EXPONENTIAL TWO MACHINE LINES WITH MULTIPLE FAILURE MODES AND FINITE BUFFER CAPACITY

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Abstract. *This paper presents an exact analytical method for the perfomance evaluation of two machine lines with random processing times, multiple failure modes and finite buffer capacity. The method models machines subject to different types of failures each failure mode being characterized by exponentially distributed mean time between failures (MTBF) and mean time to repair (MTTR). The steady state probability of the Markov chain of the system is obtained in a compact form with a computational effort that depends only on the number of failure modes considered and not on the capacity of the buffer. The improvements given by the proposed method in comparison with the existing techniques are quantified. The method can be used as a building block for the development of a new decomposition approach for the analysis of larger systems, such as long transfer lines and assembly/disassembly networks.*

Keywords: *perfomance evaluation, exponential line, building block, multiple failures*

1. Introduction

This paper presents an exact analytical method for the performance evaluation of two-machine lines with exponentially distributed processing times, multiple failure modes and finite buffer capacity. The expression *multiple failures* means that each machine can fail in different modes, each one characterized by a specific *MTBF* and *MTTR*.

In fact, in real systems machines fail in different ways with more or less severe consequences: all these kinds of failures happen with different frequencies and interrupt the station for periods of different lenght. Therefore, to evaluate the perfomance of a two machine line it is very important to be able to distinguish among failure modes. If this separation is not possible, all different failure modes must be grouped and considered as an *average failure,* as it has been done in previous works [3] [4]; this can be done by means of a first order approximation which introduces significant error in perfomance evaluation as shown in the section "Numerical Results".

The first model of unreliable two machine lines with exponential service times and finite buffer capacity was by Buzacott in 1972 [1]: he considered the case of machines with the same service rate and subject to failures occurring according to a geometric distribution in the number of operations since the last failure (i.e. *number-ODFs*). In 1980, Gershwin and Berman [3] proposed a model of exponential two machine lines which differs from the one proposed by Buzacott in that failures are represented by an exponential distribution

in time rather than a geometrical distribution in the number of parts produced and in that machines can have different service rates.

The model described in this paper is an extension to the case of muliple failure modes of Gershwin and Berman's model [3] and although limited to two machine-one buffer systems, it is very important because represents the first step for the development of a new decomposition approach for the analysis of larger systems, such as long transfer lines and assembly/disassembly networks.

2. Model Description and Assumptions

The system consists of a two-stage production line separated by a finite buffer; the behaviour of the system is based on the following assumptions (the same as in Gershwin [2] except for the multiple failure modes):

- Each part enters the system through the first machine, then goes to the buffer waiting for the availability of the second machine and then leaves the system.
- A machine whose upstream buffer is empty is said to be *starved* while a machine whose downstream buffer is full is said to be *blocked*.
- The upstream machine is never starved while the downstream machine is never blocked.
- Service, failure and repair times for the two machines are assumed to be exponential random variables with parameters representing, respectively, the service rate, failure rates and repair rates.
- A machine can be in one of the following conditions: *operational* or *failed in a certain mode*.
- An operational machine which is not processing a workpiece (i.e. starved or blocked) cannot fail: this means that operation dependent failures (*ODFs*) are assumed.
- When a machine is operational, it works on a piece if it is not starved or blocked and it continues working until either it completes the piece or one of the different failures occurs.
- A machine cannot fail in more than one mode at the same time.
- When a machine is under repair, it remains in that state for a period of time which is exponentially distributed with parameter depending on the type of failure that occurred.
- Workpieces are not destroyed or rejected at any stage in the line. Partly processed workpieces are not added into the line.

3. Notation

The system is modeled using a continuous time, discrete state Markov chain. The state of the system is given by (n, α_u, α_d) where n is an integer that indicates the number of pieces in the buffer plus the piece on the working area of the downstream machine $(0 \leq n \leq N)$ while α_u and α_d indicate the state of upstream and downstream machine, respectively. When the upstream machine is operational $\alpha_u = 1$ while $\alpha_u = u_i$ for $i = 1, ..., s$ means that the upstream machine is down due to failure mode i. Similarly, α_d can assume the values $1, d_1, d_2, ..., d_t$. The convention for blocking is $n = N = C + 2$, where C is the buffer capacity. The steady state probability of the system being in state (n, α_u, α_d) is indicated by $p(n, \alpha_u, \alpha_d)$.

If the upstream machine is working on a piece at time t, during the time interval $(t, t + \delta t)$ it can complete the piece with probability approximately $\mu_u \delta t$ or it can fail in mode u_i with probability approximately $p^{u_i} \delta t$. If the upstream machine is failed in mode

u_i at time t, it can be repaired during the time interval $(t, t+\delta t)$ with probability approximately $r^{u_i}\delta t$, for small δt. Similarly, μ_d, p^{d_j} and r^{d_j} for $j = 1, ..., t$ represent, respectively, service, failure and repair rates for the downstream machine. The total failure rate P^U of the upstream machine, i.e. the approximate probability of a failure during the time interval $(t, t+\delta t)$ regardless of the mode in which the machine fails, is given by $P^U = \sum_{i=1}^{s} p^{u_i}$. Similarly, the total failure rate P^D of the downstream machine is given by $P^D = \sum_{j=1}^{t} p^{d_j}$.

4. Performance Measures

The efficiency is defined as the steady state probability that a machine is working on a piece and it can be seen as the fraction of time in which the machine produces parts. Therefore, for the upstream machine it is possible to write:

$$E_u = prob\left[\alpha_u = 1, n < N\right] = \sum_{n=0}^{N\text{-}1} \left[p(n,1,1) + \sum_{j=1}^{t} p\left(n,1,d_j\right) \right] \tag{1}$$

since the upstream machine, being the first machine of the line, cannot be starved. Similarly, for the downstream machine:

$$E_d = prob\left[\alpha_d = 1, n > 0\right] = \sum_{n=1}^{N} \left[p(n,1,1) + \sum_{i=1}^{s} p\left(n,u_i,1\right) \right] \tag{2}$$

since the downstream machine, being the last machine of the line, cannot be blocked.

On the basis of the efficiency, it is possible to evaluate the production rate of the upstream and downstream machines as:

$$\theta_u = \mu_u E_u \qquad \theta_d = \mu_d E_d \tag{3}$$

Moreover, since there is no mechanism of creation or destruction of workpieces it can be shown, extending to the case of multiple failures the proof given by Gershwin in [2], that part flow in the line is conserved:

$$\theta = \mu_u E_u = \mu_d E_d \tag{4}$$

Finally, the average buffer level can be written as:

$$\overline{n} = \sum_{n=0}^{N} n \left[p(n,1,1) + \sum_{i=1}^{s} p\left(n,u_i,1\right) + \sum_{j=1}^{t} p\left(n,1,d_j\right) + \sum_{i=1}^{s}\sum_{j=1}^{t} p\left(n,u_i,d_j\right) \right] \tag{5}$$

5. Balance Equations

Since we have to deal with a continuos time, discrete space Markov process, to obtain the steady state distribution we make use of balance equations, which equate the rate of leaving a state with the rate of entering it.

We distinguish four sets of equations on the basis of the different values of the state variables α_u and α_d. Moreover, having defined as "internal" the states (n, α_u, α_d) where $1 \leq n \leq N-1$, we can further classify balance equations that do not involve boundary states as internal equations while all the others are boundary equations. The complete set of balance equations for the Markov chain that models the two machine line are listed in the following:

- $\boldsymbol{\alpha_u}$**=** $\mathbf{u}_i$ **for** $\mathbf{i} = \mathbf{1}, ..., \mathbf{s}$ **and** $\boldsymbol{\alpha_d}$**=** $\mathbf{d}_j$ **for** $\mathbf{j} = \mathbf{1}, ..., \mathbf{t}$**.**

$$p\left(n,u_i,d_j\right)\left(r^{u_i} + r^{d_j}\right) = p\left(n,u_i,1\right)p^{d_j} + p\left(n,1,d_j\right)p^{u_i},\ 1 \leq n \leq N-1 \tag{6}$$

$$p\left(0,u_i,d_j\right)\left(r^{u_i}+r^{d_j}\right)=p\left(0,1,d_j\right)p^{u_i} \tag{7}$$

$$p\left(N,u_i,d_j\right)\left(r^{u_i}+r^{d_j}\right)=p\left(N,u_i,1\right)p^{d_j} \tag{8}$$

- $\boldsymbol{\alpha_u}$**= u$_i$ for i = 1, ..., s and** $\boldsymbol{\alpha_d}$**= 1.**

$$p\left(n,u_i,1\right)\left(r^{u_i}+\mu_d+\sum_{j=1}^{t}p^{d_j}\right)=\sum_{j=1}^{t}p\left(n,u_i,d_j\right)r^{d_j}+ \tag{9}$$
$$+p(n,1,1)p^{u_i}+p(n+1,u_i,1)\mu_d$$

for $1 \leq n \leq N-1$.

$$p\left(0,u_i,1\right)r^{u_i}=\sum_{j=1}^{t}p\left(0,u_i,d_j\right)r^{d_j}+p(0,1,1)p^{u_i}+p(1,u_i,1)\mu_d \tag{10}$$

$$p\left(N,u_i,1\right)\left(r^{u_i}+\mu_d+\sum_{j=1}^{t}p^{d_j}\right)=\sum_{j=1}^{t}p\left(N,u_i,d_j\right)r^{d_j} \tag{11}$$

- $\boldsymbol{\alpha_u}$**= 1 and** $\boldsymbol{\alpha_d}$**= d$_j$ for j = 1, ..., t.**

$$p\left(n,1,d_j\right)\left(\mu_u+\sum_{i=1}^{s}p^{u_i}+r^{d_j}\right)=\sum_{i=1}^{s}p\left(n,u_i,d_j\right)r^{u_i}+ \tag{12}$$
$$+p\left(n,1,1\right)p^{d_j}+p(n-1,1,d_j)\mu_u$$

for $1 \leq n \leq N-1$.

$$p\left(0,1,d_j\right)\left(\mu_u+\sum_{i=1}^{s}p^{u_i}+r^{d_j}\right)=\sum_{i=1}^{s}p\left(0,u_i,d_j\right)r^{u_i} \tag{13}$$

$$p\left(N,1,d_j\right)r^{d_j}=\sum_{i=1}^{s}p\left(N,u_i,d_j\right)r^{u_i}+p\left(N,1,1\right)p^{d_j}+p(N-1,1,d_j)\mu_u \tag{14}$$

- $\boldsymbol{\alpha_u}$**= 1 and** $\boldsymbol{\alpha_d}$**= 1.**

$$p\left(n,1,1\right)\left(\mu_u+\sum_{i=1}^{s}p^{u_i}+\mu_d+\sum_{j=1}^{t}p^{d_j}\right)=p\left(n+1,1,1\right)\mu_d+ \tag{15}$$
$$+p(n-1,1,1)\mu_u+$$
$$+\sum_{i=1}^{s}p\left(n,u_i,1\right)r^{u_i}+\sum_{j=1}^{t}p\left(n,1,d_j\right)r^{dj}$$

for $1 \leq n \leq N-1$.

$$p\left(0,1,1\right)\left(\mu_u+\sum_{i=1}^{s}p^{u_i}\right)=p\left(1,1,1\right)\mu_d+\sum_{j=1}^{t}p\left(0,1,d_j\right)r^{d_j}+\sum_{i=1}^{s}p\left(0,u_i,1\right)r^{u_i} \tag{16}$$

$$p\left(N,1,1\right)\left(\mu_d+\sum_{j=1}^{t}p^{d_j}\right)=p(N-1,1,1)\mu_u+\sum_{j=1}^{t}p\left(N,1,d_j\right)r^{d_j}+\sum_{i=1}^{s}p\left(N,u_i,1\right)r^{u_i} \tag{17}$$

6. Solution Methodology

The solution methodology is an extension to the case of multiple failure modes of the one

originally proposed by Gershwin and Berman [3]. The first step is to analyse the Markov chain and make a guess on the form of the steady state probabilities of the internal states. Then, substituting this guess into the internal balance equations, a set of $s+t+2$ solutions (i.e. vectors that satisfy the internal equations) can be found. At this point, if the guess is correct it must be possible to find a linear combination of these internal solutions that also satisfies the boundary conditions. The advantage of this approach is that the computational effort depends only on the total number of failure modes taken in account and not on the buffer capacity C.

7. Analysis of Internal Equations

Following the analysis carried out in Tolio and Gershwin [7], we propose a solution for the internal equations of the form:

$$p\left(n,\alpha_u,\alpha_d\right) = X^n \Phi_u \Phi_d,\ 1 \leq n \leq N-1 \tag{18}$$

where Φ_u (Φ_d) indicates the constant corresponding to the upstream (downstream) machine state which takes the value 1 if the machine is operational or the value U_i (D_j) if the machine is down in mode i (j), for $i = 1, ..., s$ $(j = 1, ..., t)$.

Substituting equation (18) into the internal equations, after some manipulations we obtain:

$$\frac{p^{u_i}}{U_i} - r^{u_i} = -\left(\frac{p^{d_j}}{D_j} - r^{d_j}\right),\ i = 1, ..., s,\ j = 1, ..., t \tag{19}$$

$$\mu_d\left(X-1\right) + \frac{p^{u_i}}{U_i} - r^{u_i} + \sum_{j=1}^{t}\left(D_j r^{d_j} - p^{d_j}\right) = 0,\ i = 1, ..., s \tag{20}$$

$$\mu_u\left(\frac{1}{X}-1\right) + \frac{p^{d_j}}{D_j} - r^{d_j} + \sum_{i=1}^{s}\left(U_i r^{u_i} - p^{u_i}\right) = 0,\ j = 1, ..., t \tag{21}$$

$$\mu_d\left(X-1\right) + \mu_u\left(\frac{1}{X}-1\right) + \sum_{i=1}^{s}\left(U_i r^{u_i} - p^{u_i}\right) + \sum_{j=1}^{t}\left(D_j r^{d_j} - p^{d_j}\right) = 0 \tag{22}$$

Since equation (19) is a linear combination of equations (20), (21) and (22), these three last equations form a system of $s+t+1$ equations in $s+t+1$ unknowns, i.e. the constants X, U_i and D_j we introduced to formulate the guess in (18).

In equation (19) it is possible to notice that all parameters and unknowns referring to the upstream machine are on the left side while all parameters and unknowns referring to the downstream machine are on the right side. This fact implies:

$$\frac{p^{u_i}}{U_i} - r^{u_i} = K,\ i = 1, ..., s \tag{23}$$

$$\frac{p^{d_j}}{D_j} - r^{d_j} = -K,\ j = 1, ..., t \tag{24}$$

where K is not a function of i or j. From (23) and (24) we can write:

$$U_i = \frac{p^{u_i}}{K + r^{u_i}},\ i = 1, ..., s \tag{25}$$

$$D_j = \frac{p^{d_j}}{r^{d_j} - K},\ j = 1, ..., t \tag{26}$$

Then, isolating X from equation (20) and from equation (21) and equating the two

expressions found out, after the introduction of equations (23) and (24) we obtain:

$$\left[1-\frac{K}{\mu_d}\left(1+\sum_{j=1}^{t}\frac{p^{d_j}}{r^{d_j}-K}\right)\right]\left[1+\frac{K}{\mu_u}\left(1+\sum_{i=1}^{s}\frac{p^{u_i}}{r^{u_i}+K}\right)\right]=1 \tag{27}$$

which is a polynomial in K of degree $R=s+t+2$.

If we indicate with K_m ($m=1,...,s+t+2$) the m^{th} root of the polynomial, we can find the corresponding values of $U_{i,m}$, $D_{j,m}$ and X_m by means of equations (25), (26) and (20) which can be rewritten in the following way:

$$U_{i,m}=\frac{p^{u_i}}{K_m+r^{u_i}},\ i=1,...,s \tag{28}$$

$$D_{j,m}=\frac{p^{d_j}}{r^{d_j}-K_m},\ j=1,...,t \tag{29}$$

$$X_m=\left[1-\frac{K_m}{\mu_d}\left(1+\sum_{j=1}^{t}\frac{p^{d_j}}{r^{d_j}-K_m}\right)\right] \tag{30}$$

It is important to notice that all the K_m roots of the polynomial are real: indeed, if some roots were complex the steady state probabilities would be either complex, which is impossible, or they would show a periodic pattern not coherent with the behaviour of the real system.

It is simple to demonstrate that one of the roots of (27) is always equal to 0: indeed, if we introduce $K=0$ in the polynomial we obtain the identity $1=1$; moreover, defining this root as K_R and substituting into equations (28), (29) and (30) we obtain:

$$U_{i,R}=\tfrac{p^{u_i}}{r^{u_i}},\ i=1,...,s\quad D_{j,R}=\tfrac{p^{d_j}}{r^{d_j}},\ j=1,...,t\quad X_R=1 \tag{31}$$

Since it is possible to find different values for the parameters X, U_i and D_j such that the guess proposed in (18) satisfies the internal equations, by definition also all the possible linear combinations of these solutions satisfy the internal equations. Therefore, it is possible to write:

$$p\left(n,u_i,d_j\right)=\sum_{r=1}^{R}C_r X_r^n U_{i,r} D_{j,r},\ r=1,...,R,\ i=1,...,s,\ j=1,...,t \tag{32}$$

The next step is to find out if it is possible to set the values of the constant C_r, for $r=1,...,R$, so that also the boundary conditions are satisfied.

8. Analysis of Boundary Equations

The assumptions of the model imply that certain boundary states are transient, that is they have zero steady state probability:

$$p\left(0,1,d_j\right)=p\left(0,u_i,d_j\right)=p\left(N,u_i,d_j\right)=p\left(N,u_i,1\right)=0,\ i=1,...,s,\ j=1,...,t \tag{33}$$

This result can be obtained extending to the case of multiple failures the proof given by Gershwin in [2]. Regarding non-transient boundary states, since it can be shown that the balance equations of boundary states $(0,1,1)$ (16) and $(N,1,1)$ (17) involve only internal states, also the corresponding steady state probabilities can be expressed in internal form, namely:

$$p\left(0,1,1\right)=\sum_{r=1}^{R}C_r \tag{34}$$

$$p\left(N,1,1\right)=\sum_{r=1}^{R}C_{r}X_{r}^{N} \tag{35}$$

Moreover, introducing equations (33) and (34) in (10) and equations (33) and (35) in (14) we obtain the following expressions for the steady state probabilities of boundary states $(0,u_i,1)$ and $(N,1,d_j)$:

$$p\left(0,u_{i},1\right)=\frac{1}{r^{u_{i}}}\sum_{r=1}^{R}C_{r}\left[p^{u_{i}}+\mu_{d}X_{r}U_{i,r}\right],\ i=1,...,s \tag{36}$$

$$p\left(N,1,d_{j}\right)=\frac{1}{r^{d_{j}}}\sum_{r=1}^{R}C_{r}X_{r}^{N-1}\left[X_{r}p^{d_{j}}+\mu_{u}D_{j,r}\right],\ j=1,...,t \tag{37}$$

In this way the steady state probabilities of all the non-transient boundary states can be expressed by means of the parameters of the two machines, of the R different solutions of the internal equations and of the constants C_r, for $r=1,...,R$, which are the only unknowns that need to be evaluated.

To find the values of the constant C_r, for $r=1,...,R$, we take into account all the boundary equations that are not identically satisfied when the expressions (32), (34), (35), (36) and (37) are introduced. In particular, equations (16) and (17) become:

$$\sum_{r=1}^{R}C_{r}\left[\mu_{u}-\mu_{d}X_{r}\left(1+\sum_{i=1}^{s}U_{i,r}\right)\right]=0 \tag{38}$$

$$\sum_{r=1}^{R}C_{r}X_{r}^{N-1}\left[\mu_{d}X_{r}-\mu_{u}\left(1+\sum_{j=1}^{t}D_{j,r}\right)\right]=0 \tag{39}$$

while introducing equation (32) into equations (12) for $n=1$ and into equations (9) for $n=N-1$, we obtain:

$$\sum_{r=1}^{R}C_{r}X_{r}\left[\left(\mu_{u}+\sum_{i=1}^{s}p^{u_{i}}+r^{d_{j}}\right)D_{j,r}-\sum_{i=1}^{s}U_{i,r}D_{j,r}r^{u_{i}}-p^{d_{j}}\right]=0,\ j=1,...,t \tag{40}$$

$$\sum_{r=1}^{R}C_{r}X_{r}^{N-1}\left[\left(\mu_{d}+r^{u_{i}}+\sum_{j=1}^{t}p^{d_{j}}\right)U_{i,r}-\sum_{j=1}^{t}U_{i,r}D_{j,r}r^{d_{j}}-p^{u_{i}}\right]=0,\ i=1,...,s \tag{41}$$

Considering also the normalization equation, i.e. the fact that the sum of the steady state probabilities of all the states must be equal to 1, we obtain:

$$1=\sum_{n=0}^{N}\left[p(n,1,1)+\sum_{i=1}^{s}p\left(n,u_{i},1\right)+\sum_{j=1}^{t}p\left(n,1,d_{j}\right)+\sum_{i=1}^{s}\sum_{j=1}^{t}p\left(n,u_{i},d_{j}\right)\right] \tag{42}$$

Equations (38), (39), (40), (41) and (42) represent a system of $s+t+3$ equations in the $s+t+2$ unknowns C_r, for $r=1,...,R$.

Introducing equation (21) in (40) and simplifying every term that can be simplified, we obtain:

$$\sum_{r=1}^{R}C_{r}D_{j,r}=0,\ j=1,...,t \tag{43}$$

Similarly, introducing equation (20) in (41) and simplifying every term that can be

simplified, we obtain:

$$\sum_{r=1}^{R} C_r X_r^N U_{i,r} = 0,\ i = 1, ..., s \tag{44}$$

Adding up all equations (43) for $j = 1, .., t$ to equation (38) we obtain:

$$\sum_{r=1}^{R} C_r \left[\mu_u \left(1 + \sum_{j=1}^{t} D_{j,r} \right) - \mu_d X_r \left(1 + \sum_{i=1}^{s} U_{i,r} \right) \right] = 0 \tag{45}$$

Similarly, adding up all equations (44) for $i = 1, ..., s$ to equation (39) we obtain:

$$\sum_{r=1}^{R} C_r X_r^{N-1} \left[\mu_d X_r \left(1 + \sum_{i=1}^{s} U_{i,r} \right) - \mu_u \left(1 + \sum_{j=1}^{t} D_{j,r} \right) \right] = 0 \tag{46}$$

It is possible to show that:

$$\mu_u \left(1 + \sum_{j=1}^{t} D_{j,r} \right) - \mu_d X_r \left(1 + \sum_{i=1}^{s} U_{i,r} \right) = 0 \ \forall r \neq R \tag{47}$$

Therefore, equations (45) and (46) are both satisfied by setting $C_R = 0$.

At this point, we have exactly $s + t + 1$ linear conditions given by the equations (42), (43) and (44) in the $s + t + 1$ unknowns C_r, for $r = 1, ..., R - 1$.

9. Algorithm

To obtain the performance measures of the system the following steps must be performed:

1. Numerical evaluation of the roots K_m $(m = 1, ..., R)$ of the polynomial (27). One of these roots is always given by $K_R = 0$.
2. Evaluation of the constants $U_{i,m}$ $(i = 1, ..., s)$, $D_{j,m}$ $(j = 1, ..., t)$ and X_m by means of equations (28), (29) and (30) respectively for each root K_m.
3. Solution of the linear system of $s+t+1$ equations in the unknowns C_r $(r = 1, ..., R - 1)$ formed by equations (44), (43) and (42).
4. Evaluation of the steady state probability of all the states. For the internal states they are given by equation (32) while for the boundary states they can be obtained using equations (34), (35), (36) and (37).
5. Evaluation of the performance measures by means of equations (1), (2), (4) and (5).

10. Numerical Results

The possibility to distinguish among different failures for each machine leads to a better evaluation of the perfomance measures of two machine lines where machines are subject to different kinds of failures. To quantify this improvement, we make a comparison with the method proposed by Gershwin and Berman [3] that considers only one failure for each machine and, therefore, it requires the introduction of an "average" failure for each machine, chosen so that the efficiency in isolation of the machine is preserved. The comparison is done by considering the behaviour of a two machine line in which the upstream machine can fail in two different ways and the downstream machine has only one failure mode. Since the efficiency in isolation of a machine with F failure modes is given by:

$$e = \frac{1}{1 + \sum_{i=1}^{F} \frac{p_i}{r_i}} \tag{48}$$

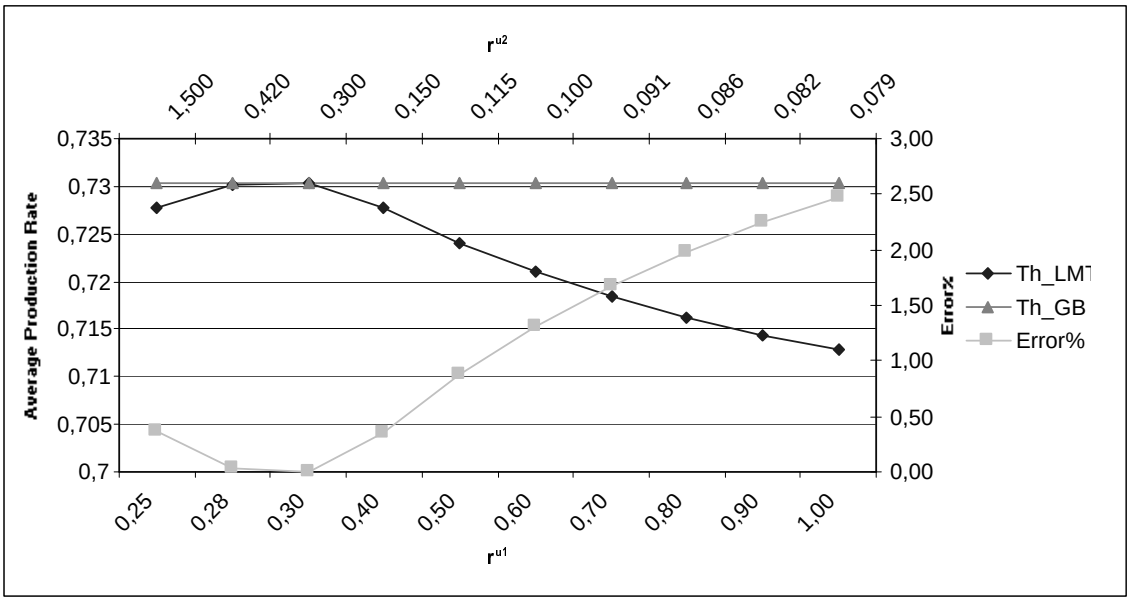

Figure 1: Average Production Rate versus r^{u1}(with $\mu_u = 1.2$, $p^u = 0.1$, $r^u = 0.3$, $\mu_d = 1.0$, $p^d = 0.04$, $r^d = 0.2$, $N = 10$)

the "average" failure mode of the first machine can be evaluated (preserving the efficiency in isolation) by the following equations:

$$\begin{cases} p^u = p^{u_1} + p^{u_2} \\ \frac{1}{r^u} = \frac{p^{u_1}}{p^{u_1}+p^{u_2}} \cdot \frac{1}{r^{u_1}} + \frac{p^{u_2}}{p^{u_1}+p^{u_2}} \cdot \frac{1}{r^{u_2}} \end{cases} \tag{49}$$

In the experiments, we consider various cases; in all the cases the efficiency in isolation and the failure rates of both machines are kept constant while the repair rates of the two failure modes of the upstream machine are varied. The graph of the average production rate versus r^{u1} is shown in Fig.1: as r^{u1} increases, r^{u2} decreases in order to maintain e^u constant. It can be observed that the Gershwin-Berman model tends to overstimate the throughput rate of the system (Th_GB) in comparison with the exact solution given by the model proposed (Th_LMT). Indeed, when $r^{u1} = r^{u2} = r^u$ the two methods give the same results. Also as it can be seen in Fig.2 that the Gershwin-Berman model tends to underestimate the average buffer level of the system.

Other tests have been carried out choosing different values for e^u and e^d which have shown similar results: therefore, we can conclude that the simplification used by the method that considers only one average failure mode can lead to significant errors in the estimate of both the average production rate and of the average buffer level.

11. Conclusions

An analytical method for the perfomance evaluation of two machine lines with random processing times, multiple failure modes and finite buffer capacity has been developed. The method is able to obtain the steady state probability of the Markov chain of the system in a compact form with a computational effort that depends only on the total number of failure modes considered and not on the capacity of the buffer. The improvement in performance estimation given by the possibility to distinguish among different failure modes has been quantified. The model presented can be used as a building block for the devel-

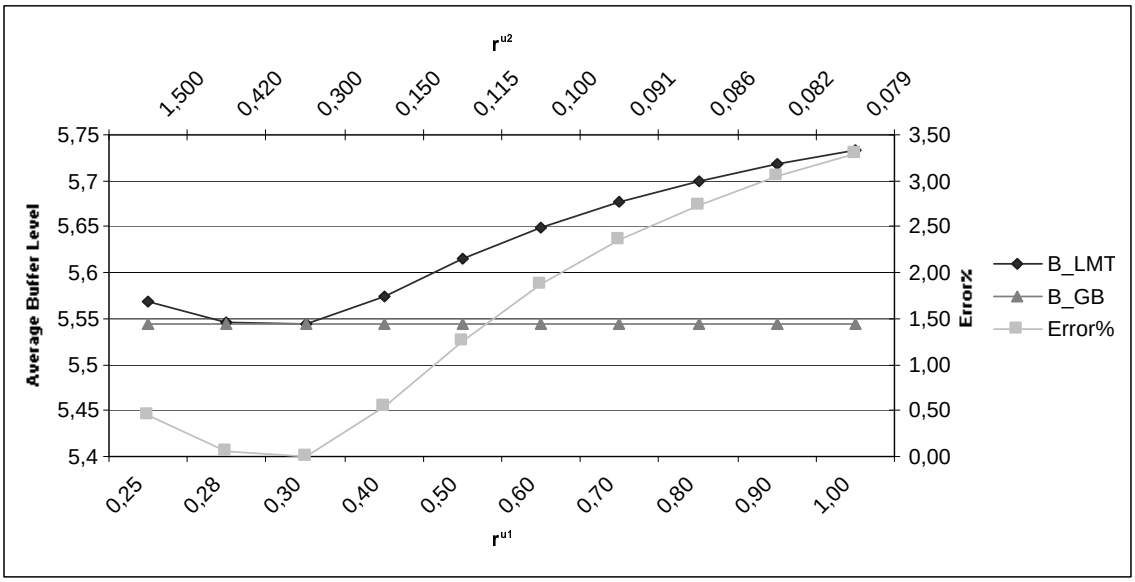

Figure 2: Average Buffer Level versus r^{u1}(with $\mu_u = 1.2$, $p^u = 0.1$, $r^u = 0.3$, $\mu_d = 1.0$, $p^d = 0.04$, $r^d = 0.2$, $N = 10$)

opment of a new decomposition method for the analysis of larger systems, such as long transfer lines and assembly/disassembly networks.

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