., i-1 \tag{36}$$

$$r_{j,0}^{u}(i) = \mu_j,\ j = 1, ..., i-1 \tag{37}$$

$$\mu^{d}(i-1) = \mu_i \tag{38}$$

$$p_{i,f}^{d}(i-1) = p_{i,f},\ f = 1, ..., F_i \tag{39}$$

$$r^d_{i,f}(i-1) = r_{i,f},\ f = 1, ..., F_i \tag{40}$$

$$p^d_{j,f}(i-1) = \alpha,\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{41}$$

$$r^d_{j,f}(i-1) = r_{j,f},\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{42}$$

$$p^d_{j,0}(i-1) = \alpha,\ j = i+1, ..., K \tag{43}$$

$$r^d_{j,0}(i-1) = \mu_j,\ j = i+1, ..., K \tag{44}$$

2. For $i = 2, ..., k-1$: evaluation of the two machine $l(i)$ using for the parameters of $M^d(i)$ the most recent values from the previous iteration while updating parameters of $M^u(i)$ with the results of the evaluation of the upstream building block $l(i-1)$:

$$p^u_{j,f}(i) = \frac{S_{j,f}(i-1)}{E^u(i)} \cdot r_{j,f},\ j = 1, ..., i-1,\ f = 1, ..., F_j \tag{45}$$

$$p^u_{j,0}(i) = \frac{S_{j,0}(i-1)}{E^u(i)} \cdot \mu_j,\ j = 1, ..., i-1 \tag{46}$$

3. For $i = K-1, ..., 2$: evaluation of the two machine $l(i-1)$ using for the parameters of $M^u(i-1)$ the most recent values from the previous iteration while updating parameters of $M^d(i-1)$ with the results of the evaluation of the downstream building block $l(i)$:

$$p^d_{j,f}(i-1) = \frac{B_{j,f}(i)}{E^d(i-1)} \cdot r_{j,f},\ j = i+1, ..., K,\ f = 1, ..., F_j \tag{47}$$

$$p^d_{j,0}(i-1) = \frac{B_{j,0}(i)}{E^d(i-1)} \cdot \mu_j,\ j = i+1, ..., K \tag{48}$$

4. Steps 2 and 3 are repeated alternatively until the termination condition is satisfied, that is until the equations converge: the misure of convergence is given by the maximum relative change in any of the unknown parameters.

10. Numerical Results

In order to show the accuracy of the new analytical method developed (method LMT) a set of numerical tests has been carried out comparing the analytical results with those obtained applying the method developed by Gershwin [4] (method Ger.) and those obtained running simulation experiments.

For each simulation experiment, to ensure statistical significance 20 replications have been performed. Each run entails a simulation period of $2.000.000$ time units preceeded by a warm-up period of $1.000.000$ time units. The first set of experiments considers transfer lines with machines subject to one failure mode to comply with the hypoteses of the Gershwin method. The second set of experiments, on the contrary, considers machines characterized by two failure modes rather different in terms of repair rates.

In the case of machines characterized by one failure the difference between the two methods is not very big even if the proposed method seems to be more precise in throughput estimation as shown in the examples reported in table 1. On the other hand, considering lines with machines affected by two failure modes the existing method, being forced to include in an average failure all different causes of interruption of flow, can lead to a considerable error in comparison with the simulation results while the proposed method, being able to distinguish among all the different failures gives rather accurate results.

Case 1							Case 2						
M	μ	p	r	C	-	Th.	M	μ	p	r	C	-	Th.
1	1.5	0.03	0.3	8	Sim.	0.9401	1	1.5	0.03	0.3	8	Sim.	0.7983
2	1.2	0.05	0.5	4	Ger.	0.9599	2	1.2	0.05	0.5	4	Ger.	0.8240
3	1.8	0.01	0.2	6	$\Delta\%$	2.11	3	1.8	0.01	0.2	6	$\Delta\%$	3.22
4	1.6	0.06	0.4	8	LMT	0.9456	4	1.6	0.06	0.4	8	LMT	0.8073
5	2.0	0.02	0.1	-	$\Delta\%$	0.58	5	1.5	0.06	0.1	-	$\Delta\%$	1.13
Case 3							Case 4						
M	μ	p	r	C	-	Th.	M	μ	p	r	C	-	Th.
1	1.5	0.03	0.3	8	Sim.	1.0667	1	1.2	0.03	0.1	8	Sim.	0.6798
2	1.6	0.02	0.4	4	Ger.	1.0886	2	1.0	0.05	0.3	8	Ger.	0.6999
3	1.8	0.01	0.2	6	$\Delta\%$	2.05	3	1.4	0.01	0.2	8	$\Delta\%$	2.96
4	1.6	0.06	0.4	8	LMT	1.0678	4	1.2	0.06	0.4	8	LMT	0.6838
5	2.0	0.02	0.1	-	$\Delta\%$	0.11	5	1.0	0.02	0.09	-	$\Delta\%$	0.59
Case 5							Case 6						
M	μ	p	r	C	-	Th.	M	μ	p	r	C	-	Th.
1	1.2	0.03	0.01	8	Sim.	0.1806	1	1.2	0.03	0.01	12	Sim.	0.1918
2	1.0	0.05	0.03	8	Ger.	0.1827	2	1.0	0.05	0.03	12	Ger.	0.1953
3	1.4	0.01	0.02	8	$\Delta\%$	1.16	3	1.4	0.01	0.02	12	$\Delta\%$	1.83
4	1.2	0.06	0.04	8	LMT	0.1820	4	1.2	0.06	0.04	12	LMT	0.1942
5	1.0	0.02	0.09	-	$\Delta\%$	0.78	5	1.0	0.02	0.09	-	$\Delta\%$	1.25

Table 1: Five machines, one failure parameters and results

11. Conclusions

A decomposition method for the performance evaluation of production lines with random processing times, multiple failure modes and finite buffer capacity has been developed.

The main advantage of this new method in comparison with existing techniques is the ability to take in account machines subject to different failure modes that allow to distinguish (and to keep separate) the various causes that can interrupt machine operations. Simulation experiments and numerical results show that this approach is able to increase sensitively the accuracy of evaluation over existing techniques.

Further research should consider the possibility to extend this new decomposition method to more general networks such as assembly/disassembly networks.

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Case 7									Case 8								
M	μ	p_1	p_2	r_1	r_2	C	-	Th.	M	μ	p_1	p_2	r_1	r_2	C	-	Th.
1	1.2	0.08	0.02	0.6	0.08	8	Sim.	0.4919	1	1.8	0.09	0.01	0.8	0.073	8	Sim.	0.7089
2	1.0	0.09	0.03	0.7	0.09	6	Ger.	0.5454	2	1.6	0.08	0.04	0.8	0.200	8	Ger.	0.7745
3	1.2	0.07	0.01	0.9	0.07	8	$\Delta\%$	10.87	3	2.0	0.06	0.03	0.6	0.086	8	$\Delta\%$	9.25
4	1.4	0.06	0.04	0.5	0.06	6	LMT	0.4960	4	1.8	0.06	0.02	0.9	0.900	8	LMT	0.7169
5	1.2	0.05	0.01	0.8	0.10	-	$\Delta\%$	0.83	5	1.6	0.10	0.04	0.5	0.060	-	$\Delta\%$	1.13
Case 9									Case 10								
M	μ	p_1	p_2	r_1	r_2	C	-	Th.	M	μ	p_1	p_2	r_1	r_2	C	-	Th.
1	0.8	0.09	0.01	0.8	0.073	8	Sim.	0.3105	1	1.0	0.02	0.01	0.2	0.05	6	Sim.	0.5804
2	0.6	0.08	0.04	0.8	0.200	8	Ger.	0.3361	2	1.0	0.01	0.01	0.3	0.15	6	Ger.	0.5935
3	1.0	0.06	0.03	0.6	0.086	8	$\Delta\%$	8.24	3	1.2	0.02	0.02	0.3	0.1	6	$\Delta\%$	2.26
4	0.8	0.06	0.02	0.9	0.900	8	LMT	0.3181	4	1.0	0.01	0.03	0.3	0.18	6	LMT	0.5823
5	0.6	0.10	0.04	0.5	0.060	-	$\Delta\%$	2.45	5	1.2	0.01	0.02	0.2	0.4	-	$\Delta\%$	0.33
Case 11									Case 12								
M	μ	p_1	p_2	r_1	r_2	C	-	Th.	M	μ	p_1	p_2	r_1	r_2	C	-	Th.
1	1.0	0.02	0.01	0.2	0.05	6	Sim.	0.6145	1	1.0	0.02	0.01	0.2	0.05	10	Sim.	0.6709
2	1.0	0.01	0.01	0.3	0.15	6	Ger.	0.6333	2	1.0	0.01	0.01	0.3	0.15	10	Ger.	0.6876
3	1.2	0.02	0.02	0.3	0.6	6	$\Delta\%$	3.06	3	1.2	0.02	0.02	0.3	0.6	10	$\Delta\%$	2.49
4	1.0	0.01	0.03	0.3	0.18	6	LMT	0.6185	4	1.0	0.01	0.03	0.3	0.18	10	LMT	0.6749
5	1.2	0.01	0.02	0.2	0.4	-	$\Delta\%$	0.65	5	1.2	0.01	0.02	0.2	0.4	-	$\Delta\%$	0.59

Table 2: Five machines, two failures parameters and results

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