Extending the Scope of Analytical Methods for Performance Evaluation of Manufacturing Flow Systems

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Abstract

Manufacturing flow systems consisting of machines separated by buffers of finite capacity have been studied a lot in the literature because systems of this type are widely encountered in industry. The basic assumptions are that the processing time of each machine is deterministic and machines are subject to random failures. Failure and repair times of each machine are characterized by probabilistic distributions. The corresponding model is referred to as the asynchronous model. The basic configuration that has been widely studied is the so-called production lines. Analytical methods for performance evaluation of production lines have been developed for two different models: the synchronous model and the continuous flow model. For the range of parameters usually encountered in practical applications, the behavior of either model is close to that of the asynchronous model, and therefore, these models can be used as approximations of asynchronous models. Moreover, for both models, efficient analytical approximation technique have been developed. Most of them methods are based on the decomposition method originally proposed by Gershwin and further developed and improved by him and other researchers. Such decomposition methods have been extended to other topologies, namely closed-loop production lines and assembly/disassembly systems. However, there are other topologies and/or features that are encountered in real systems for which very little work on analytical methods has been done so far. In this paper, we show that some of these features can easily be incorporated into analytical methods based on the continuous flow model. These features include random yield, split and merge mechanisms, and multi-items assembly systems.

Keywords: Manufacturing flow systems, unreliable machines, finite buffers, performance evaluation, analytical methods, continuous flow model, random yield, split and merge mechanisms, multi-items assembly systems.

1. Introduction

Manufacturing flow systems consisting of machines separated by buffers of finite capacity have been studied a lot in the literature because systems of this type are widely encountered in industry; see (Dallery and Gershwin 1992) and the references therein. The basic assumptions (especially corresponding to the situation of automated manufacturing systems) are that the processing time of each machine is deterministic (i.e., a constant amount of time is necessary to perform the operation on the part) and machines are subject to random failures. Failure and repair times of each machine are characterized by probabilistic distributions. The corresponding model is referred to as the *asynchronous model* in (Dallery and Gershwin 1992). Failures can be classified in two categories: operation-dependent failures (ODFs) and time-dependent failures (TDFs) depending on whether or not a machine may fail only when it is working (Buzacott and Hanifin 1978).

The basic configuration that has been widely studied is the so-called production lines. Production lines consist of a set of machines in series separated by finite capacity buffers. The usual assumption relevant for capacity analysis of production lines is that there are always raw parts available at the input and that there is always room to accommodate the finished parts at the output (i.e., the first machine is never starved and the last machine is never blocked). The behavior of such systems is complex because of the random nature of failures and repairs of the machines and their effect on the behavior of the whole production line due to blocking and starvation phenomena occurring as a consequence of the finiteness of intermediate storage areas. Therefore, performance evaluation methods are of high importance in the design and operation of production lines. Although simulation if the most widely used tool in industry, analytical methods provide an alternative (and complementary) approach for performance evaluation of production lines, and more generally of manufacturing flow systems. An excellent illustration of the usefulness of analytical methods can be found in the case study reported in (Burman et al. 1998).

A number of methods have been developed for analyzing production lines with unreliable machines and finite buffers; refer to (Dallery and Gershwin 1992) for a survey and a list of references as well as to (Buzacott and Shanthikumar 1993). Obtaining exact analytical solutions of asynchronous models of production lines with deterministic processing times is in general not feasible. As a result, different models have been used to approximate the behavior of asynchronous models: the *synchronous model*; see e.g. Buzacott (1967) and Gershwin (1994) and the *continuous flow model*; see e.g. Alvarez et al. (1994) and Suri and Fu (1994). These models provide a good approximation of the original asynchronous model as long as the average times to failure are significantly larger than the processing times, which is usually the case in production systems (Alvarez et al. 1994). The basic assumptions are that the distributions of times to failure and times to repair are geometric in the case of the synchronous model and exponential in the case of the continuous flow model. For both models, exact solutions of a line consisting of two machines separated by a finite buffer can be obtained; see e.g. Buzacott (1967) and Gershwin (1994) in the case of the synchronous model and Gershwin and Schick (1990) and Dubois and Forestier (1982) in the case of the continuous flow model. The major advantage of the continuous flow model over the synchronous model is that it applies to any production line, whereas the synchronous model is restricted to homogeneous lines, i.e., production lines in which all machines have the same processing time.

The analysis of longer lines is based on approximation methods. Among these methods, the *decomposition method* originally proposed by Gershwin (1987) in the context of the synchronous model appears to be the method of choice. Moreover, using the iterative algorithm proposed by Dallery et al. (1988) and referred to as the *DDX algorithm*, this method can be made very efficient and reliable. The decomposition method and the DDX algorithm were then adapted to the continuous flow model of homogenous lines by Dallery et al. (1989). Several extensions of these basic works have then been developed. The first one pertains to non-homogenous lines. Two approaches have been proposed : a direct approach and a two-step approach. The principle of the direct approach is to extend the decomposition method to non-homogenous models. (It only applies to the continuous flow model since there is no synchronous model for non-homogenous lines.) Different variants of the decomposition method have been proposed for the continuous flow model of non-homogenous lines (Alvarez et al. 1991; Burman 1995; Burman and Gershwin 1997). The method proposed by Burman, has the advantage of always converging. Recently, Le Bihan (1998) has shown that the derivation of Burman’s method can significantly be simplified. The alternative approach to the analysis of non-homogenous lines consists of two steps. In the first step, the non-homogeneous line is transformed into an approximately equivalent homogeneous line. The resulting homogeneous line is then analyzed using the decomposition method for homogeneous lines; see Dallery and Le Bihan (1997) and the references therein for more details. Another extension pertains to the improvement of the accuracy of decomposition methods in situations where the original decomposition method may not provide accurate results. Such a situation is encountered when the reliability parameters (mean times to failure and mean times to repair) of the different machines have different orders of magnitude, which can be the case in real production lines. Such improvements of the original decomposition method are based on the replacement of the first-moment approximation of the repair time of the equivalent machines in the decomposition by a two-moment approximation (Dallery and Le Bihan 1999) or a three-moment approximation (Le Bihan and Dallery 1998).

Some work has been devoted to extending decomposition methods to handle manufacturing flow systems more general than production lines. Two important topologies derived from production lines have been studied. Extensions of the decomposition methods to tree-structured assembly/disassembly flow systems have been proposed by Gershwin (1991) in the context of the synchronous model, by Di Mascolo et al. (1991) in the context of the continuous flow model of homogeneous lines, and by Gershwin and Burman (1998) in the context of the continuous flow model of non-homogeneous lines. An extension of the decomposition method has also been made in the context of the continuous flow model of closed-loop production lines by Frein et al. (1996) for homogenous lines and by Patchong (1997) for non-homogenous lines. Although this has not been explicitly done yet, by combining these two extensions, decomposition methods can in principle be extended to general assembly/disassembly systems (general in the sense that they may contain several closed loops).

However, there are other topologies and/or features that are encountered in real systems but for which very little work on analytical methods has been done so far (for more details, refer to Dallery and Gershwin (1992) and Helber (1997 and 1998) and the references therein). This includes random yield, split and merge mechanisms, and multi-items assembly mechanisms. One approach, that has been followed by Helber (1997 and 1998) for instance, would be to extend decomposition methods to handle one or more of these features. In this paper, we present an alternative approach. We show that some of these features can easily be incorporated into analytical methods based on the continuous flow model. The idea is that a continuous flow model of a general assembly/disassembly flow system with one or more of the above-mentioned features can be transformed into an equivalent continuous flow model with none of the features. Moreover, this transformation is an exact equivalence and therefore does not involve any approximation. Such a transformation is of high interest since it allows a model with random yield, split and merge mechanisms, and multi-items assembly mechanisms to be transformed into a regular manufacturing flow model for which analytical methods (mostly decomposition methods) exist. Thus, this approach significantly extends the applicability of analytical methods. For the sake of simplicity, we have chosen to present the different results on specific illustrative examples rather than on general models. However, it is worth noting that the approach proposed in this paper is totally general. Also, we have chosen to present the results of this paper in an informal way and therefore, we do not provide formal proofs of these results although this can easily be done.

Our approach is of interest for two reasons. First it does not require extending existing decomposition methods since our approach transforms a model with non-classical features such as random yield, split and merge mechanisms, and multi-items assembly mechanisms into a basic model for which decomposition methods have been developed. Note also that this is done without introducing any further approximation since our transformation is exact. Secondly, it shows some interesting properties, in the context of models of manufacturing flow systems, on the equivalence between mechanisms that at first glance would appear to be of very different nature, for instance split and merge mechanisms on the one hand and assembly/disassembly on the other hand.

2. Continuous Flow Model of a Production Line

Consider a production line consisting of three machines in series, M_1, M_2 and M_3 and separated by two intermediate buffers B_{12} and B_{23}, as illustrated in Fig. 1. Parts flow from outside the system to Machine M_1, then to Buffer B_{12}, then to Machine M_2, then to Buffer B_{23}, then to Machine M_3, after which they leave the system. We assume that there are always parts available at the input of the system and space available at the output of the system. Buffers B_{12} and B_{23} have finite capacity. We do not make any specific assumption on the blocking mechanism (blocking-after-service or blocking-before-service). The processing time of each machine is deterministic and machines are subject to random failures. Let T_i be the processing time of Machine M_i, i = 1, 2, 3. We do not make any specific assumptions on the failures and repairs of the different machines. Namely, failures can be ODFs or TDFs and the times to failure and times to repair of each machine may have any general distributions. This model is referred to the asynchronous model.

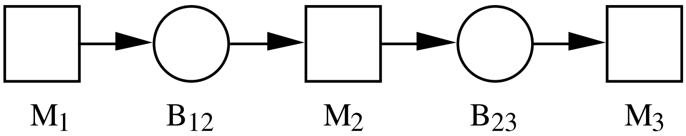

Figure 1: Illustration of a production line

For this basic asynchronous model, the corresponding continuous flow model (CFM) has the following features. Buffers B_{12} and B_{23} have finite capacity. The finite capacity of these buffers are set to some values C_{12} and C_{23}. Although different choices can be considered, the capacity of a buffer of the CFM is usually set equal to that of the corresponding buffer in the asynchronous model if the blocking mechanism is blocking-before-service, and equal to that of the corresponding buffer augmented by one (the space on the machine) if the blocking mechanism is blocking-after-service; see Alvarez et al. (1994), Dallery and Gershwin (1992) and David et al. (1990) for a discussion of this issue. The speed of machine M_i, denoted by U_i, is defined as the inverse of the processing time of the corresponding machine in the asynchronous model, i.e.: $U_i = 1/T_i$, i = 1, 2, 3.

The quantity of material in buffer B_{12} (resp. B_{23}) at any time is a real number $h_{12}(t)$, where $0 \le h_{12}(t) \le C_{12}$ (resp. $h_{23}(t)$, where $0 \le h_{23}(t) \le C_{23}$). Each machine can be in two states: operational (not in a failure condition) or down (under repair). When operational, it can be either working or idle. A machine is idle if it is starved or blocked. For the purpose of illustration, let us consider the behavior of Machine M_2 (the behavior of Machine M_1 and M_3 is similar). Machine M_2 is starved at time t if Machine M_1 is down and buffer B_{12} is empty. Machine M_2 is blocked at time t if Machine M_3 is down and buffer B_{23} is full. When Machine M_2 is operational and neither starved nor blocked, it is working. When Machine M_2 is working and not slowed down by another machine, it transfers material from its upstream buffer, B_{12}, to its downstream buffer, B_{23}, at a constant rate U_2. That is a quantity of material U_2dt is transferred in time dt. However, Machine M_2 can also be slowed down by Machine M_1, if $U_2 > U_1$, or slowed down by Machine M_3, if $U_2 > U_3$. Machine M_2 is slowed down by Machine M_1 at time t if Machine M_1 is working and buffer B_{12} is empty (and $U_2 > U_1$). Similarly, Machine M_2 is slowed down by Machine M_3 at time t if Machine M_3 is working and buffer B_{23} is full (and $U_2 > U_3$). While slowed down, Machine M_2 is still working but transfers material from its upstream buffer, B_{12}, to its downstream buffer, B_{23}, at a reduced rate (either U_1 or U_3).

Let us finally discuss the failure/repair behavior of the CFM. Consider first the case where the failures in the original asynchronous model are TDFs. In this case, the failure/repair mechanism in the CFM is exactly identical to that of the asynchronous model. In particular, the times to failure and times to repair of each machine have the same distributions as those of the corresponding machine in the asynchronous model. This also holds when the failures in the original asynchronous model are ODFs under the condition that the production line is homogeneous, i.e., $T_1 = T_2 = T_3 = T$ for the asynchronous model, or equivalently $U_1 = U_2 = U_3 = U$ for the CFM. On the other hand, the characterization of failures in the case of non-homogenous production lines under the ODF assumption is slightly more involved. This is because machines may be slowed down. The most general way to define the failure mechanism of the machines of the CFM is to transform the original *time distributions* in the asynchronous model into *quantity distributions* for the CFM. That is instead of characterizing the *time to the next failure* by a random variable, the idea is to characterize the *amount of work to the next failure*, expressed in terms of a (continuous) quantity of material, by a random variable; see Suri and Fu (1994) for more details. Note that when the times to failure are exponentially distributed, an equivalent interpretation if to say that the failure rate of the machine is reduced proportionally to the speed reduction; see Alvarez et al. (1994) and Dallery and Gershwin (1992) for more details.

The CFM approximates the discrete flow of parts of the asynchronous model by a continuous flow of material. The continuous model can be interpreted as obtained from the asynchronous model by splitting each part into a set of N identical subparts, each one being processed independently of the others by the machines of the production lines. The processing times of the machines as well of the capacity of the buffers have to be adjusted accordingly. Namely the processing time of each machine is divided by N (since it corresponds to the processing time of a subpart) whereas the capacity of each buffer is multiplied by N (since a buffer being able to store C parts has to be replaced by a buffer being able to store NC subparts). The CFM is the model obtained when this transformation is applied with N going to infinity (after a rescaling of the model pertaining to the buffer sizes and the processing speeds of the machines). As a result, the discrete processing of parts in the asynchronous model is replaced by a continuous processing of material in the CFM. Note that Theoretical (David et al. 1990) and numerical (Alvarez et al. 1994) results have shown that the CFM provides a good approximation of the original asynchronous model as long as the average times to failure are significantly larger than the processing times. This condition is referred by Dallery and Gershwin (1992) as the *multiple time scale* (MTS) assumption.

3. Production Lines with Random Yield

In this section, we are interested in production lines where some of the machines have random yield. Consider first the asynchronous model. The behavior of the model is the same as the one described in Section 2, except for the following feature. Some of the parts produced in the production line have quality problems. We assume that those parts cannot be reworked and are therefore scrapped. We also assume that quality problems occur randomly with the following specific behavior. Each part has a probability to be good and we assume independence of the quality outcome between successive parts. This behavior will be referred to as *random yield*. To be specific, consider again a three-machine production line where the second machine has random yield, as illustrated in Fig. 2. Let q_2 be the random yield probability of Machine M_2. This means that after completion of its operation on Machine M_2, the part is either good (with probability q_2), in which case it is transferred to Buffer B_{23}, or bad (with probability $1-q_2$), in which case it is scrapped and therefore removed from the production line.

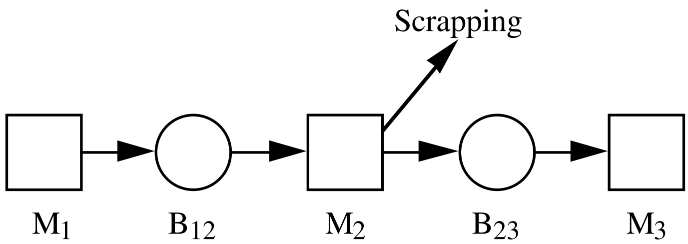

Figure 2: Illustration of a production line with random yield

Let us now define the corresponding CFM. Again, the behavior of the model is the same as the one described in Section 2, except for the random yield feature. In the CFM, the scrapping phenomenon is modeled as follows. Whenever material flows out of Machine M_2, a proportion q_2 of this flow is transferred to Buffer B_{23}, whereas a proportion $1-q_2$ of this flow leaves the system. Consider for instance the situation where Machine M_2 is working and not slowed down. It then processes material at a rate U_2. As a result, it transfers material at a rate q_2U_2 into buffer B_{23} and transfers material at a rate $(1-q_2)U_2$ outside of the production line (this external flow corresponds to the scrapping phenomenon). We refer to this behavior as the *proportional yield* mechanism. Note that in the continuous flow, the routing of material out of Machine M_2 is no longer described in a probabilistic way. This can be explained if we again look at the continuous model as the transformation of an asynchronous model where parts are split into subparts (as described in Section 2). The random yield is now applied at the level of the subparts and as N goes to infinity, things are averaging and as a result, at any instant a proportion q_2 of material is good while a proportion $1-q_2$ is bad and therefore scrapped. Again, the CFM does provide a good approximation of the behavior of the original asynchronous model under the MTS condition.

Let us now turn to our claim that a CFM with random yield can be equivalently transformed into a CFM with no random yield mechanism. Let $\mathcal{M}$ be the CFM with random yield as defined above. Let $\mathcal{M}^e$ denote the equivalent CFM with no random yield mechanism.

Definition 1: Equivalent model to the proportional yield CFM
Model $\mathcal{M}^e$ is a classical CFM with the following characterization:

- Speed of Machine M_1: $U^e_1 = U_1$
- Speed of Machine M_2: $U^e_2 = U_2$
- Speed of Machine M_3: $U^e_3 = U_3/q_2$
- Capacity of Buffer B_{12}: $C^e_{12} = C_{12}$
- Capacity of Buffer B_{23}: $C^e_{23} = C_{23}/q_2$

If failures are TDFs, the characterization of failures and repairs (expressed in term of time distributions) in Model $\mathcal{M}^e$ is identical to that in Model $\mathcal{M}$.

If failures are ODFs, the characterization of repairs (expressed in term of time distributions) in Model $\mathcal{M}^e$ is identical to that in Model $\mathcal{M}$. As for the characterization of failures (expressed in terms of quantity distributions), it has to be adjusted proportionally to the change of the processing speed of the machines. Namely for Machine M_3, the amount of work to the next failure (which is a random variable) is multiplied by $1/q_2$. (If failures are ODFs and times to failure are exponentially distributed, the failure rates are the same in Model $\mathcal{M}^e$ as in Model $\mathcal{M}$.)

♦

Property 1. Model $\mathcal{M}^e$ is equivalent to Model $\mathcal{M}$ in the following sense:

- Failures and repairs of the different machines occur exactly at the same instant in both models.
- The quantity of material in each buffer of Model $\mathcal{M}^e$ is identical (up to a scaling factor) to that in the corresponding buffer of Model $\mathcal{M}$, at any time t, i.e.:

$$h^e_{12}(t) = h_{12}(t)$$
$$h^e_{23}(t) = h_{23}(t)/q_2$$

Proof. Here we give an informal proof of this property. A simple way of understanding this equivalence is the following one. Model $\mathcal{M}^e$ can be seen as a modification of Model $\mathcal{M}$ in which we decide to keep the bad parts in the production line and only get rid of them at the output of the system. To accommodate the bad parts without affecting the behavior of the production line, we need to add some space in Buffer B_{23} since when there is a quantity of material h_{23} (corresponding to good parts) in Buffer B_{23} of Model $\mathcal{M}$, there is a quantity of material h_{23}/q_2 (corresponding to good and bad parts) in Buffer B_{23} of Model $\mathcal{M}^e$. Thus, the capacity of Buffer B_{23} in Model $\mathcal{M}^e$ must be set to $C^e_{23} = C_{23}/q_2$, part of it (namely $q_2C^e_{23} = C_{23}$) being devoted to good parts and the rest (namely $(1-q_2)C^e_{23} = ((1-q_2)/q_2)C_{23}$) being devoted to bad parts. Also, Machine M_3 must now process the bad parts in addition to the good ones and therefore its speed must be increased in proportion, i.e.: $U^e_3 = U_3/q_2$. Part of the capacity (namely $q_2U^e_3 = U_3$) of the machine is devoted to good parts while the rest (namely $(1-q_2)U^e_3 = ((1-q_2)/q_2)U_3$) is devoted to bad parts. It is then easy to see that these modifications do not change the behavior of the CFM and in particular: failures and repairs, blocking and starvation, and slowing down phenomena occurs at the same time in both models. As a result, the behavior of both models is identical (up to a scaling factor for the quantity of material in Buffer B_{23}).

♦

The equivalence between the two models stated in property 1 is a so-called *sample-path equivalence*. It is thus a strong equivalence since it means that the sample-path behavior of the two models is identical (up to a scaling factor). This sample-path equivalence of course implies a probabilistic equivalence of the two models. Thus, all performance parameters are the same in both models (up to a scaling factor for some of them). In particular, let P_i denote the production rate of Machine M_i, let $Q_{i,i+1}$ denote the average amount of material in Buffer $B_{i,i+1}$, and let s_i and b_i de note the starvation and blocking probabilities of Machine M_i. We have:

$$
\begin{array}{ll}
P^e_i = P_i & i = 1, 2 \\
P^e_3 = P_3/q_2 & \\
b^e_i = b_i & i = 1, 2, 3 \\
s^e_i = s_i & i = 1, 2, 3 \\
Q^e_{12} = Q_{12} & \\
Q^e_{23} = Q_{23}/q_2 &
\end{array}
$$

The above equivalence straightforwardy extends the scope of analytical methods in the analysis of manufacturing flow lines. Indeed, any CFM with random yield can be equivalently transformed into a CFM with no random yield. This second model can then be analyzed using the decomposition methods reviewed in Section 1. The performance parameters of the CFM with random yield can then easily be derived from those obtained for the CFM with no random yield.

4. Other Features

The above approach actually applies to a much larger class of manufacturing flow systems. For the purpose of illustration, let us consider some examples. The first example consists of a production line with splitting, as illustrated in Fig. 3. The behavior is the same as in a classical production line except that the flow of parts outside of Machine M_2 is split into two flows. Some of the parts are transferred to Buffer B_{23} while other parts are transferred to Buffer B_{24}. Let q_{23} be the proportion of parts that join Buffer B_{23} and q_{24} be the proportion of parts that join Buffer B_{24} (such that $q_{23} + q_{24} = 1$). In the asynchronous model, this splitting can be done in different ways: either in a probabilistic way or in a deterministic way. In the corresponding CFM, this issue is irrelevant. Indeed, in the CFM, whenever Machine M_2 is working, it transfers a proportion q_{23} (resp. q_{24}) of its output flow to Buffer B_{23} (resp. B_{24}). We refer to this mechanism as *proportional splitting*. Our assumption is that Machine M_2 needs to respect this proportional splitting at any time. As a result, if for instance Machine M_3 is down and Buffer B_{23} is full, then Machine M_2 is blocked and therefore cannot transfer material to Buffer B_{24}, even if this buffer is not full.

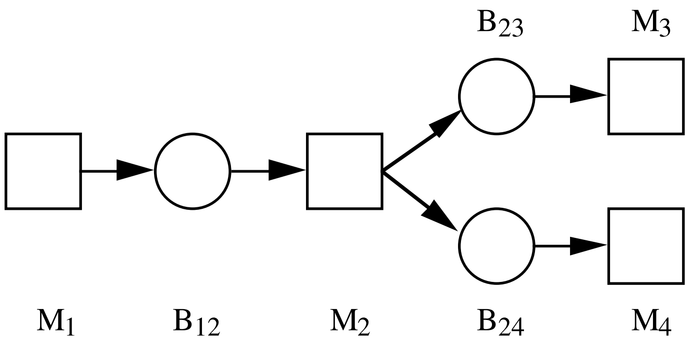

Figure 3: Illustration of a production line with a split mechanism

It turns out that for this CFM with splitting, there is again an equivalence with a CFM with no splitting mechanism. In this case, the equivalent model is a disassembly system with the same structure as that appearing in Fig. 3. The transformation is that the splitting mechanism is replaced by a disassembly mechanism. Thus when Machine M_2 is working at a given rate, say U_2, it transfers material in both buffers B_{23} and B_{24} at the same rate U_2, which is the classical behavior of disassembly CFM. Extending the approach developed for production lines with random yield in Section 3, it appears that the two CFMs will now have identical behavior (up to a scaling factor) if the equivalent disassembly CFM is defined as follows:

Definition 2: Equivalent model to the proportional splitting CFM

Model $\mathcal{M}^e$ is a classical disassembly CFM with the following characterization:

- Speed of Machine M_1: $U^e{}_1 = U_1$
- Speed of Machine M_2: $U^e{}_2 = U_2$
- Speed of Machine M_3: $U^e{}_3 = U_3/q_{23}$
- Speed of Machine M_4: $U^e{}_4 = U_4/q_{24}$
- Capacity of Buffer B_{12}: $C^e{}_{12} = C_{12}$
- Capacity of Buffer B_{23}: $C^e{}_{23} = C_{23}/q_{23}$
- Capacity of Buffer B_{24}: $C^e{}_{24} = C_{23}/q_{24}$

The issue pertaining to failures and repairs is the same as that in Definition 1.

♦

Property 2. Model $\mathcal{M}^e$ is equivalent to Model $\mathcal{M}$ in the following sense:

- Failures and repairs of the different machines occur exactly at the same instant in both models.
- The quantity of material in each buffer of Model $\mathcal{M}^e$ is identical (up to a scaling factor) to that in the corresponding buffer of Model $\mathcal{M}$, at any time t, i.e.:

$$h^e{}_{12}(t) = h_{12}(t)$$
$$h^e{}_{23}(t) = h_{23}(t)/q_{23}$$
$$h^e{}_{24}(t) = h_{24}(t)/q_{24}$$

♦

Again, it follows from Property 2 that the performances parameters of both models are the same (up to a scaling factor for some of them). A similar result holds for the merge mechanism illustrated in Fig. 4. The behavior pertaining to the merge mechanism in the CFM of this production line is as follows. Whenever Machine M_3 is working, it pulls a proportion q_{13} (resp. q_{23}) of its input flow from Buffer B_{13} (resp. B_{23}). We refer to this mechanism as *proportional merging*. The assumption is that Machine M_3 needs to respect this proportional splitting at any time. As a result, if for instance Machine M_1 is down and Buffer B_{13} is empty, then Machine M_3 is starved and therefore cannot pull material from Buffer B_{23}, even if this buffer is not empty. Note that the behavior of the merge mechanism is the exact dual of that of the split mechanism. The CFM with proportional merging is now equivalent to a classical assembly CFM and the behavior and performance parameters of both models are again the same (up to a scaling factor).

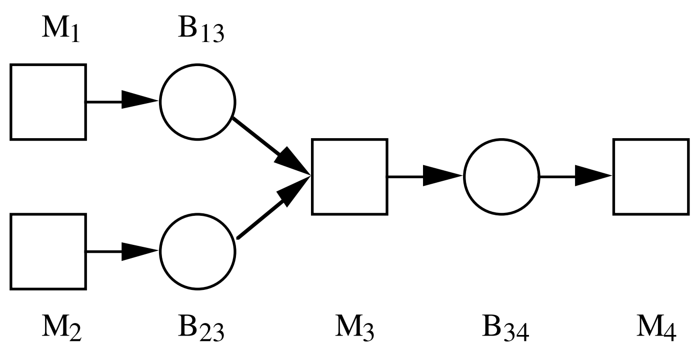

Figure 4: Illustration of a production line with a merge mechanism

Definition 3: Equivalent model to the proportional merging CFM

Model $\mathcal{M}^e$ is a classical assembly CFM with the following characterization:

- Speed of Machine M_1: $U^e{}_1 = U_1/q_{13}$
- Speed of Machine M_2: $U^e{}_2 = U_2/q_{23}$
- Speed of Machine M_3: $U^e{}_3 = U_3$
- Speed of Machine M_4: $U^e{}_4 = U_4$
- Capacity of Buffer B_{13}: $C^e{}_{13} = C_{13}/q_{13}$
- Capacity of Buffer B_{23}: $C^e{}_{23} = C_{23}/q_{23}$
- Capacity of Buffer B_{34}: $C^e{}_{34} = C_{34}$

The issue pertaining to failures and repairs is the same as that in Definition 1.

♦

Property 3. Model $\mathcal{M}^e$ is equivalent to Model $\mathcal{M}$ in the following sense:

- Failures and repairs of the different machines occur exactly at the same instant in both models.
- The quantity of material in each buffer of Model $\mathcal{M}^e$ is identical (up to a scaling factor) to that in the corresponding buffer of Model $\mathcal{M}$, at any time t, i.e.:

$$h^e{}_{13}(t) = h_{13}(t)/q_{13}$$
$$h^e{}_{23}(t) = h_{23}(t)/q_{23}$$
$$h^e{}_{34}(t) = h_{34}(t)$$

♦

Let us finally consider an example of a *multi-item assembly* system, as illustrated in Fig. 5. The behavior is as follows. To perform an assembly operation, Machine M_4 requires q_{14} (resp. q_{24} and q_{34}) units of material from Buffer B_{14} (resp. B_{24} and B_{34}). Once the assembly operation is performed, it transfers one item (one unit of material) into Buffer B_{45}. Note that the classical *single-item assembly* system corresponds to the situation where $q_{14} = q_{24} = q_{34} = 1$. It turns out that in this case, there is an equivalence between the multi-item assembly CFM and a single-item assembly CFM defined as follows.

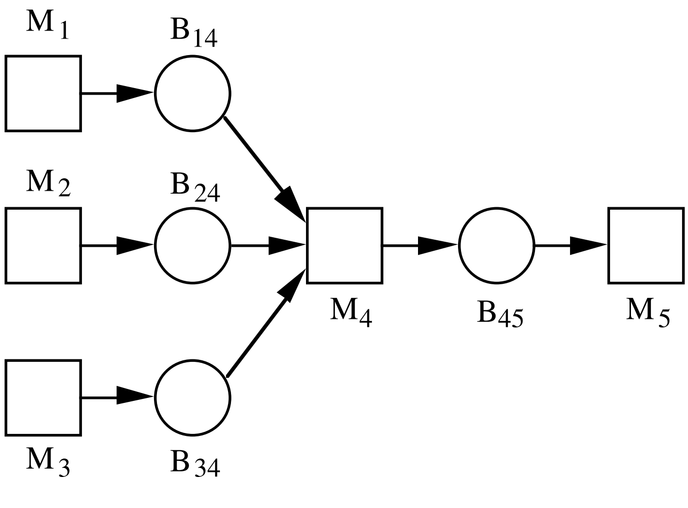

Figure 5: Illustration of a multi-item assembly systems

Definition 4: Equivalent model to the multi-item assembly CFM

Model $\mathcal{M}^e$ is a classical single-item assembly CFM with the following characterization:

- Speed of Machine M_1: $U^e{}_1 = U_1/q_{14}$
- Speed of Machine M_2: $U^e{}_2 = U_2/q_{24}$
- Speed of Machine M_3: $U^e{}_3 = U_3/q_{34}$
- Speed of Machine M_4: $U^e{}_4 = U_4$
- Speed of Machine M_5: $U^e{}_5 = U_5$
- Capacity of Buffer B_{14}: $C^e{}_{14} = C_{14}/q_{14}$
- Capacity of Buffer B_{24}: $C^e{}_{24} = C_{24}/q_{24}$
- Capacity of Buffer B_{34}: $C^e{}_{34} = C_{34}/q_{34}$
- Capacity of Buffer B_{45}: $C^e{}_{45} = C_{45}$

The issue pertaining to failures and repairs is the same as that in Definition 1.

♦

Property 4. Model $\mathcal{M}^e$ is equivalent to Model $\mathcal{M}$ in the following sense:

- Failures and repairs of the different machines occur exactly at the same instant in both models.
- The quantity of material in each buffer of Model $\mathcal{M}^e$ is identical (up to a scaling factor) to that in the corresponding buffer of Model $\mathcal{M}$, at any time t, i.e.:

$$h^e{}_{14}(t) = h_{14}(t)/q_{14}$$
$$h^e{}_{24}(t) = h_{24}(t)/q_{24}$$
$$h^e{}_{34}(t) = h_{34}(t)/q_{34}$$
$$h^e{}_{45}(t) = h_{45}(t)$$

♦

Finally, let us emphasize that the different features can easily be combined. That is, using the approach presented in this paper, it is always possible to transform a CFM with features such as random yield, split and merge mechanisms, and multi-items assembly mechanisms, into a classical CFM. For example, one could consider a manufacturing flow system having the same topology as that in Fig. 5, but combining the merge and assembly mechanism. This can easily be done by changing the values of the parameters q_{14}, q_{24} and q_{34}. Consider for instance a situation such that: $q_{14} = 0.4$, $q_{24} = 0.6$ and $q_{34} = 1$. This would correspond to a situation where Machine M_4 performs an assembly operation between two items; one of these items comes from Buffer B_{34}, while the other comes either from Buffer B_{14} (40% of the time) or from Buffer B_{24} (60% of the time). Again, this CFM can be transformed into an equivalent classical assembly CFM with the same transformation as that given in Definition 4.

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